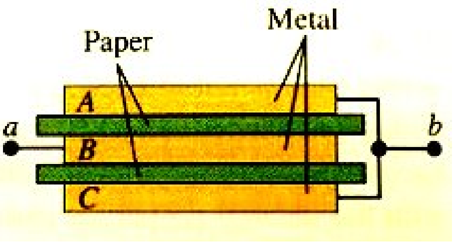
Question 1. (Marks 15)

Three square metal plates A, B, and C, each 15.0 cm on a side and 1.50 mm thick, are arranged as shown in the Figure. The plates are separated by insulating sheets of paper, 0.45 mm thick and with a dielectric constant of 4.2. The two outer plates are connected to point *b*. The inner plate is connected to point *a*.

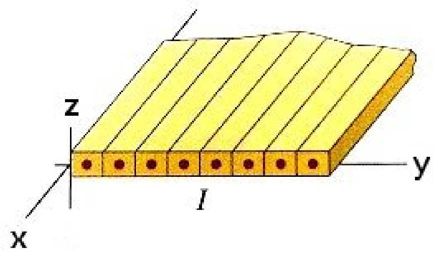


(a) Redraw the Figure to a large scale in your answer book, and show with + and − symbols the charge distribution in the apparatus, assuming that point *a* is maintained at a positive potential with respect to point *b*. Clearly indicate with a written description how the charges are distributed in/on the metal plates (e.g., are they uniformly distributed throughout the metal? Are they on one or more surfaces of the metal?).

(b) Showing all reasoning and working, calculate the capacitance between points *a* and *b*.

Question 2. [Marks 15]

Consider an infinite current sheet in the *xy*-plane composed of long straight conductors, each carrying a current I in the +x direction (see the Figure, which shows just part of the infinite sheet). Assume that the conductors are of square section with side a, and are closely packed side by side.



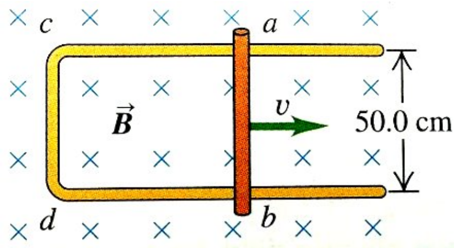
(a) Showing all working, derive an expression for the magnetic field (magnitude and direction) at a distance $d > a$ above the current sheet. Your answer should be given in terms of the unit vectors $\hat{i}, \hat{j}, \hat{k}$.

(b) Repeat part (a), but for a distance $d > a$ below the sheet.

(c) Now remove the conductors and replace them with an infinite insulating sheet with surface charge density σ moving at velocity v in the +x direction. What is the magnitude of the magnetic field at a distance d above the insulating sheet?

Question 3. [Marks 15]

The conducting rod *ab* shown in the Figure makes contact with the metal rails *ca* and *db*. The apparatus is in a uniform magnetic field of 0.800T, perpendicular to the plane of the Figure, as shown.



(a) Find the magnitude of the emf induced in the rod when it is moving towards the right with a speed of 7.50 m/s.

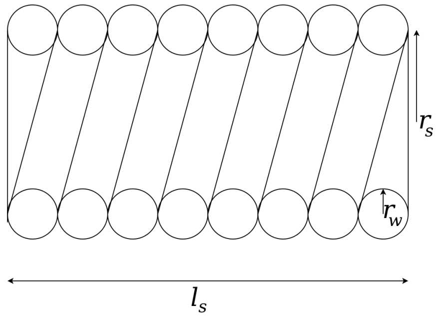
(b) In what direction does current flow in the rod? Give reasons.

(c) If the resistance of the circuit *abdc* is 1.50 Ω (assumed to be constant), find the force (magnitude and direction) required to keep the rod moving to the right with a constant speed of 7.50 m/s. Ignore friction.

(d) Calculate and compare the rate at which mechanical work is done by the force (Fv) with the thermal energy dissipated in the resistance of the circuit (I^2R).

Question 4. [Marks 15]

Suppose that you are given a fixed mass m of copper, with density ρ and electrical conductivity σ. You are also given a battery with emf V. You are asked to use all the copper to make a wire of radius r_w and then to use the wire to make a solenoid of radius r_s. The length of the wire, l_w, and length of the solenoid, l_s, follow from geometrical considerations from the choices you make for r_w and r_s. The solenoid is then placed in series with the battery, causing, at long times, a steady current I to flow.



When answering the following parts of the question, ignore end-effects when calculating the magnetic field in the solenoid, and assume that the solenoid consists of a single layer of wire, closely wound (i.e., neighbouring loops of wire touch each other, see the Figure). You may assume that the solenoid is a perfect inductor with a series resistor (with resistance equal to that of the copper wire). Also assume that $r_w << r_s$ to simplify the geometrical calculations. The battery is a perfect voltage source, with no internal resistance.

(a) Derive an expression for l_w in terms of r_w, m, ρ.

(b) Using the fact that the resistance R of a wire of length l, conductivity σ and cross-sectional area A is

$$R = \frac{l}{\sigma A},$$

derive a formula to give the current I flowing through the solenoid at long times, as a function of V, r_w, m, ρ, σ. [Hint: your expression should involve the fourth power of r_w].

(c) From geometrical considerations derive an expression for l_s in terms of l_w, r_w, r_s,

(d) By combining your results from parts (a) and (c) derive an expression for l_s in terms of r_s, r_w, m, ρ. [Hint: your expression should involve the reciprocal of $r_w r_s$].

(e) Derive an expression for n, the number of turns per unit length of the solenoid, in terms of r_w.

(f) Derive an expression for the magnetic field strength B in the solenoid as a function of V, r_w, m, ρ, σ. [Hint: your expression should involve the third power of r_w].

(g) Finally, express the total amount of magnetic energy as a function of the two choices you can make: r_s and r_w; the properties of the copper: m, ρ, and σ; and the voltage of the battery: V. [Hint: your expression should involve the fifth power of r_w, and the first power of r_s].

Question 5. (Marks 15)

(a) The Sun's radiant energy flux at the top of the Earth's atmosphere is about 1.3 kW m^{-2}. What are the rms values of the corresponding electric and magnetic fields?

(b) A "space yacht" has a sail of mass 10 g per m^2, which is aligned to face the Sun, and absorbs all sunlight falling on it. If the rest of the spacecraft is of negligible mass compared to the sail, what acceleration is achievable in the neighbourhood of the Earth?

Question 6. (Marks 15)

Light from a Helium-Neon laser has a vacuum wavelength of 633 nm.

(a) What is the frequency of this light?

(b) What is the wavelength in glass? (Refractive index of glass, n=1.5)

(c) In a vacuum, the light shines at normal incidence on a diffraction grating with 1000 lines/mm. What is the angle of the first-order maximum?

Question 7. (Marks 15)

(a) In a low-energy light bulb, 10% of the electrical power is converted to visible light (compared to <2% for ordinary incandescent bulbs.) How many photons/second are emitted by a low-energy light bulb consuming 8.0 W of electrical power? (You can assume all the light is at wavelength 550 nm.)

(b) Early black-and-white photographic plates contained crystals of silver bromide, AgBr. Assuming the energy required to sensitise these crystals, so that an image can be developed later, was about 1.8 eV, calculate the cut-off wavelength of light, such that light of longer wavelength was unable to form an image.

Question 8. (Marks 15)

Electrons incident on a certain crystal are refracted because they experience an attractive potential inside the crystal. This changes their kinetic energy, and hence their velocity, inside the crystal.

(a) If the electrons have an energy of 100 eV outside the crystal, and the attractive potential is 15 V, calculate the velocities outside and inside the crystal.

(b) If the electrons are incident at 45^0 to the surface, what is the angle of refraction?