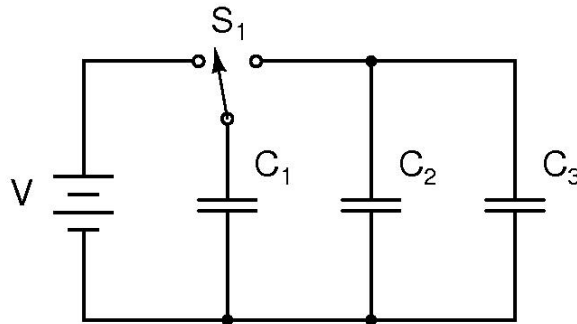


Question 1. (Marks 15)

In the figure below, all capacitors are initially uncharged. Switch S_1 is then thrown to the left until the capacitor C_1 is at equilibrium (i.e., the voltage across it will now equal the battery voltage V). The switch is then thrown to the right until a new equilibrium is reached.



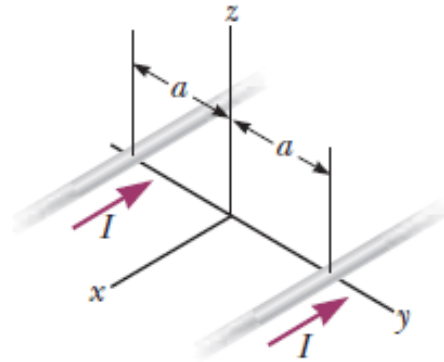
Showing all working,

- Derive an expression for the final voltage on the capacitors, with switch S_1 in the rightmost position. Your answer should be given in terms of V , C_1 , C_2 , and C_3 .
- Derive expressions for the final charge stored in each of the three capacitors. Your answer should be given in terms of V , C_1 , C_2 , and C_3 .
- If switch S_1 is thrown to the left again, and then back to the right, waiting for equilibrium in both cases, is the final voltage on the capacitors now different? If so, what is the new final voltage? Once again, express your answer only in terms of V , C_1 , C_2 , and C_3 .

Question 2. [Marks 15]

In the figure, both currents in the infinitely long wires are 9.45 A in the negative x direction.

The wires are separated by a distance $2a = 7.75$ cm.

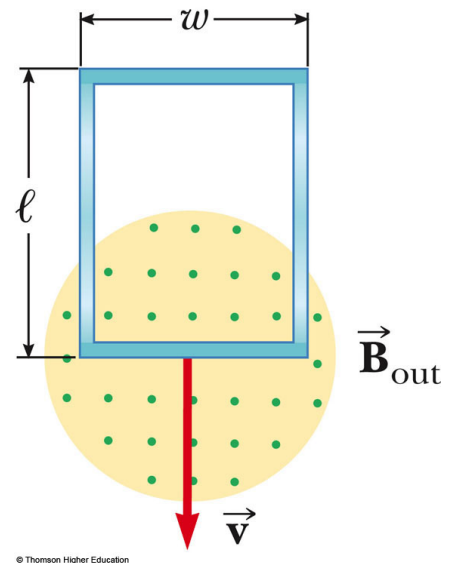


- Sketch the magnetic field pattern in the yz plane.
- What is the value of the magnetic field at the origin?
- Derive an expression, showing all working, for the magnetic field (magnitude *and* direction) at points along the z axis as a function of z .
- At what distance d along the positive z axis is the magnetic field a maximum?
- What is this maximum value?

Question 3. [Marks 15]

The figure shows a rectangular loop of width w , length ℓ , mass M and resistance R falling through a region of uniform magnetic field strength B , directed out of the page. The magnitude of the velocity of the loop at any instant is v . The loop is made from conducting wire.

During the time interval before the top edge of the loop reaches the field, and before any part of the bottom edge of the loop leaves the field,



- State, with reasoning, whether the direction of the induced current I in the loop is clockwise or counterclockwise.
- Derive an expression for the rate of change of magnetic flux through the loop.
- Derive an expression for the power dissipated in the loop.
- The loop will approach a terminal speed v_T . Derive an expression for this speed in terms of the properties of the loop, the magnetic field strength B , and the acceleration due to gravity g .
- How would the numerical value of v_T change if the rectangular loop was replaced by a coil of N rectangular loops of the same dimensions as the original loop, made from the same resistivity wire as used for the original loop, and with the ends of the coil connected together so as to produce a continuous circuit?

Question 4. [Marks 15]

An induction stove uses induction heating to warm a heavy-based metal pan. For the purpose of this question we will model the base of the pan as a flat conducting disc of radius a , thickness b and resistivity ρ . A sinusoidal magnetic field with vertical component $B_z = B_{\max} \cos \omega t$ is applied to the disc. The horizontal components of B are zero. Assume that the resultant eddy currents occur in circles concentric with the disc.

- (a) Draw a diagram showing the pan, the magnetic field at time $t = 0$, and arrows indicating the flow of the eddy currents at that time.
- (b) Repeat part (a), but now for time $t = \pi/(2\omega)$.
- (c) Showing all working, calculate the average power delivered to the disk.

[Hint: the resistance of a ring of radius r , radial thickness dr , resistivity ρ , and height b is $R = (2\pi r \rho)/(b dr)$].

Question 5. (Marks 15)

Electromagnetic waves are described by the wave equation, which for the electric field takes the form:

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

- (a) Based on this equation, state the speed of this wave. What is its numerical value?
- (b) What is a non-trivial solution of the wave equation above assuming $E(x=0, t=0) = E_0$?
- (c) An electromagnetic wave consists of an electric and a magnetic field. Describe such a wave in words and provide a sketch to illustrate your answer.
- (d) If the electric field is directed along x and the magnetic field along y , what is the direction of energy flow?
- (e) A solar sail exploits the fact that electromagnetic waves carry momentum. Suppose such a sail has an area of $9 \times 10^4 \text{ m}^2$ and a mass of 900 kg, and is exposed perpendicularly to sunlight having an intensity of 1400 W/m^2 . Assuming the sail reflects all the light incident on it, calculate the force on the sail and the corresponding acceleration.
- (f) If the sail starts from rest, how long would it take it to cover a distance of 100000 km?

Question 6. (Marks 15)

- (a) State Huygens' principle describing how waves propagate. Make a sketch illustrating this principle for a plane wave.
- (b) Red light from a laser, having a wavelength of 650nm, is propagating inside an optical fibre with a refractive index $n_{\text{fibre}} = 1.49$ for red light. What is the wavelength of the light in the fibre?
- (c) Suppose the red laser light exits the flat-end face of the optical fibre onto air with a refractive index of $n_{\text{air}} = 1$. The beam incident onto the flat-end face of the fibre makes an angle $\theta_1 = 30$ degrees with respect to the normal at the end of the fibre. Sketch the interface between the fibre and air, indicating both the reflected and refracted rays, and calculating both the angles of reflection and refraction.
- (d) What is the numerical value for the critical angle θ_c for the 650nm laser light attempting to exit the fibre?
- (e) What happens to the light at angles of incidence larger than θ_c ? Sketch the path of the light inside the fibre when the angle of incidence is just larger than θ_c .
- (f) Because of dispersion, the index of refraction for blue light is $n_{\text{blue}} = 1.51$. Calculate the critical angle in this case.

Question 7. (Marks 15)

- (a) A monochromatic beam of light of wavelength λ is incident onto a single slit of width a . Sketch the diffraction pattern produced by the light after it has passed through the slit.
- (b) Explain how the interference of light waves gives rise to this pattern.
- (c) The atoms in a solid can be viewed as a diffraction grating. Consider a cubic crystal in which adjacent planes are separated by a distance d , illuminated by X rays of wavelength λ . What are the angles at which maxima will occur?
- (d) MgO has the rock salt lattice structure. You can visualise this as a cube of side a , with 8 atoms at its corners: 4 Mg atoms (molar weight 24 g/mol) and 4 oxygen atoms (molar weight 16 g/mol). The density of MgO is 3.58 g/cm³. What is the lattice constant a ?
- (e) What is the separation between adjacent planes?
- (f) Assume the X-rays have a wavelength $\lambda = 1.5$ Angstroms. What is the angular separation between the maxima? [Note: 1 Angstrom = 10^{-10} m]
- (g) The diffracted X-rays are projected onto a screen 1 m away. What is the separation between adjacent maxima on the screen?

Question 8. (Marks 15)

Consider a quantum mechanical simple harmonic oscillator.

- (a) If the movement is in the x direction, and the oscillator is envisaged as mass m on a spring with elastic constant k , what is its potential energy?
- (b) Using this result, write down the Schrodinger equation for the simple harmonic oscillator.
- (c) One possible wave function is ae^{-bx^2} . Show that this satisfies the Schrodinger equation. Identify the energy and the constant b .
- (d) Another possible wave function is cxe^{-dx^2} . Show that this satisfies the Schrodinger equation. Identify the energy and the constant d .
- (e) Which levels do the wave functions in (c) and (d) correspond to?
- (f) Use this information to deduce the energy spectrum of the simple harmonic oscillator.