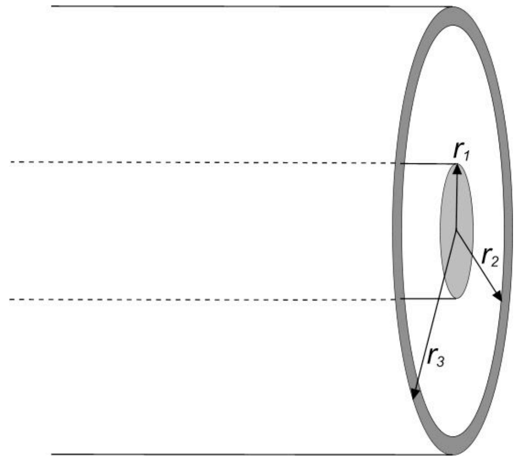
Question 1. (Marks 15)

The figure shows a cross-section through a long, straight wire with radius r_1 and charge per unit length λ. Co-axial with the wire is a hollow metal cylinder with internal radius r_2, external radius r_3, and a net charge per unit length 2λ. Use Gauss's law and appropriate Gaussian surfaces to find:



(a) where the charges are distributed, and the charge per unit length on each surface,

(b) the electric field at a distance $r < r_1$ from the axis,

(c) the electric field at a distance $r_1 < r < r_2$ from the axis,

(d) the electric field at a distance $r2 < r < r_3$ from the axis,

(e) the electric field at a distance $r > r_3$ from the axis, and finally

(f) plot the electric field as a function of radius from the axis, from r = 0 to r > r_3.

Question 2. [Marks 15]

Suppose you have two 3000 F capacitors (initially uncharged) and a 120 V battery.

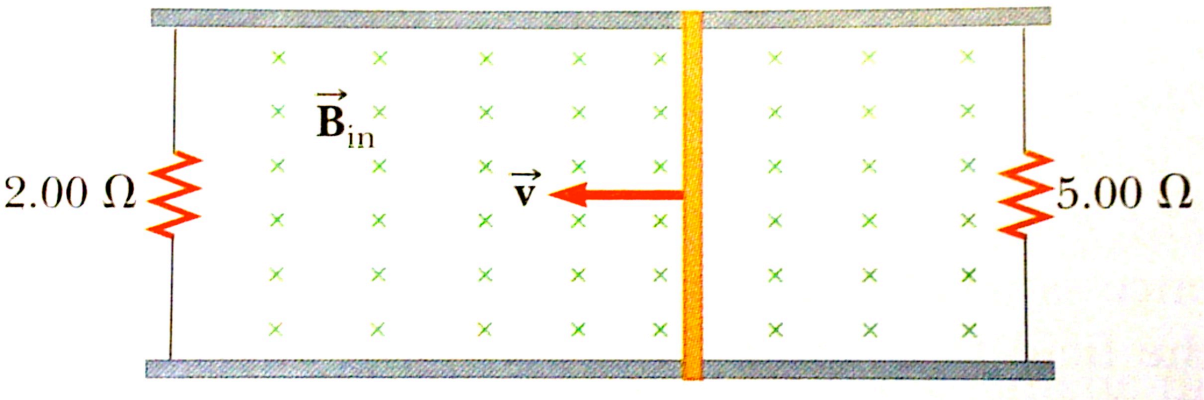
(a) Calculate the total energy stored in both capacitors if they are connected in *series* across the battery. Draw a schematic diagram showing how the capacitors are connected to the battery.

(b) Calculate the total energy stored in both capacitors if they are connected in *parallel* across the battery. Draw a schematic diagram showing how the capacitors are connected to the battery.

(c) Now, charge both capacitors to 120 V, then join their two +ve terminals together, and then connect their free -ve terminals across the battery. Draw a schematic diagram showing how the capacitors are connected to the battery, and the voltages on all the components *just prior to the final connection being made*. After the final connection is made, the capacitors will be in series with the battery, and current will flow until a new equilibrium is reached. Calculate the final voltage across each capacitor, and the total energy stored in both capacitors.

Question 3. (Marks 15)

A conducting rod of length $\ell = 0.50$ m is free to slide on two parallel conducting rails as shown in the figure. Two resistors are connected across the rails as shown. A constant magnetic field $B_{in} = 0.85$ T is directed perpendicularly into the page as shown. The conducting rod moves to the left at a constant velocity v = 4.0 m/s due to the action of an external force.



(a) Calculate the current flow in each resistor.

(b) Find the total power dissipated in the two resistors.

(c) Find the magnitude and direction of the applied external force that is required to maintain the velocity of the conducting rod.

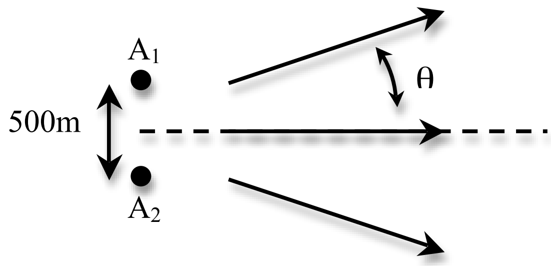
Question 4. (Marks 16)

(a) Consider a laser that emits sinusoidal electromagnetic (EM) waves that travel in the negative x-direction. EM waves of wavelength $\lambda = 10{,}600$ nm are emitted from the laser into a vacuum with the **E** field parallel to the z-axis; the **E** field amplitude is $1.5\text{x}10^6$ Vm^{-1}. Write vector equations for **E** and **B** as a function of time and position. (10 marks)

(b) In a CD ROM drive, light from a semiconductor diode laser having wavelength $\lambda = 780$ nm travels a distance 125 nm in a polycarbonate layer. Polycarbonate is a transparent medium of refractive index 1.58. Calculate,
(i) the optical path length (3 marks)
(ii) the wavelength of the light in the transparent medium (3 marks)

Question 5 (Marks 10)

A pair of antennas, A_1 and A_2, spaced 500 m apart, broadcast a radio signal at a frequency 1200 kHz. The signals broadcast from the antennas are of equal power and in phase. Calculate the angular directions θ in which the resultant intensity in the radiation pattern is greatest.
[Note: θ can range from 0 to 2π radians]
(10 marks)

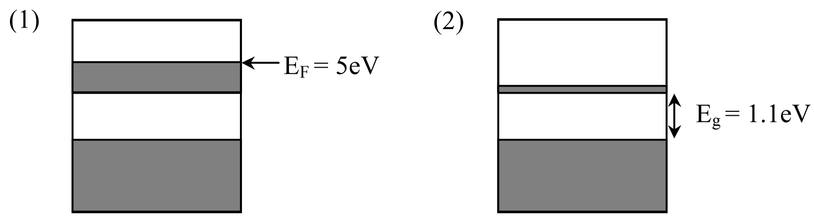


Question 6 (Marks 19)

(a) For an electron confined to a region of width 0.5 nm in the x direction estimate the minimum uncertainty in the x-component of the electron's momentum. (5 marks)

(b) Calculate the de Broglie wavelength of a 25kV electron. State whether relativistic corrections are significant. (Support your statement with a numerical estimate.) (3 marks)

(c) Two materials have the energy band structures shown schematically in the diagram below representing (1) a metal and (2) an n-type doped semiconductor. The shaded areas indicate occupied (by electrons) energy ranges.



(i) For the metal shown in (1), find the Fermi velocity. (3 marks)
(ii) Find the wavelength of EM radiation that will cause a sharp increase in the electrical conductivity of material (2). (4 marks)
(iii) Comment on the expected electrical conductivity of materials (1) and (2) at very low temperatures, as the temperature tends towards 0K, giving your reasons. (4 marks)