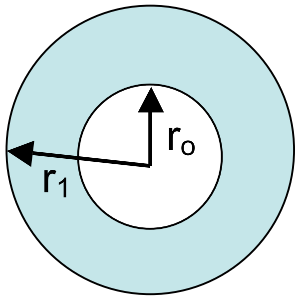
Higher Physics 1A PHYS11131 T2-2008

<u>Questions and Answers</u>
Total Marks: 75

Question 1: 12 marks
1. (a) Define the average coefficient of linear expansion for a material in terms of its fractional change in length and the change in temperature. If your definition includes an equation, define all terms used in it.

(b) A circular disk has a concentric circular hole cut out of its middle, as in the diagram. Explain, giving your reasoning, what happens to both the disk and the hole as it is heated.



Suppose that r_0=5.0cm, r_1=10cm and the coefficient of linear expansion is 2.0×10^{-5} °C^{-1}. What is the change in the area of the material that makes up the disk for a 30°C increase in its temperature?

(c)
Water at temperature 25°C flows from a tap T into a heated container C. The container has a heating element (a resistor R) which is supplied with electrical power, P, that may be varied.

The rate of flow of water F = 0.030 litres per minute. The electrical power is sufficient that the water in the container is boiling. What is the minimum power that must be supplied in the steady state so that the amount of liquid water in the container neither increases nor decreases with time? You may neglect other losses of heat, such as conduction and radiation from the container to the air.

For water, c=4,200 J kg^{-1} K^{-1}, L_{vap}=2.30×10^6 J kg^{-1}, ρ=1,000 kg m^{-3}.

<u>Solution</u>

(a) The coefficient of linear expansion, α, for a material is defined as the fractional change in length, divided by the change in temperature.

i.e. $\alpha = \dfrac{\Delta L / L_i}{\Delta T}$ where ΔL is the change in length, $L_f - L_i$, ΔT is the change in temperature, $T_f - T_i$ and i, f refer to the initial and final values for the relevant quantities.

(b) Both the disk and hole expand in radius (and in area) as the disk is heated. All elements expand by the same fraction, so that both the radius of a circle, and its circumference, will expand by the same fraction. The areas will also expand. Thus both the hole and the disk expands.

We have $\Delta A = 2\alpha\, A_i\, \Delta T$ for area expansion (or derive directly from square of linear expansion). i.e. twice the rate of linear expansion.

With $A_{disk} = \pi\, (r_1^2 - r_0^2)$ for the disk and $A_{hole} = \pi\, r_0^2$.

Thus, ΔA_{disk} = $2 \times 2.0\ 10^{-5} \times \pi \times [10^2 - 5.0^2] \times 30 = 0.2827\ cm^2$ for disk
= $0.28\ cm^2$ for disk to 2SF

And ΔA_{hole} = $2 \times 2.0\ 10^{-5} \times \pi \times 5.0^2 \times 30 = 0.09425\ cm^2$ for hole
= $0.094\ cm^2$ for hole to 2SF

(c) We must have, in the steady state,

Energy supplied = Heat to raise water to boiling + Heat to boil the water

i.e. $P\, t = m\, L + m\, c\, \Delta T$

where m is the mass of the water in the container, L and c are the latent and specific heats, respectively, and ΔT is the temperature rise, in time t.

The flow rate, F, given by $m = \rho\, F\, t$ where ρ is the density.

Thus $P\, t = \rho\, F\, t\, [L + c\, \Delta T]$

or $P = \rho\, F\, [L + c\, \Delta T]$

i.e. $P = 1000 \times 0.030 \times 10^{-3}/60 \times [2.3\ 10^6 + 4200 \times (100\text{-}25)]$ W
= 1307.5 W = 1,300W to 2SF.

Question 2: 15 Marks

2. (a) Write down an equation that describes the First Law of Thermodynamics. What quantity is being conserved? Make sure that you fully describe any symbols that you use.

(b) In the last four columns of the following table fill in the boxes with a –, + or 0 depending on whether the quantities decrease, increase, or remain (approximately) unchanged, respectively. Provide brief explanations as to your choices.

Situation	System	Heat	Work Done	Change in Internal Energy	Change in Temperature
Rapidly letting air out of a bicycle tyre	Air in the tyre				
Covered pan of hot water sitting on room temperature bench top	Water in the pan				

(c) Figure from tutorial 5, Q4, left side. The figure shows a cylinder containing a gas which is closed by a moveable piston. The cylinder is submerged in an ice-water mixture, and the gas is initially in thermal equilibrium with the ice-water. The piston is pushed down very quickly from position 1 to position 2. It is then held at position 2 until the gas is again in thermal equilibrium with the ice-water. The piston then is slowly raised until it returns to position 1. Sketch a Pressure-Volume diagram showing the changes that have taken place, and indicate briefly what thermodynamic processes are taking place at each stage around one complete cycle. If 200g of ice are melted during one cycle, how much work has been done on the gas? L_{fusion}=3.33 x 10^5 J/kg.

Solution

(a) The first law of thermodynamics states that the increase in internal energy of a system, ΔE_{int}, is equal to the flow of energy into the system, Q, plus the work done on the system, W.

i.e. $\Delta E_{int} = Q + W$.

This is a statement about the conservation of energy.

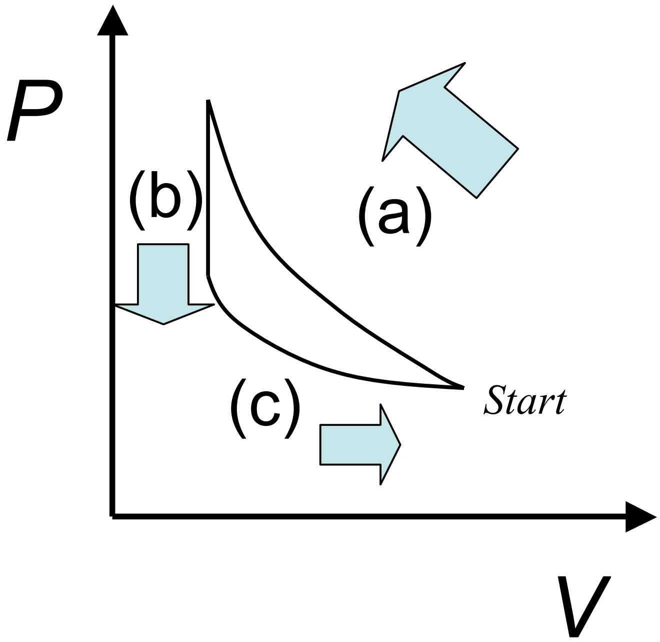
(b)

Situation	System	Heat	Work Done	Change in Internal Energy	Change in Temperature
Rapidly letting air out of a bicycle tyre	Air in the tyre	**0**	**–**	**–**	**–**
Covered pan of hot water sitting on room temperature bench top	Water in the pan	**–**	**0**	**–**	**–**

When letting air rapidly out of a bicycle tyre, the change is adiabatic, so Q=0. The work done is negative since the gas expands (the gas does work). Thus, there is a decrease in internal energy, applying the 1st Law. Hence a decrease in temperature, since this depends on the internal energy.

When allowing the pan of hot water to cool, there is no work done as the water is simply cooling. It loses energy as it does so. Thus Q<0. So, from the 1st Law, ΔE_{int} is < 0, and hence so is ΔT.

(c)



There is rapid compression from position 1 to position 2 along (a)

There is a decrease in pressure at constant volume along (b)

There is a slow increase in volume along (c).

Path (a) is adiabatic, as the process is fast and there is no time for heat to flow. So Q=0, but P and T both rise.

Path (b) is isovolumetric (or isochoric) and heat flows out of the gas as the temperature falls to 0°C.

Path (c) is isothermal because the expansion is slow enough to remain in thermal equilibrium with the bath. i.e. it takes place at constant temperature.

When we return to the start, the gas is at the same state, so at the same internal energy.

Thus $\Delta E_{int} = 0$, so from the 1st Law $Q + W = 0$.

Thus W $= -Q$ = heat to melt the ice
$= -(-0.2 \times 3.33\ 10^5$ J/kg$) = 6.66 \times 10^4$ J/kg.

Since Q is < 0, as heat is lost from the gas to the ice-water, so that the work done on the gas, W, is positive.

Question 3 15 Marks

3. (a) Show that $x = A\cos(\omega t+\phi)$ describes the displacement of a system where the restoring force is proportional to the distance from an equilibrium position, x.

(b) What do the symbols A, ω and ϕ correspond to?

(c) Hence show that, for a stretched spring of spring constant k, with block of mass m attached at the free end, that the period of oscillation, T, of the block is given by $T = 2\pi\sqrt{m/k}$.

(d) For this system, give expressions for the kinetic energy and potential energy of the block at position x, in terms of A, m, k and ϕ.

(e) What also is the total energy of the system at this position? Simplify your expression and comment on this value.

(f) Suppose that k=5.00 N/m and the block has mass m=0.200kg, and is oscillating with an amplitude of 10.0cm. If initially the velocity is 0.100 m/s in the negative x-direction, determine the equation describing the distance x from equilibrium as a function of time. What is the maximum speed and acceleration experienced by the block?

(g) If the oscillator also experiences an additional restoring force $R = -b\,v$, where v is the speed and b is a positive constant, how does Newton's Second Law describe the motion of the system? Describe qualitatively the effect on the subsequent motion of this additional force.

Solution

(a) Let that $x = A\cos(\omega t+\phi)$

Then $v = x' = -\omega A\sin(\omega t+\phi)$

And $a = x'' = -\omega^2 A\cos(\omega t+\phi) = -\omega^2 x$

So that $F_x = mx'' = -m\omega^2 x$, as required for the restoring force being proportional to the distance from the equilibrium.

(b) A is the amplitude of the oscillation

ω is the angular frequency

ϕ is the phase angle (i.e. the phase when t=0)

(c) For SHM $T=2\pi/\omega$.

For a spring, where $F_x = mx'' = -kx$ with $\omega^2=k/m$.

Thus $T = 2\pi\sqrt{\dfrac{m}{k}}$

(d) KE at position x is $1/2mv^2 = 1/2m[-\omega A\sin(\omega t+\phi)]^2 = 1/2m\omega^2 A^2\sin^2(\omega t+\phi)$.

PE at position x is $1/2kx^2 = 1/2kA^2\cos^2(\omega t+\phi)$

Thus, since $k=m\omega^2$, then KE = $1/2kA^2\sin^2(\omega t+\phi)$

and PE = $1/2kA^2\cos^2(\omega t+\phi)$

i.e. these are independent of the mass, and only involve A, k and ϕ.

(e) The total energy E = KE + PE = 1/2k A^2 since $\sin^2 + \cos^2 = 1$.
This is fixed, as it must be for conservation of energy.

(f) k=5.00 N/m, m=0.200 kg, A=10.0cm
At t=0 v=–0.100 m/s with ω^2=k/m so that $\omega=(5.00/0.200)^{0.5}$= 5 rad/s.
So $v = \dot{x} = -\omega A \sin(\omega t + \phi)$

$$v[0] = -0.1 = -\sqrt{\frac{5.00}{0.200}}\,0.10\sin(\phi)$$

i.e. sin(ϕ)=0.2 so that ϕ=0.20 rads = 11.5°

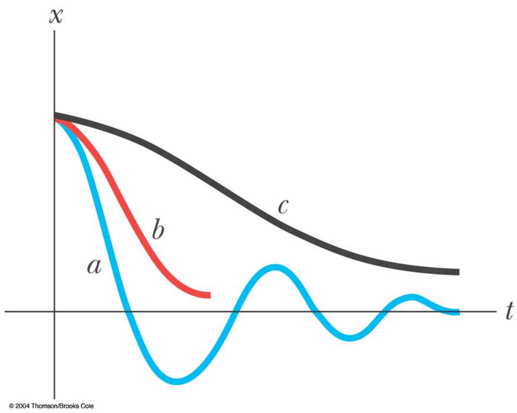
Thus
$$\begin{aligned} x &= A\cos(\omega t + \phi) \\ &= 10.0\cos(5t + 0.20)\ \text{cm} \end{aligned}$$

Maximum speed is $|\omega A| = 5 \times 0.1 = 0.5\text{m/s}$

Maximum acceleration is $|\omega^2 A| = 5^2 \times 0.1 = 2.5\text{m/s}^2$

(g) If there is an additional restoring force given by R=-bv, where b is a positive constant, then the equation of motion is given by the solution to F=ma, i.e. to $m\ddot{x} = -kx - b\dot{x}$.

This leads to damped oscillations, decaying to zero. How rapidly depends on the magnitude of the additional restoring force.



(a) is damped oscillations (under-damped)
(b) is critically damped (not an oscillation, comes to rest in quickest time)
(c) is overdamped

Question 4. 19 Marks

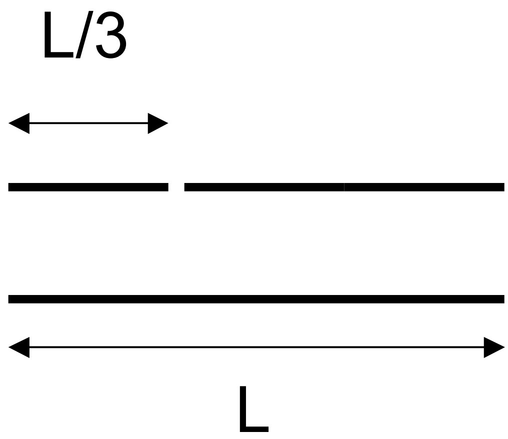
(a) Two travelling waves, moving in the $+x$ and $-x$ directions, are described by the equations $y=C\sin(kx-\omega t)$ and $y=C\sin(kx+\omega t)$, respectively. What do the symbols k and ω represent, and how are they related to the wave speed along the x-direction?

(b) Derive an expression for the wave resulting from the linear superposition of these two travelling waves. Interpret this equation quantitatively with the aid of a labelled sketch.

(c) For a thin cylindrical pipe of length L, open at both ends, and sound speed c, show that the frequency of the resonant modes of oscillation is given by $f=\frac{nc}{2L}$, where n is an integer, ≥ 1. Sketch the first three resonant modes.

(d) Suppose that a small loudspeaker with frequency adjustable from 1 to 2 kHz is placed near to, but not touching, one end of the pipe. The pipe has length 0.60m. At what frequencies will resonance occur? Take the speed of sound to be 330 m/s.

(e) If the pipe now has a small hole drilled in it, a distance $L/3$ from one end, determine how the first three resonances will change. Sketch the first of these.

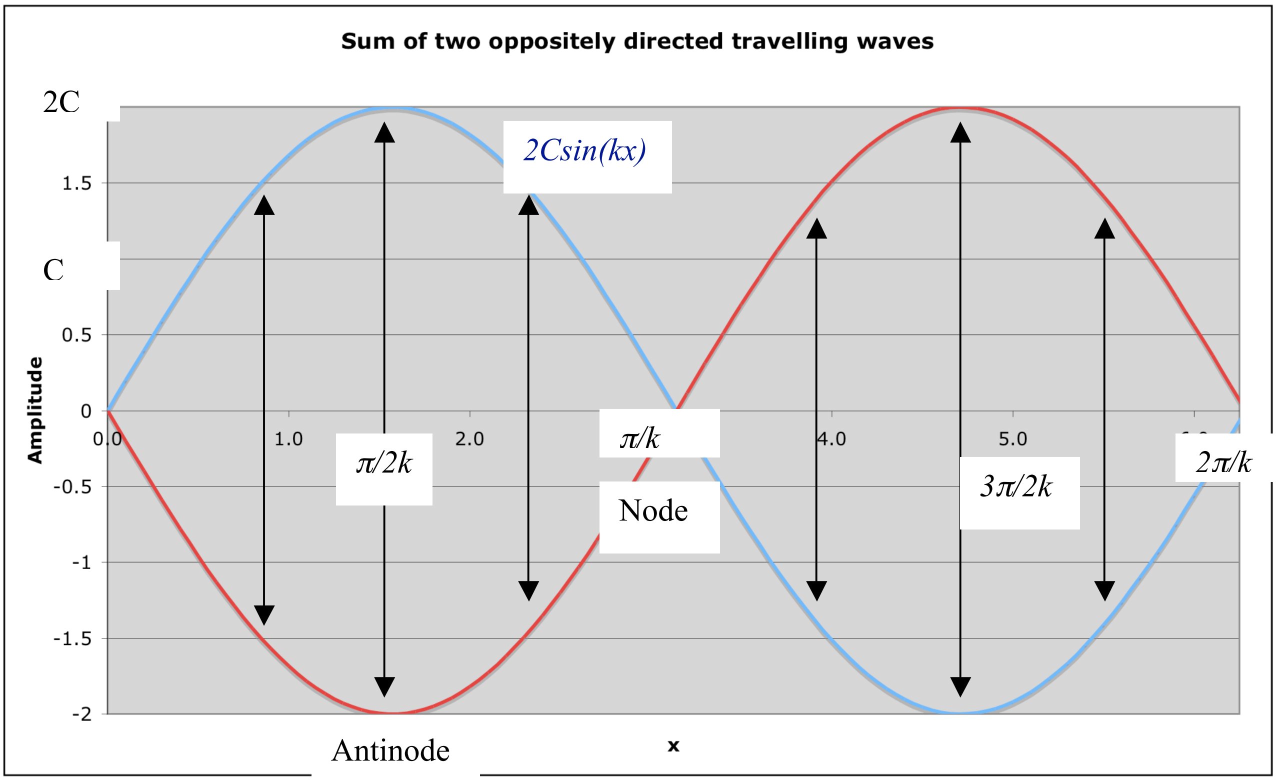


(f) Describe how taking into account end effects will alter the resulting resonant frequencies from part (d).

Solution

(a) k is the wavenumber $= 2\pi/\lambda$, where λ is the wavelength
ω is the angular frequency $= 2\pi/T$, where T is the period
$v = \omega/k$ is the wave speed, for the disturbance travelling in either the $+x$ or $-x$ direction.

(b)
Let $y_1=C\sin(kx-\omega t)$ and $y_2=C\sin(k+\omega t)$.
Then the sum, $y=y_1+y_2=C[\sin(kx-\omega t)+\sin(kx+\omega t)]$
Making use of $\sin A + \sin B = 2\sin(A+B)/2\cos(A-B)/2$ then,
With $A=kx-\omega t$ and $B=kx+\omega t$, we have
$Y=2\sin(kx)\cos(-\omega t) = 2\sin(kx)\cos(\omega t)$.

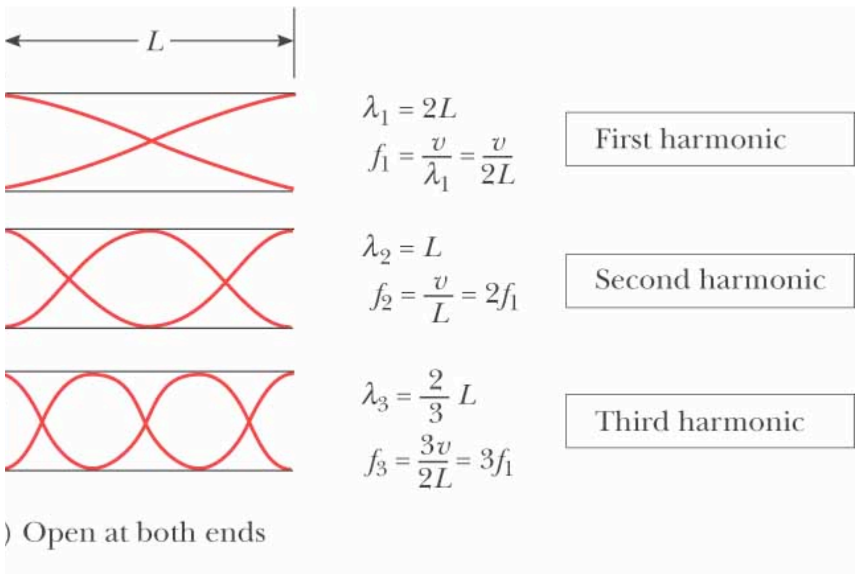


Nodes are positions with no oscillation, so sin(kx)=0, hence kx=nπ, with n=0, 1, 2, …

Antinodes are positions with maximum oscillation, sin(kx)=+/-1, hence kx=(n+1/2) π with n=0, 1, 2…

Each element of the string oscillates in SHM given by cos(*ωt)* within an envelope of amplitude *2Csin(kx*).

(c)



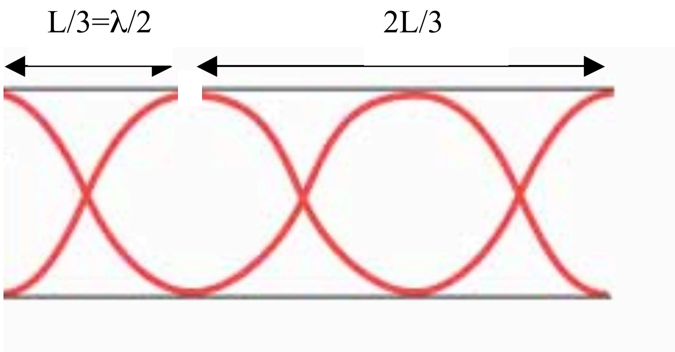
Shown are the first three resonances (or harmonics). Each end of the pipe is an antinode since it is open. We see that $f_n = nc/2L$ where c is the wavespeed and n=1, 2, 3, ….

(d)
We have to find f in the range 1–2 kHz.
that $f_n = nc/2L$ = n(330/2/0.6) Hz = n x 275 Hz.
So, the solutions between 1 and 2 kHz are, for

n=	4,	5,	6,	7
f=	1100	1375	1650	1925Hz

(e) If a hole is cut in the pipe then this will correspond to the position of an antinode.

This will thus be a distance L/3 from one end, 2L/3 from the other end.



First resonance when hole cut at L/3 from one end.

At the antinode $L/3=\lambda/2$ so that $L=3\lambda/2$. So $f_1=c/\lambda=3c/2L$

2^{nd} resonance: $L/3=\lambda$ so that $L=3\lambda$. So $f_2= 6c/2L$

3^{rd} resonance: $L/3=3\lambda/2$ so that $L=9\lambda/2$. So $f_3= 9c/2L$

i.e. we have $f_n=n(3c/2L)$, n=1, 2, 3

Thus, the frequencies are increased by a factor 3 from previously.

(f)
Including end effects we add a small distance, ε, to each open end to indicate where the antinode occurs

i.e. we effectively increase the length of the cylinder to L+2ε for resonance.

Thus $\lambda/2=(L+2\varepsilon)$ with $f=c/\lambda$, so that $f=2c/(L+2\varepsilon)$

i.e. the frequency is decreased by a factor $(L+2\varepsilon)/L = 1+2\varepsilon/L$.

Question 5. 14 Marks

(a) Suppose that a person (the observer) is moving at speed v_O directly towards a stationary source of sound which emits waves of frequency f at wavespeed c in still air. Show that the frequency experienced by the person, f', is given by $f' = f\left(\frac{c + v_O}{c}\right)$.

(b) Show also that a person at rest, hearing sound waves from the same source moving directly towards them at speed v_S in still air, hears that sound at a frequency f'' given by $f'' = f\left(\frac{c}{c - v_s}\right)$.

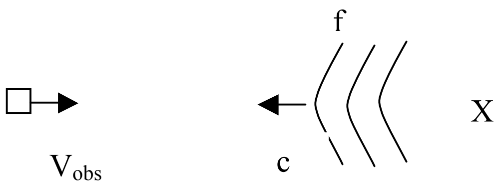
(c) A person walks at 5 m/s directly towards a stationary siren emitting sound at frequency 1,500 Hz in still air. What frequency will the person hear? The speed of sound may be taken as 330 m/s.

(d) Suppose now that the siren is also moving directly away from that person at a speed of 20 m/s. What frequency will the person now experience?

(e) Suppose now that the siren is moving directly towards a wall, which also reflects the sound waves back towards the observer. What frequency will the observer hear for the reflected sound waves?

Solution

(a)



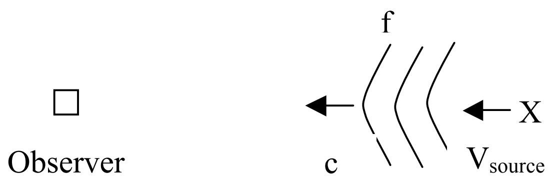
Speed of sound $c = f\lambda$

Observer travels towards source at speed $V_{obs.}$

Speed of waves relative to observer is now $c+V_{obs}$

Wavelength is unchanged. Thus observed frequency, f' given by $f' = \frac{c + V_{obs}}{\lambda} = f\left(\frac{c + V_{obs}}{c}\right)$

(b)



In this case the distance between wavefronts is now given by $\lambda'' = \lambda - {V_{source}}/{f} = \frac{c - V_{Source}}{f}$ since the source moves a distance V_{source}/f between the emission of 2 wavefronts.

Thus the perceived frequency by the Observer is $f'' = \frac{c}{\lambda''} = c\frac{f}{c - V_{Source}} = f\frac{c}{c - V_{Source}}$

(c) We have V_{Obs}= 5 m/s, f=1500 Hz and c=330 m/s

So $f' = f\left(\frac{c + V_{obs}}{c}\right) = 1500\left(\frac{330+5}{330}\right)$ Hz

Thus f'=1523 Hz

(d) The general Doppler effect equation is $f' = f\frac{c + V_{Obs}}{c - V_{Source}}$ where V_{Obs} and V_{Source} are directed towards each other.

In this case, V_{Obs}= 5 m/s and V_{Source}= -20 m/s as the source is moving <u>away</u> from the observer.

Thus $f' = 1500\left(\frac{330+5}{330--20}\right) = 1436$ Hz

(e) In this case the reflected signal is equivalent to the Doppler effect for a source moving towards the observer at +20 m/s.

i.e. $f' = 1500\left(\frac{330+5}{330-20}\right) = 1621$ Hz