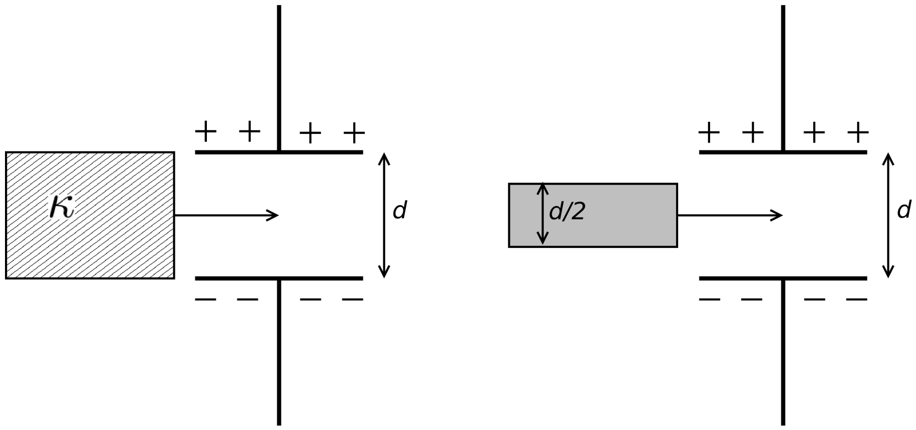
Question 1. (Marks 15)

A parallel-plate capacitor, of capacitance *C* and plate separation *d*, is charged with *Q* columbs using a battery to a voltage *V* . The battery is then disconnected. A dielectric slab of permittivity κ is then slipped between the plates, displacing the air that was previously present, as shown in the Figure below on the left. The energy stored in the capacitor is *U*. The electric field between the plates is *E*.

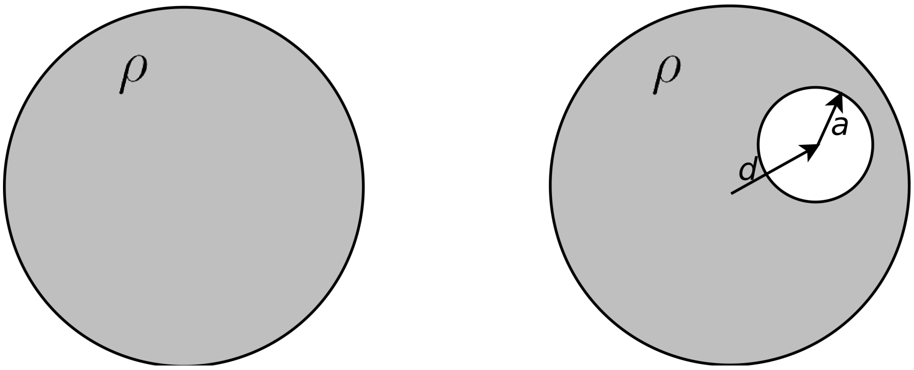


(a) Write down expressions, with reasoning if needed, for the final values (Q', C′, V′, E′, U′) for the charge, capacitance, potential difference, electric field, and stored energy, of the capacitor, once the dielectric slab on the left is fully inserted.

(b) Suppose that a conductive slab (solid grey in the Figure on the right) was used instead of the dielectric slab, and that the slab thickness was one-half of the separation of the capacitor plates. How would your answers to part (a) change?

Question 2. [Marks 15]

Consider a spherical region with a uniform volume charge density ρ, as shown in the Figure on the left (the grey area represents uniform charge density). Showing all your working,

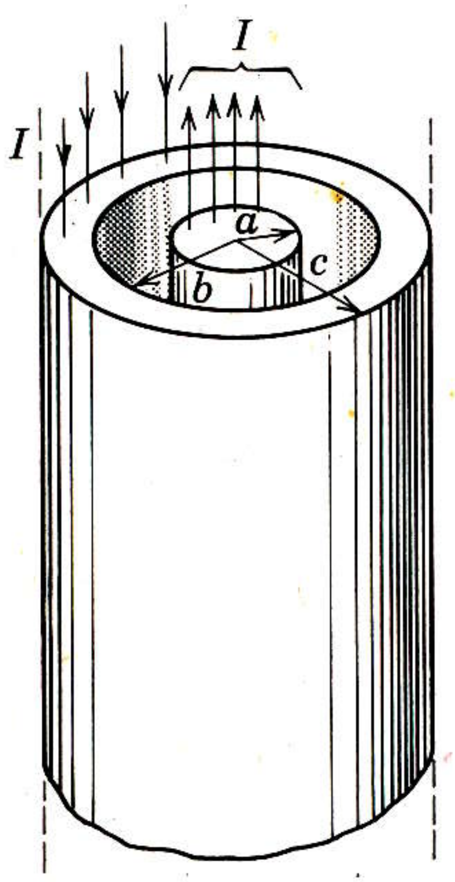


(a) Calculate the electric field as a function of position within the sphere on the left in the Figure.

(b) Now suppose that a spherical cavity of radius a is formed entirely within the original sphere, with a distance d between the centres of the two spheres, as shown in the Figure on the right. Calculate the electric field within the cavity. [Hint: use the principle of superimposition; add a sphere of charge density $-\rho$.]

Question 3. [Marks 15]

A pair of infinitely-long coaxial circular conductors are located in free space as shown in the Figure. They have the dimensions shown. Equal and opposite currents I flow in the conductors. The current density is uniform across each of the conductors.



(a) Derive the magnetic fields (magnitude and direction) for the four regions $r < a$, $a < r < b$, $b < r < c$, and $r > c$, where r is the radial distance from the central line, showing all working and with a clear description of your derivation.

(b) Sketch a graph showing the magnetic field as a function of r over the range $(0, c)$. Your graph should show the values of the magnetic field at the points $r = a$, $r = b$, and $r = c$.

Question 4. [Marks 15]

Solar radiation falling normally on the Earth's surface has a power of 1.37 kW/m^2.

(a) Calculate the maximum magnitude of the electric and magnetic fields in this radiation.

(b) What is the pressure on a surface that completely absorbs this radiation?

Question 5. (Marks 15)

Two transmitters, producing microwaves of wavelength 5.0 cm, with identical polarization, are placed a distance 20 m apart. A receiver can move freely along the line joining them. You may assume the receiver is close to the mid-point, so that the waves are plane-parallel and the amplitudes of the two waves can be taken as equal.

(a) Write an equation for the electric field (maximum value $2E_0$) as a function of position (x) along the line joining the transmitters, and the time (t). How far apart are the maxima of intensity observed in the receiver?

(b) How far do these maxima move if a slab of wax (n=1.5 at this wavelength) of thickness 2.0 cm is placed in front of one of the transmitters?

Question 6. (Marks 15)

(a) Many streetlights use sodium atoms to give (yellow) light of wavelength λ = 588nm, in a very sharp spectral line. However, newer versions use sodium at high pressure, where the line is broadened to $\Delta\lambda$ = 34 nm. Calculate the average time between collisions of the sodium atoms.

(b) Calculate the wavelength of (i) an electron, and (ii) a photon, each of energy 10 eV.