Question 1. **[Marks 12]**

One particle, mass m_1, is fired vertically upwards with an initial velocity v_0 from the ground (height $y = 0$). At the same time ($t = 0$), another particle, mass m_2, is released from rest at a height $y = h$, directly above the first, where $h > 0$. The particles are then in free fall near the earth's surface. Air resistance and the rotation of the earth may be neglected.

a) Write equations for the vertical positions y_1 and y_2 of the particles as functions of time t from their launch, before any collision.

b) Write an equation for the position y_{cm} of the centre of mass of the two particles as a function of time. Simplify your expression by removing any common factors.

c) Derive an expression for the velocity $\underline{\mathbf{v}}_{cm}$ of the centre of mass.

d) Hence derive an expression for the acceleration $\underline{\mathbf{a}}_{cm}$ of the centre of mass.

e) Assume that the particles collide and that the collision is very brief and completely inelastic. Write an expression for the position of the particles after the collision but before they hit the ground.

f) From the results above, derive an expression for the total momentum of the two particles both before and after the collision, but before they hit the ground.

g) Is momentum conserved during the flight of the particles? Comment briefly: one clear sentence will be sufficient.

Question 2. **[Marks 8]**

a) Define the moment of inertia. If your definition is an equation, define the terms used.

b) State the law of conservation of mechanical energy, including any conditions.

c) A ball bearing (which is solid, uniform and spherical) of mass m and radius r rolls without slipping down a plane inclined at angle α to the horizontal. The bearing starts from rest at height h.

Derive an expression for its speed at height zero. Be careful to justify any principle you use, and to state it carefully. In particular, mention friction.

Question 3 [Marks 10]

(a) Describe three assumptions associated with the description of an ideal gas enclosed within a container.

(b) Describe in words the equation of state for an ideal gas, explaining the meaning of any symbols that you use.

(c) Thus show that the total kinetic energy, K, of an ideal gas is given by $\mathrm{K} = (3/2)\mathrm{nRT}$, where you describe the meaning of the symbols.

(d) Calculate the average speed of oxygen molecules (which are comprised of two oxygen atoms) in air at room temperature (T=20°C, Mass of Oxygen atom =16 m_p).

Question 4 [Marks 8]

(a) Describe the *superposition principle* for travelling waves passing through a linear medium. What is meant by the *interference* of two travelling waves. Describe two examples of interference for two travelling waves.

(b) Suppose two waves that give rise to pressure variation at a certain point in space are described by $P_1 = P_0 \cos(2\pi ft)$ and $P_2 = P_0 \cos(2\pi[ft - \alpha])$, where P_0 is the amplitude, f the frequency, t is the time and α is a constant. Determine the resultant wave.

(c) Calculate the amplitude of the resultant wave when (i) α=0, (ii) α=1/4 and (iii) α=1/3.

Question 5 [Marks 10]

(a) Suppose there are two musicians playing guitars, one seated and the other walking towards you at a speed of 3 m s^{-1}. One string on each of the guitars is vibrating with a frequency of 262.0 Hz. You are recording the sound with a microphone. What difference in frequency will you measure between the sounds from each guitar? The speed of sound in air may be taken to be 340 m s^{-1}.

(b) What length of cylindrical organ pipe (which is open at both ends) is needed to give a fundamental frequency of 262 Hz?. Derive the formulae used to determine this. You may neglect end effects.

(c) What new frequency would this organ pipe have if the temperature were to drop from 27°C to 10°C? You may assume that the thermal contraction of the pipe itself is negligible.

(d) What changes would occur in the frequency of the note produced by the organ pipe if a hole were now drilled in it one-third the distance from one end.

Question 6 [12 Marks]

(a) An object is oscillating in simple harmonic motion with amplitude A. What fraction of the energy is in kinetic energy when the object has displacement $x = -A/2$ from equilibrium?

(b) A block of mass $M = 65.0$ kg is thrown off a high bridge attached to a mass-less spring of length $L = 11.0$ m. The cord has an unknown spring constant, k. The block falls a distance $h = 36.0$ m below the bridge before bouncing back. Assume that the motion may be separated into an 11.0m free-fall followed by a 25.0 m section of simple harmonic motion (which lasts for less than one complete period). You may also ignore air resistance.

(i) For what time interval is the block in free fall?

(ii) Assuming conservation of energy, determine the spring constant, k.

(iii) Show that simple harmonic motion takes place while the cord is extended.

(iv) What is the angular frequency ω during this oscillatory portion of the block's motion?

(v) What is the equilibrium extension of the spring?