Question 1 (Marks 18)

(a) The vector position of a particle varies in time according to the expression $\mathbf{r} = 3.00\,\hat{\mathbf{i}} - 6.00t^2\hat{\mathbf{j}}$.

Find:

(i) Expressions for the velocity and acceleration as functions of time.

(ii) Determine the particle's position and velocity at $t = 1.00$ s.

(b) A particle initially located at the origin has an acceleration of $\mathbf{a} = 3.00\,\hat{\mathbf{j}}$ m.s^{-2} and an initial velocity of $\mathbf{v}_i = 5.00\,\hat{\mathbf{i}}$ m.s^{-1}.

Find:

(i) The vector position and velocity at any time t and

(ii) The coordinates and speed of the particle at $t = 2.00$ s.

(c) An automobile whose speed is increasing at a rate of 0.600 m.s^{-2} travels along a circular road of radius 20.0 m. When the instantaneous speed of the automobile is 4.00 m.s^{-1}, find:

(i) The tangential acceleration component,

(ii) The centripetal acceleration component, and

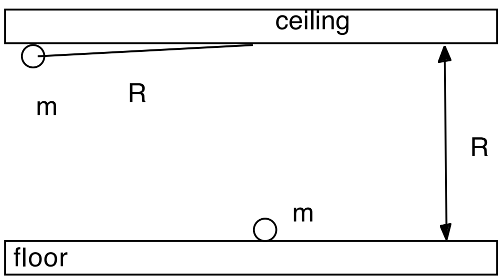
(iii) The magnitude and direction of the total acceleration.

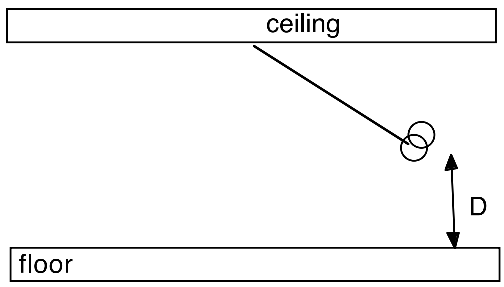
Question 2 (Marks 24)

(i) State clearly the conditions under which mechanical energy is conserved.

(ii) State clearly the conditions under which momentum is conserved.

(iii) One end of a light, inextensible string, length *R*, is attached rigidly to a point in the ceiling. The other is attached to a mass *m*, whose size is very much less than *R*, so you may treat it as a particle. Initially, the string is extended horizontally and straight so that the mass touches the ceiling, as shown in the sketch at left (not to scale). The floor is distance *R* below the ceiling. Directly below the point of attachment in the ceiling lies another small mass *m*.



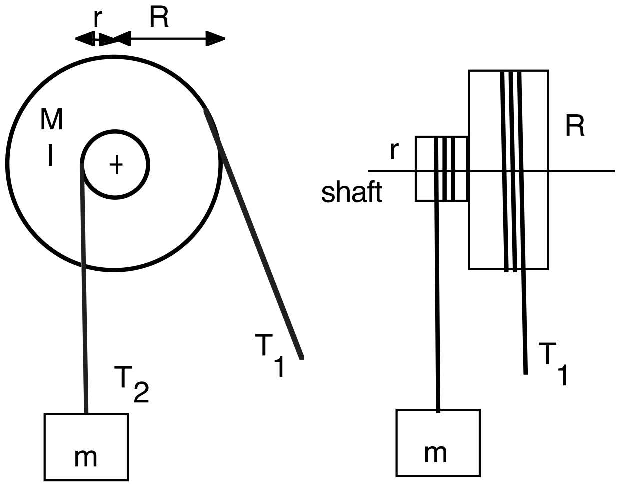


The mass on the string is released. It travels in an arc, does not quite touch the floor, but strikes the second mass. Air resistance is negligible. The mass that swung down from the ceiling is made of sticky clay and, during and after the brief collision, the masses stick together. Together, they swing upwards in an arc and rise to a height *D* above the floor.

Showing all working, derive an expression for *D*. In your derivation, state clearly any approximations you make and any principles you use.

(iv) Referring to part (iii) above: What is the change in the mechanical energy from when the first mass is released until the two masses are at height *D*?
In no more than two clear sentences, comment on your result.

Question 3 [Marks 18]



The sketch shows, from two views, a simple pulley system for raising large masses. A light, inextensible cord attached to a mass *m* is wound, without slipping, on a cylinder of radius *r*. Rigidly joined to this cylinder and mounted on the same central shaft is a cylinder of radius *R*. The friction between the cylinders and the shaft is negligible. The two cylinders together have mass *M*, moment of inertia, *I* and radius of gyration *k*. Around the larger cylinder is wound, without slipping, anther light, inextensible cord, as shown. An operator pulls with tension T_1 on this cord to raise the mass *m*.

The mass *m* accelerates upwards with vertical acceleration *a*.

(i) Using Newton's laws or otherwise, write an expression for T_2, the tension in the cord attached to mass *m*, in terms of *m, g* and *a*.

(ii) Using Newton's laws for rotation or otherwise, derive an expression for the angular acceleration, α, of the cylinders, in terms of variables given in the diagram.

(iii) Showing all working, derive an expression for the acceleration *a* of mass *m* in terms of some or all of the parameters T_1, *m, M, I, r, R* and the gravitational acceleration *g*.

Question 4 [Marks 30]

In this section make use of the data provided in these tables.

Specific Heats and Thermal conductivities of selected metals

Substance	Specific Heat c, (J kg^{-1} K^{-1})	Thermal conductivity k, (W m^{-1} K^{-1})
Aluminium	910	205.0
Brass	377	109.0
Copper	390	385.0
Lead	130	34.7
Steel	456	50.2

Water

Quantity	**Value**
Specific Heat (liquid)	4186 Jkg^{-1}K^{-1}
Latent heat of Fusion	3.33×10^5 Jkg^{-1}
Latent heat of vapourization	2.26×10^6 Jkg^{-1}
Density (at 4^o C)	1000 kgm^{-3} ≡ 1.00 kg/L
Melting point (at 1 atm)	0.000 °C
Boiling point (at 1 atm)	100.0 °C

(a) A copper calorimeter can with mass 0.100 kg contains 160 mL of water and 0.00180 kg of ice in thermal equilibrium at atmospheric pressure. 0.750 kg of lead at a temperature of 255 ˚C is dropped into the calorimeter can. Assume that no heat is lost to the surroundings.

(i) What is the initial temperature of the copper calorimeter?

(ii) How much energy is needed to melt the ice?

(iii) What is the final temperature of the system?

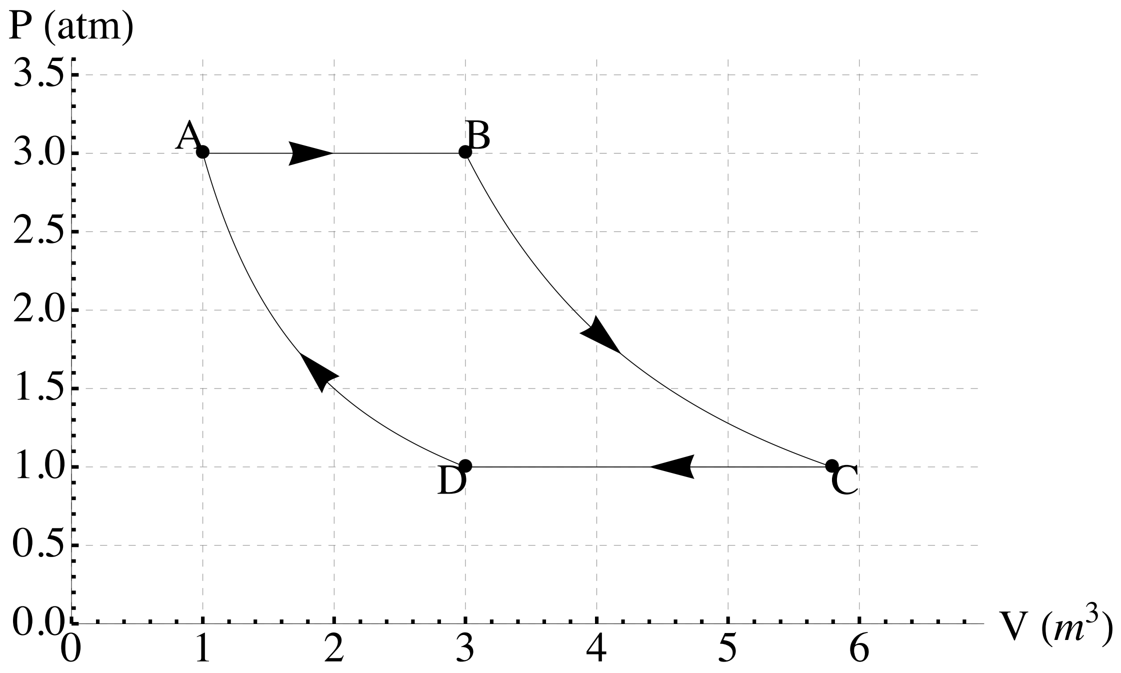
(b) 3.00 kg of Oxygen gas, O_2, with a molar mass of 32.0 g/mol, is pumped into an empty container. The container is shaped like a cube with each side having an area 1.00 m^2.

(i) The gas in the container reaches thermal equilibrium at -70.0^0 C. What is the pressure of the gas inside the container?

(ii) What is the root mean squared (rms) velocity of the oxygen molecules in the gas?

(iii) What is the total internal energy stored within five moles of gas molecules?

(c) The diagram below shows a monatomic ideal gas undergoing a cyclic process. The states A, B, C and D are marked on the diagram. As the gas goes from state A to state B 400.0 kJ of heat energy enter the system. As the gas goes from state C to state D 407 kJ of heat energy enter the system. The process from B to C is adiabatic and from D to A is isothermal. The volume at C is 5.79 m^3.



(i) Calculate the work done on the gas as it goes from state A to state B.

(ii) What is the change in internal energy as the gas goes from state D to state A?

(iii) What is the change in internal energy as the gas goes from state B to state C?

(iv) Write down an equation relating P and V as the system goes from state B to state C. You should calculate as many of the constants in the equation as possible, giving an explanation of how you have calculated each one.

Question 5 [Marks 30]

(a) (i) A car of mass M=1.5x10^3 kg is constructed so that its frame is supported by four springs. Each spring has a force constant k=8.0x10^3 N/m. There are two people in the car, each of mass m=80kg. The car drives over a pothole in the road. Assuming that the car then oscillates with simple harmonic motion in the vertical direction then, showing your working, determine its frequency of oscillation after striking the pothole.

(ii) What would be the frequency of oscillation if there were no people in the car?

(iii) Sketch the form of the subsequent vertical displacement and velocity, as a function of time, for the passengers in the car if the springs were lightly damped. Show two complete cycles of the motion and make clear the relationship between the velocity and the displacement in this motion.

(b) Two sinusoidal waves with angular frequency ω, wave number k and amplitude A are travelling in opposite directions along the x-axis.

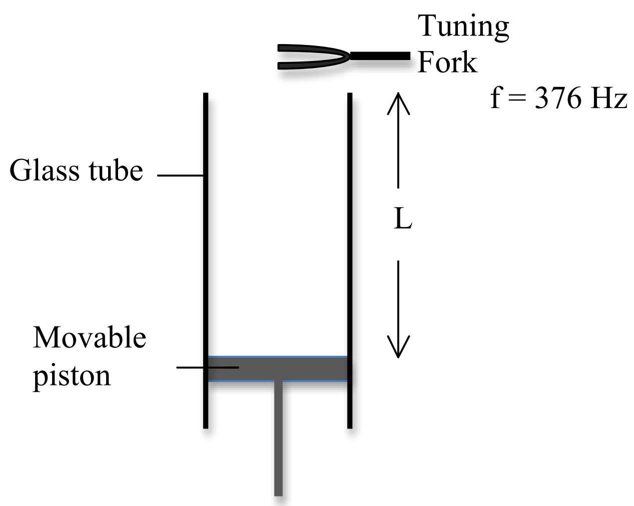
(i) Write down equations which describe the displacement of particles in the two waves, making clear the interpretation of the direction of travel of the wave.

(ii) Derive an expression for the resulting standing wave.

(iii) Explain why it is a standing wave.

(iv) Sketch its shape, including showing its variation with time.

(c) An air column in a glass tube is open at one end and closed at the other by a movable piston. The air in the tube is warmed above room temperature and a 376 Hz tuning fork is held at the open end. Resonance is heard when the piston is 10.2 cm from the open end and again when it is 56.2 cm from the open end.



(i) What speed of sound is implied by these data?

(ii) How far from the open end will the piston be when the next resonance is heard?

(iii) Suppose now that the cylinder was closed at each end. How far would the piston need to be moved from the position calculated in part (ii) in order to be placed at the closest resonance position in this case, for the same frequency and sound speed?