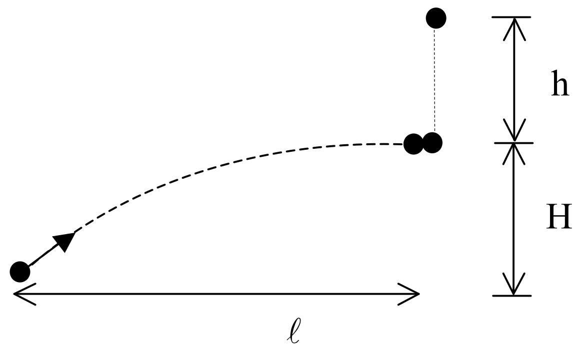
Question 1. [20 marks]

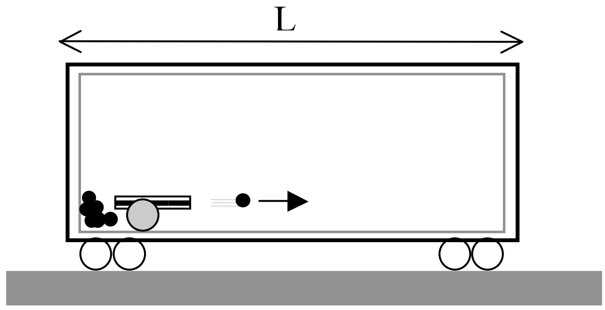
(a) A cannon-ball is fired from ground-level at the same instant a target is released from a height $h + H = 86.0$m above the ground. They collide when the cannon-ball has travelled a horizontal distance $l = 125.0$m and the target has fallen a distance $h = 4.0$m.



(i) Find the initial speed of the cannon-ball.

(ii) At what angle from the horizontal was the cannon fired?

(b) A cannon and supply of cannon-balls are inside a sealed railroad car of length L. The cannon (fixed to the car on the far left) fires to the right, ejecting a cannon-ball at speed v, and the car recoils to the left. The cannon-balls are lodged in the right wall on impact. (Assume negligible friction between the car and rails.)



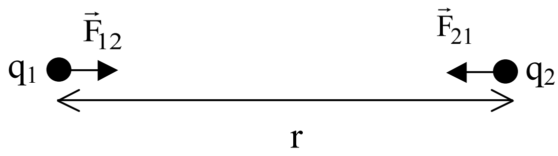
(i) What is the largest distance the car could travel after all cannon-balls have been fired?

(ii) What is the speed of the car after all cannon-balls have reached the right wall?

(iii) How do the answers to (b) (i) and (b) (ii) change when friction between the car and rails is taken into account? One or two clear sentences will suffice.

Question 2. [30 marks]

Two particles of opposite charges with magnitudes q_1 and q_2 are bound by the electrostatic force



$$|F| = k\frac{q_1 q_2}{r^2},$$

where k is a constant and r is the separation between the particles. The particles have masses m_1 and m_2 and their velocities are denoted by $\vec{v}_1$ and $\vec{v}_2$, respectively. [In the following, neglect all other forces.]

(a) Sketch the trajectories of the two particles in the centre of mass reference frame for the case when the mass of particle 1, m_1, is twice the mass of particle 2, m_2. Indicate the position of the particles at one instant.

(b) Show that the expression

$$K = \tfrac{1}{2}\mu v^2 + \tfrac{1}{2}(m_1 + m_2)v_{CM}^2$$

gives the kinetic energy of the two-particle system. Here $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$ is the “reduced mass”, $\vec{v} = \vec{v}_1 - \vec{v}_2$ is the relative velocity, and $\vec{v}_{CM}$ is the centre of mass velocity.

(c) What is the meaning of the two terms in the expression in (b)? A sentence or two will suffice.

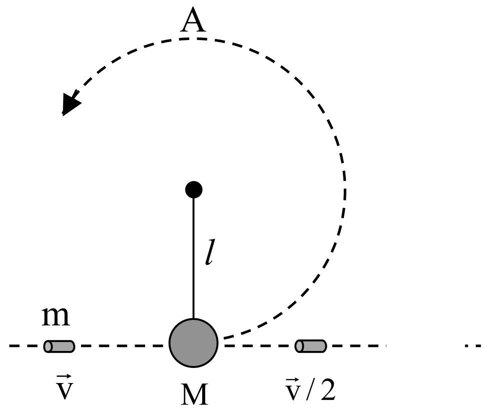
(d) Derive the potential energy U of the system of two charges and sketch its form. Choose $U = 0$ at $r = \infty$.

(e) What minimum energy is required to separate the particles to infinity?

(f) Derive a relation between the kinetic energy K and potential energy U for the two-particle system in the centre of mass reference frame for the case when the particles move in circular orbits.

Question 3. [25 marks]

A bullet of mass m and speed v is fired into a pendulum bob of mass M . The bullet passes through the bob, emerging with speed $v/2$. The pendulum bob is suspended by a massless wire of length l.



(a) What is the speed of the bob immediately after the collision?

(b) How much mechanical energy is lost in the collision?

(c) What is the speed of the bob at the highest point in its motion (at point A in the figure)?

(d) Draw a free-body diagram for the bob at the highest point, clearly identifying the forces acting.

(e) What is the minimum speed v in order for the pendulum bob to just swing a complete circle?

Now consider that after firing the bullet with speed v, it becomes lodged in the pendulum bob.

(f) What is the minimum speed v in order for the pendulum bob to just swing a complete circle?

(g) Explain why the results for (e) and (f) differ. A sentence or two will suffice.

Question 4. [25 marks]

(a) Soon after the Earth formed, heat released by the decay of radioactive elements raised the average internal temperature from 300 to 3000 K, at about which value it remains today. Assuming an average coefficient of volume expansion of $3.2\times10^{-5}\,\mathrm{K}^{-1}$, by how much has the radius of the Earth increased since its formation?

(b) Estimate the mass of the Earth's atmosphere. Express your estimate in terms of the mass of the Earth. Recall that atmospheric pressure equals 101 kPa.

(c) Gas within a chamber undergoes the processes shown in the PV diagram in the figure. Calculate the net heat added to the system during one complete cycle.

[1 MPa = 10^6 Pa; 1L = 10^3 cm^3]