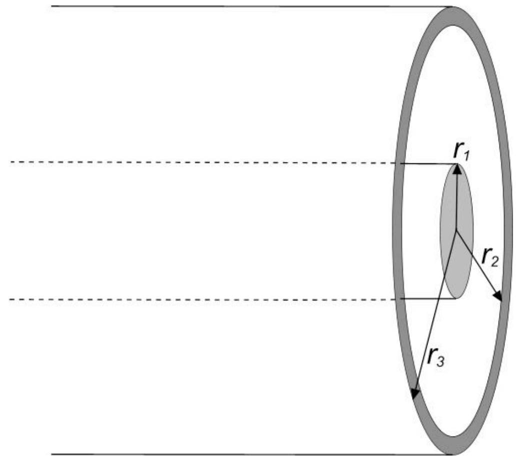
Question 1. (Marks 15)

The figure shows a cross-section through a long, straight, wire with radius r_1 and charge per unit length λ. Co-axial with the wire is a hollow metal cylinder with internal radius r_2 , external radius r_3 , and a net charge per unit length 2λ. Use Gauss's law and appropriate Gaussian surfaces to find:



(a) where the charges are distributed, and the charge per unit length on each surface,

(b) the electric field at a distance $r < r_1$ from the axis,

(c) the electric field at a distance $r_1 < r < r_2$ from the axis,

(d) the electric field at a distance $r2 < r < r_3$ from the axis,

(e) the electric field at a distance $r > r_3$ from the axis, and finally

(f) plot the electric field as a function of radius from the axis, from $r = 0$ to $r > r_3$.

Question 2. [Marks 15]

Suppose you have two 3000 F capacitors (initially uncharged) and a 120 V battery.

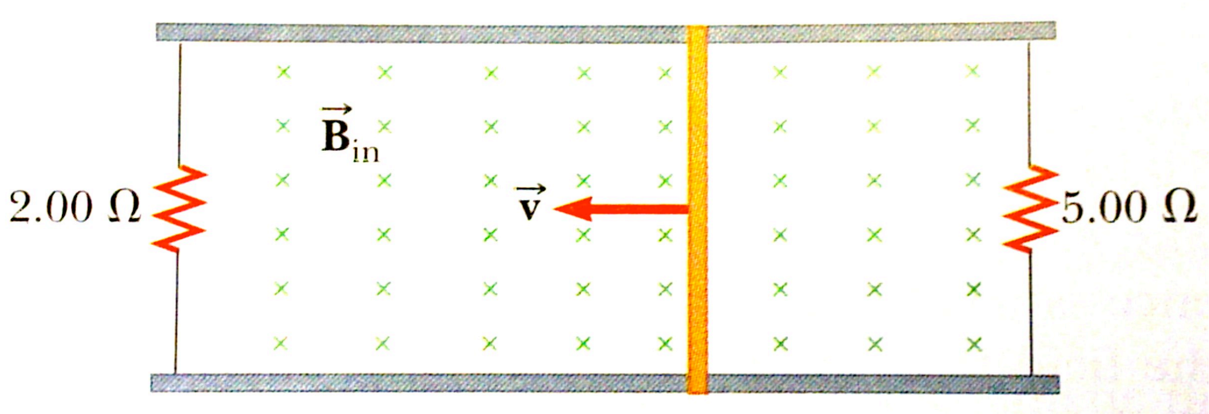
(a) Calculate the total energy stored in both capacitors if they are connected in *series* across the battery. Draw a schematic diagram showing how the capacitors are connected to the battery.

(b) Calculate the total energy stored in both capacitors if they are connected in *parallel* across the battery. Draw a schematic diagram showing how the capacitors are connected to the battery.

(c) Now, charge both capacitors to 120 V, then join their two +ve terminals together, and then connect their free -ve terminals across the battery. Draw a schematic diagram showing how the capacitors are connected to the battery, and the voltages on all the components *just prior to the final connection being made*. After the final connection is made, the capacitors will be in series with the battery, and current will flow until a new equilibrium is reached. Calculate the final voltage across each capacitor, and the total energy stored in both capacitors.

Question 3. [Marks 15]

A conducting rod of length $\ell = 0.50$ m is free to slide on two parallel conducting rails as shown in the figure. Two resistors are connected across the rails as shown. A constant magnetic field $B_{in} = 0.85$ T is directed perpendicularly into the page as shown. The conducting rod moves to the left at a constant velocity v = 4.0 m/s due to the action of an external force.



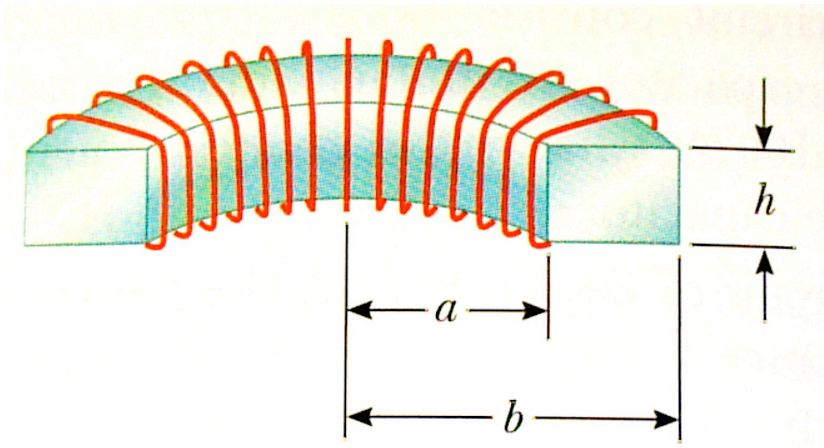
(a) Calculate the current flow in each resistor.

(b) Find the total power dissipated in the two resistors.

(c) Find the magnitude and direction of the applied external force that is required to maintain the velocity of the conducting rod.

Question 4. [Marks 15]

A toroid consisting of N turns of wire, wound with a rectangular cross section as shown in the figure (note: the toroid itself forms a complete circle, the figure is a cutaway view).



(a) By drawing an appropriate amperian loop, use Ampere's Law to derive an expression for the magnetic field B at any point within the toroid (i.e., $a < r < b$ and within the height h, where r is the radial distance from the axis).

(b) Now find the total magnetic flux passing through the toroid (i.e., through the rectangular area $h \times (b - a)$).

(c) From these two results, derive an expression for the inductance of the toroid.

Question 5. (Marks 22)

(a) Consider a laser that emits sinusoidal electromagnetic (EM) waves that travel in the negative x-direction. Suppose that EM waves of wavelength $\lambda = 10{,}600$ nm are emitted from the laser into vacuum with **E** field parallel to the z-axis; the **E** field amplitude is $1.5\text{x}10^6$ Vm^{-1}. Write vector equations for **E** and **B** as a function of time and position. (12 marks)

(b) In a CD ROM drive, light from a semiconductor diode laser having wavelength λ = 780 nm travels a distance 125 nm in a polycarbonate layer. Polycarbonate is a transparent medium of refractive index 1.58. Calculate,

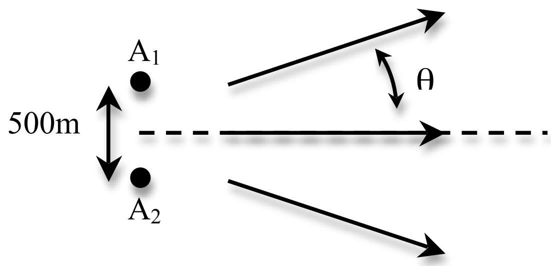
(i) the optical path length (2 marks)

(ii) the wavelength of the light in the transparent medium (3 marks)

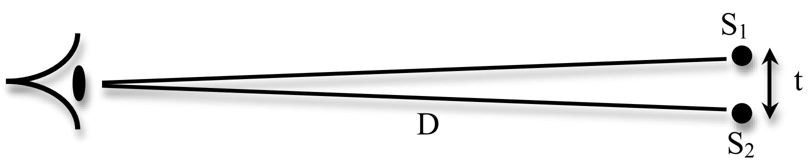
(iii) the phase difference after travelling the distance 125nm with respect to light travelling the same distance in free space. (5 marks)

Question 6. (Marks 18)

(a) A pair of antennas, A_1 and A_2, spaced 500 m apart, broadcast a radio signal at a frequency 1200 kHz. The signals broadcast from the antennas are of equal power and in phase. Calculate the angular directions θ in which the resultant intensity in the radiation pattern is greatest. [Note: θ can range from 0 to 2π radians] (8 marks)



(b) A pair of closely spaced light sources S_1 and S_2 separated by distance t, are viewed by the eye. The sources emit light with wavelength λ=550nm. The eye is D metres from the light sources, as shown schematically below. Assuming diffraction limited resolution, determine the minimum spacing t for which S_1 and S_2 may be clearly resolved as two separate sources if $D = 1200$ m. The diameter of the pupil of the eye may be taken to be 3mm. (10 marks)



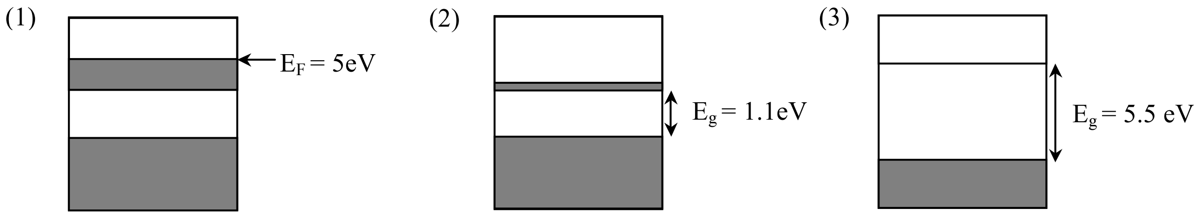
Question 7 (Marks 20)

(a) A sodium atom emits a photon of wavelength 589.0 nm (energy 2.105 eV) in a transition from an excited state to the ground state. The atom remains in the excited state for an average 'lifetime' $\tau = 0.16$ ns before the transition to the ground state. Calculate,

(i) the uncertainty in the energy of the excited state (4 marks),

(ii) the width (i.e. the spread in wavelength) of the line in the observed spectrum associated with this transition. (5 marks)

(b) Three materials have the energy band structures shown schematically in the diagram below representing, (1) a metal, (2) an n-type doped semiconductor and (3) an insulator. The shaded areas indicate occupied (by electrons) energy ranges.



(i) For the metal shown in (1), find the Fermi velocity and the thermal velocity of the electrons at 300K. (4 marks)

(ii) Find the wavelength of EM radiation that will cause a sharp increase in the electrical conductivity of material (2). (2 marks)

(iii) Comment on the expected electrical conductivity of materials (1) and (2) at very low temperatures, as the temperature tends towards 0K, giving your reasons. (5 marks)