Question 1 (Marks 20)

A car travels in a straight line, with a displacement described by the equation

$s = 4t^2 + 1$, where t is the time, in seconds, and s is the car's displacement, in metres.

(a) Find:

(i) The displacement of the car at $t = 0$;

(ii) The instantaneous velocity of the car at any time t;

(iii) The instantaneous acceleration of the car at any time t;

(b) How long does it take the car to accelerate to a speed of 50 km h^{-1}?

(c) What is the car's displacement when it has a speed of 50 km h^{-1}?

(d) After accelerating to 50 km h^{-1}, the car travels with a constant speed around a curve in the road with a radius of 30 m.

(i) Draw a diagram showing the path of the car around the curve and the direction of the velocity and acceleration vectors of the car at any moment, as it is rounding the curve.

(ii) What is the centripetal acceleration of the car as it rounds the curve?

(e) As the car is rounding the curve, a passenger drops an apple core from the car window You may neglect air resistance. Assume that the height from which the core is dropped is 1.4 m, and that the passenger drops the apple core with a velocity of zero with respect to the motion of the car.

(i) With what velocity does the apple core move at the instant it is dropped (relative to the original location of the car)? *(Remember that for a velocity you need to have both a magnitude and a direction.)*

(ii) Assume that the apple core does not collide with the anything until it hits level ground, after falling 1.4 metres. How long does the apple core take to hit the ground?

(iii) How far from the point on the ground where the apple was dropped does it hit the ground?

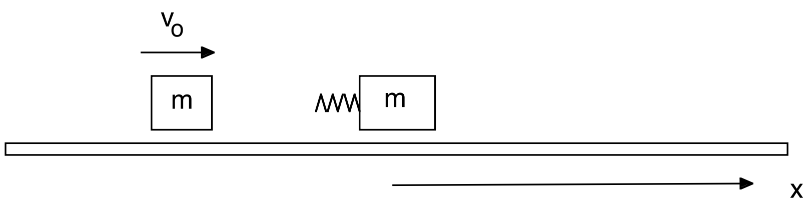
(iv) What is the magnitude and direction of the velocity of the apple core immediately before it hits the ground? Draw a sketch of the x- and y- velocity vectors, together with their resultant vector. Clearly mark on your diagram the angle that the resultant makes with the horizontal.

Question 2 (Marks 20)

(i) Under what conditions is mechanical energy conserved?

(ii) Under what conditions is momentum conserved?

Two small objects with mass m move without friction or air resistance in the x direction. (We may imagine them as sliders on an air track apparatus but, as they are small, they may be treated as particles in this question.) To one of them is attached an ideal spring with spring constant k. Initially, the one on the left travels at v_0 and the other is stationary.



(iii) Explaining your reasoning, derive an expression for the maximum compression of the spring. (Hint: at the instant when the spring is maximally compressed, what is the relative velocity of the two masses?)

(iv) After the collision, the particle on the left has velocity v to the right and that on the right velocity V to the right. Explaining your reasoning and showing all working, derive expressions for V and v.

(v) How many solutions did you get for part (iv)? In no more than three clear sentences, comment on its/ their significance.

(vi) State the definition of the centre of mass of a collection of particles. If your definition is an equation, define the terms.

(vii) Using the definition of centre of mass explicitly, derive an expression for the velocity of the centre of mass of the two mass system for the states before the collision and after the collision.

(viii) Comment on your answer to part (vii).

Question 3 [Marks 20]

Make the (good) approximation that the earth (mass M_e) has a circular orbit round the sun (mass M_s) with a radius r_e and an orbital speed v_e.

(i) State an expression for the acceleration of the earth due to its orbit around the sun in terms of r_e and v_e (remember that acceleration is a vector).

(ii) State an expression for the gravitational force exerted by the sun on the earth (remember that force is a vector) in terms of the symbols defined above, and the gravitational constant G.

(iii) Hence derive an expression for the kinetic energy of the earth in its orbit around the sun in terms of G, M_s, M_e and r_e.

(iv) State an expression for the potential energy of the earth in its orbit around the sun in terms of G, M_s, M_e and r_e.

(v) State an expression for the mechanical energy of the earth in its orbit around the sun in terms of G, M_s, M_e and r_e.

(vi) As you are doing this test, the Voyager spacecraft (launched in 1977) is leaving the solar system. It is travelling with speed v at a distance D from the sun. It has mass m. Assume that it is travelling directly away from the sun. (Repeat: it is *not* in orbit, it is *leaving* the solar system.) The radius of the earth is R_e. Derive an expression for how much higher the mechanical energy of the spacecraft is now than it was just before it blasted off. (You may neglect the rotation of the earth on its axis. You do not have to give a numerical value: just derive an expression.)

(vii) A large fraction of the energy referred to in part (vi) did not come from the engines of the rockets that lifted it off, or from its own (tiny) engines, and none from its nuclear power plant. In one or two clear sentences, explain where it did come from.

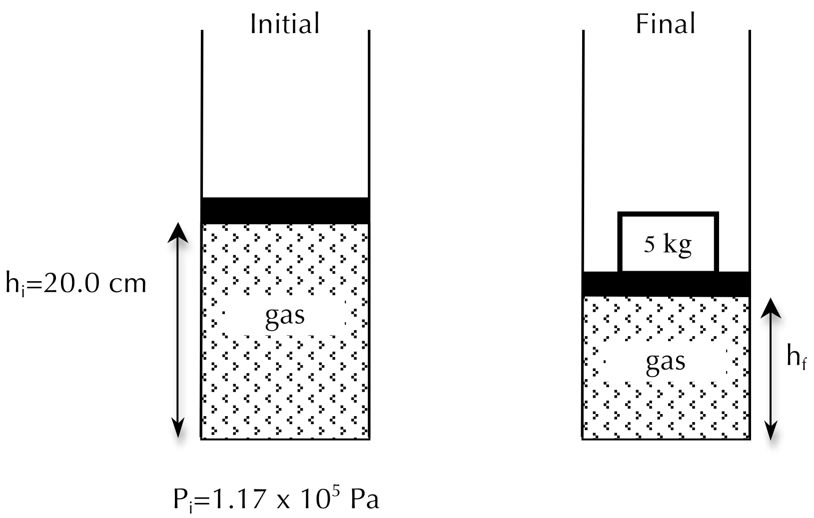
(For interest: $v = 13\ \text{km.s}^{-1}$, $D = 1.8\times10^{10}$ km and m = 720 kg).

Question 4 [Marks 30]

Specific Heats and Thermal conductivities of selected metals

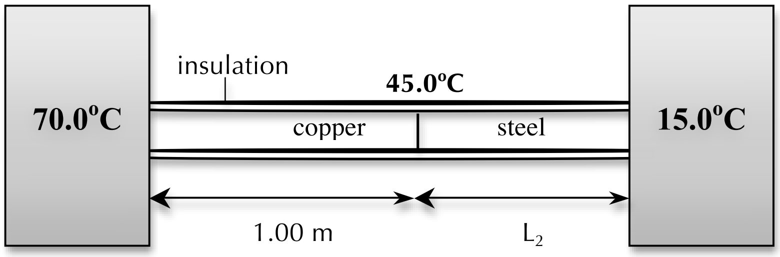
Substance	**Specific Heat** *c*, **($Jkg^{-1}K^{-1}$)**	**Thermal conductivity** *k*, **($Wm^{-1}K^{-1}$)**
Aluminium	910	205.0
Brass	377	109.0
Copper	390	385.0
Lead	130	34.7
Steel	456	50.2

(a) A piston is set up as shown below. The cylinder is filled with an ideal monatomic gas. The surface area of the piston is 12.0 cm². The piston is initially 20.0 cm above the base of the cylinder and the absolute initial pressure, $P_i = 1.17 \times 10^5$ Pa.



(i) How many degrees of freedom does the gas filling the cylinder have?
(ii) What pressure is exerted on the gas when a 5.00 kg block is placed on the piston?
(iii) Assume that the block is placed on very quickly. What will the new height of the piston above the cylinder be?
(iv) The piston is left for long enough for it to return to thermal equilibrium with its surroundings, what is the height of the piston above the cylinder?

(b) A long rod, insulated to prevent heat loss along its sides, has one end maintained at 70.0° C and the other end at 15.0° C. The rod consists of a 1.00 m section of copper (with one end at 70.0° C) joined end to end to a length L_2 of steel. Both sections of the rod have cross sectional areas of 10.00 cm². The temperature of the copper-steel junction is 45.0°C after a steady state has been set up.

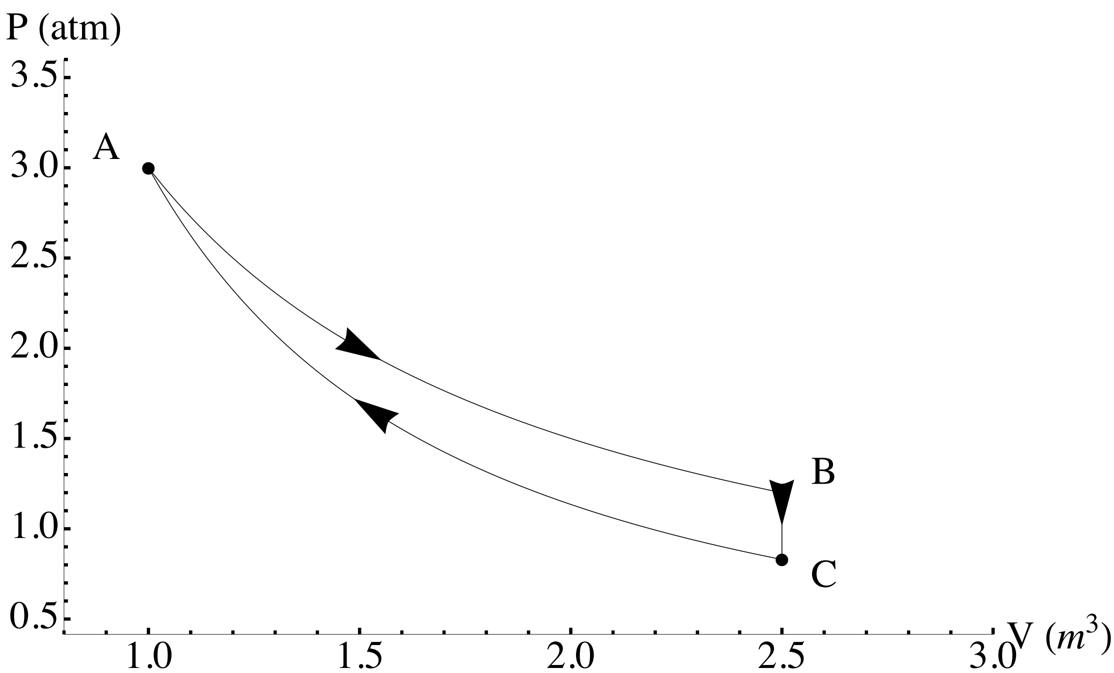


The diagram is not to scale.

(i) What is the rate of heat flow from the hot end of the rod to the cold end?

(ii) What is the length L_2 of the steel section?

(c) The figure shows a thermodynamic process followed by an ideal diatomic gas. Process A → B is isothermal with 278 kJ of heat energy entering the system, process B → C is isovolumetric with 233 kJ of heat energy leaving the system and process C → A is adiabatic. Point A has V = 1.0 m^3 and P = 3.0 atm, point B has V = 2.5 m^3 and P = 1.2 atm and point C has V = 2.5 m^3 and P = 0.83 atm.



(i) What is the change in internal energy as the gas goes from A to B?

(ii) What is the change in internal energy as the gas goes from B to C?

(iii) How much work is done on the gas as it goes from C to A?

Question 5 [Marks 30]

(a) A 5.00 kg block is attached to a spring with spring constant k = 125 N/m. The block is pulled from its equilibrium position at x = 0 to a position at x = +0.687 m and released from rest. The block then executes simple harmonic motion along the x-axis. Showing all your reasoning, calculate:

(i) The magnitude of the acceleration of the block upon release at x = +0.687 m

(ii) The period of the oscillations

(iii) The work done by the spring force between x = 0.687 m and x = 0

(iv) Sketch a displacement time graph for two oscillations showing how the position of the block varies with time. Include numbers on your axes.

(v) On the same graph sketch what would happen if a small damping force, proportional to the velocity, was applied to the spring.

(b) Two loudspeakers S1 and S2, placed 5.0 m apart, are driven in phase by an audio oscillator. A boy stands at point P, which is 12.0 m from S1 and 13.0 m from S2. A right angle triangle is formed by P, S1 and S2. The wave from S2 arrives at point P two periods later than the wave from S1. The speed of sound is 350 m/s.

(i) Calculate the frequency of the oscillator

(ii) The boy walks away from S1 along the S1-P line, until destructive interference occurs. At that point the wave from S2 arrives 1.5 periods later than the wave from S1. Calculate the new distance of the boy from S2.

(c) A man is travelling on a bicycle at 10.0 m/s along a straight road that runs close to and parallel to railroad tracks. He hears the whistle of a train that is behind him. The frequency of the train whistle is 600 Hz, but the frequency the man hears is 558 Hz. Take velocity of sound to be 340 m/s.

(i) What frequency is heard by a stationary observer, located between the train and the man on the bicycle?

(ii) What is the speed of the train and is it travelling towards or away from the man on the bicycle?

(iii) If the sound of the whistle was reflected by a stationary wall in front of the man on the bicycle, what will be the beat frequency of waves reaching the cyclist directly from the train and reflected from the wall. Show your working.