Question 1 (Marks: 20)

(a) On a straight, flat section of road, a car follows a truck. Initially ($t < 0$), both are travelling at speed v_0 = 90 k.p.h. The truck maintains constant speed throughout.

The car is initially 2.0 seconds behind the truck, *i.e.* for $t < 0$ the car passes a given point on the road two seconds later than the truck.

At some point, which we will call $x=0, t=0$, the car begins accelerating forwards at a = 2.0 ms^{-2}.

The car subsequently overtakes the truck and when the car is 50 metres ahead of the truck (at time t_1), the car driver realises he is over the legal speed limit, so decelerates at 2.0 ms^{-2}.

When the car has slowed down to 90 k.p.h. (at time t_2), it then maintains that constant speed.

(i) Draw a clear, well-labelled *displacement-time graph* for the car and the truck for all t and x relevant to this question (*i.e.* include $t < 0$ and $t > t_2$). The car and truck should be illustrated in the same figure. Do not draw them on two separate figures. Ensure that the time axis is horizontal and the displacement axis is vertical. Illustrate the times $t = t_1$ and $t = t_2$ on the time axis of your graph.

In the region $t < 0$, show an example of the 2.0 s by which the car is behind the truck.

You may draw the graphs in pencil and erase as necessary. (You may also cross out and start again.)

(ii) Draw a clear, well-labelled *velocity-time graph* for the car and the truck for all t and x relevant to this question. The car and truck should be illustrated in the same figure. Do not draw them separate figures. Ensure that the time axis is horizontal and the velocity axis is vertical. Illustrate the times $t=0$, t_1 and t_2 on the time axis of your graph.

(iii) How far (in metres) is the car behind the truck at $t < 0$?

(iv) Calculate the time t_1 when the car is 50 m ahead of the truck

(v) Calculate x_1, the position of the car at time t_1.

(vi) Show t_1 and x_1 on your graph.

Question 2 (Marks: 20)

(a) (i) Relative to the ground, a wind blows with speed v_w from a direction θ North of East. Take East and North as the **i** and **j** directions respectively, where **i** and **j** are unit vectors. Draw the vector diagram representing this situation.

(ii) Write the velocity of the wind relative to the ground in terms of v_w, θ and the unit vectors **i** and **j**.

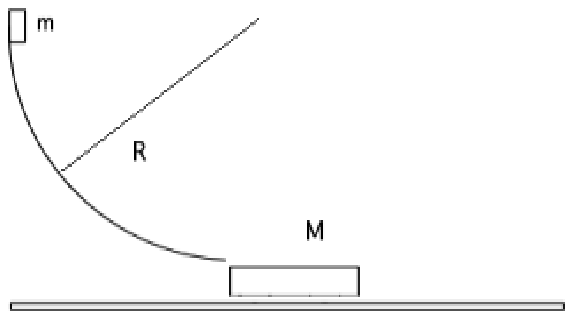
(iii) You are travelling North with speed v. Derive an expression for the apparent wind velocity you observe, i.e. the velocity of the wind measured in your frame. Give your answer in terms of v, v_w, θ, **i** and **j**.

(b) One way of discovering an exoplanet is as follows. An astronomer observes a periodic variation in the Doppler shift of the light from a distant star with mass M. From the amplitude and duration of these cycles she knows that the star is moving in a (relatively small) circle with radius R and period T. This circular motion is assumed to be entirely due to the gravitational effect of a planet (an exoplanet) with mass m ($\ll M$) in a circular orbit at distance r ($\gg R$) from the centre of the star.

(i) Draw a clearly labelled sketch of the path of the star and the exoplanet, showing the radius of each.

(ii) Stating any laws or assumptions used, derive an expression for m in terms of any of G, r, M and T. (Hint: $r \gg R$)

Question 3 (Marks: 10)



A small mass *m* slides without friction on a surface making a quarter-circle with radius *R*, as shown. Then it lands on the top surface of a cart, mass *M*, that slides without friction on a horizontal surface. (In practice, this cart could be a slider on an air-track.) The mass m slides a distance d along the top of the cart, but doesn't fall off.

i) Derive an expression for the velocity *v* of the mass *m* immediately before the collision in terms of *g* and *R*.

ii) Hence derive an expression for the velocity *V* of the combined masses immediately after the mass *m* has come to rest relative to the cart, in terms of *g, R, m* and *M*.

Show all working and assumptions and state carefully and explicitly any relevant laws or principles. (Hint: you will find it helpful to break the problem up into separate stages and to draw diagrams for each.)

Question 4 (Marks: 25)

In this section make use of the data provided in these tables.

Specific Heats and Thermal conductivities of selected metals

Substance	Specific Heat c, ($J\ kg^{-1}\ K^{-1}$)	Thermal conductivity k, ($W\ m^{-1}\ K^{-1}$)
Aluminium	910	205.0
Brass	377	109.0
Copper	390	385.0
Lead	130	34.7
Steel	456	50.2

Water

Quantity	Value
Specific Heat (liquid)	4186 $Jkg^{-1}K^{-1}$
Latent heat of Fusion	3.33×10^5 Jkg^{-1}
Latent heat of vapourization	2.26×10^6 Jkg^{-1}
Density (at 4.00° C)	1000 kgm^{-3}
Melting point (at 1 atm)	0.000 °C
Boiling point (at 1 atm)	100.0 °C
Volume expansion coefficient (β) (at 20°C: you may assume it is constant between 15°C and 100° C)	207×10^{-6} $(°C)^{-1}$

(a) Blacksmiths temper steel by heating it to a specific temperature and then "quenching" (rapidly cooling) it in water or oil. This gives the metal the desired toughness.

To get a steel axe head to turn a light blue colour a blacksmith heats the metal to 337 °C. The axe head weighs 755 g and he quenches it in 12.5 L buckets of water that is initially at 24.3 °C. The bucket is made from 500.0 g of steel.

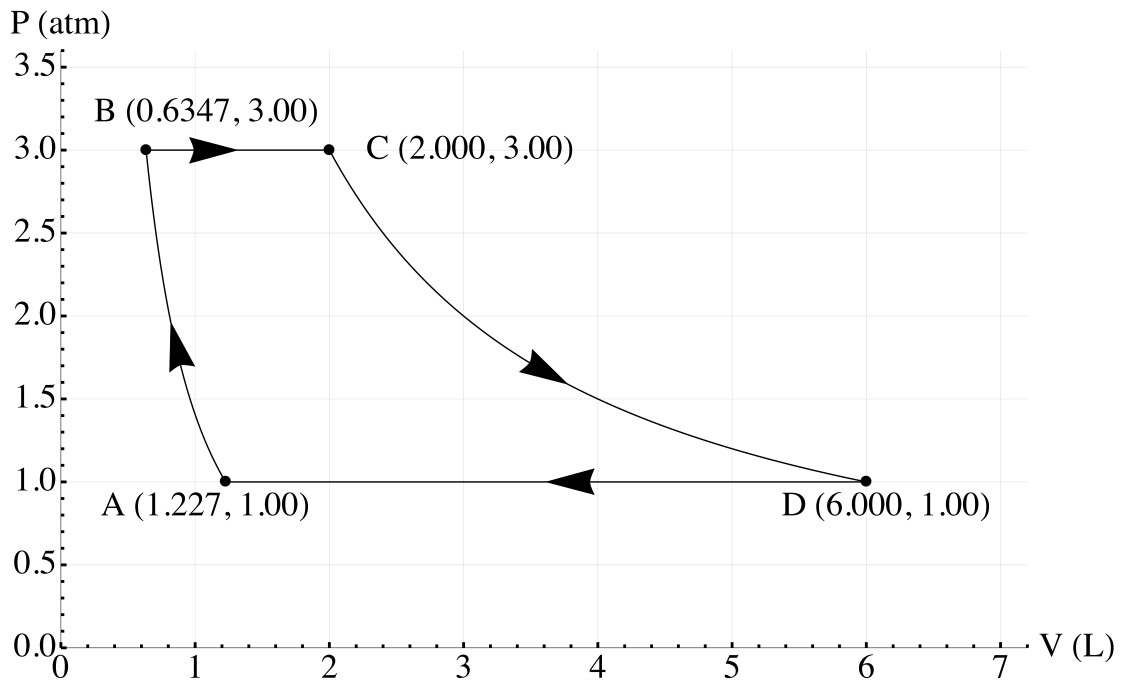
Assume that no steam is produced when the axe is quenched and no heat is lost to the surroundings. What is the final temperature of the bucket of water?

(b) The main components of air are N_2 and O_2. Assume that we can approximate air as only made from these two gasses.

(ii) At room temperature how many degrees of freedom does air have? State what type of movement (vibrational, rotational and translational) each of these degrees of freedom corresponds to.

(iii) A walk-in refrigerator (filled with air) has an air-tight seal, and does not lose heat to the surrounding rooms. The refrigerator has dimensions 5.00 m x 5.00 m x 2.00 m. To defrost the refrigerator a heater is placed inside it. The temperature is raised from 0.00 °C to 24.5 °C. How much energy does the heater provide during this process? Show all the steps in your working. You may assume that the door is opened at 0.00 °C to place the heater in but in then left closed until the temperature inside is 24.5 °C.

(c) A monatomic ideal gas undergoes a cycle from A➔B➔ C➔D ➔A. The process from A➔B is adiabatic, the process from B➔ C is isobaric with 1033 J of heat energy flowing into the system, C➔D is isothermal with 664 J of heat energy flowing into the system and D ➔A is isobaric with 1203J of heat energy flowing out of the system. On the graph, the points are written as (x, y), where x = volume in litres, y = pressure in atmospheres.



(i) How much work is done by the gas as it goes from B to C?
(ii) What is the change in internal energy of the gas as it goes from B to C?
(iii) How much work is done on the gas as it goes from C to D?
(iv) What is the change in internal energy of the gas as it goes from A to B?

Question 5 (Marks: 25)

(a) An oscillating block-spring system has a mechanical energy of 1.00 J, an amplitude of 10.0 cm, and a maximum speed of 1.20 m/s. Find

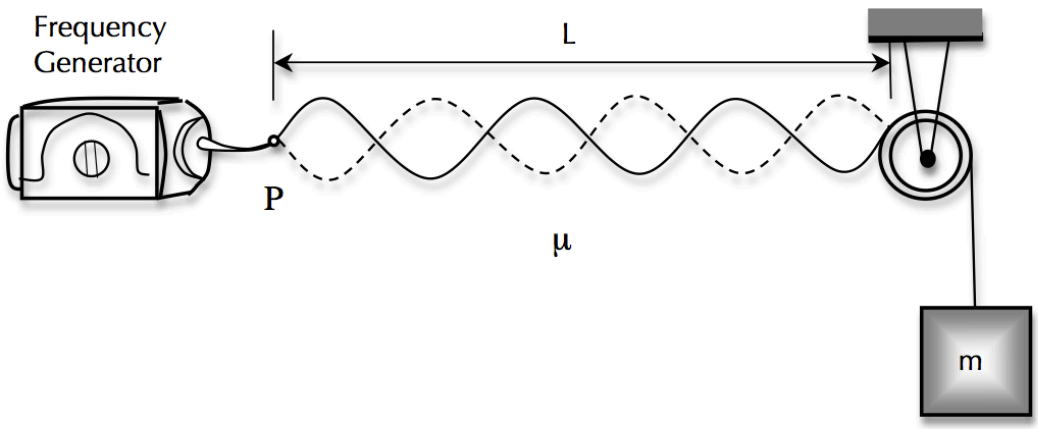
(i) the spring constant,

(ii) the mass of the block,

(iii) the frequency of oscillation, and

(iv) write down an equation to describe how the position, x, of the block changes with time (assume that it starts (t = 0) at the origin).

(b) A hanging mass is hung by a string over a pulley and tied to a signal generator as shown in the diagram (The mass is not accelerating and is located on Earth). The length of the string is L = 2.40 m. At the moment shown in the diagram the frequency is set to 50 Hz.



(i) What is the wave speed of the travelling wave generated by the frequency generator that sets up the standing wave in the string at the moment shown in the diagram?

(ii) The mass per unit length of the string, μ, is 0.234 gm^{-1}. What is the mass of the hanging mass, m, shown in the diagram?

(iii) What mass needs to be removed from or added to the mass hanger to produce the next harmonic (ie. Increase the number of loops on the string)?

(c) Stationary Stan is listening to the sound from an airplane flying along his line of sight. The frequency that Stan perceives is two-thirds the frequency emitted by the plane.

(i) Is the airplane approaching or receding from Stan?

(ii) Calculate the speed of the airplane if the speed of sound in air is 340 ms^{-1}.

(iii) Stan now drives towards the airplane at 40.0 km/h. What frequency does he now detect? Give your answer in terms of f_p, the frequency of sound produced by the plane.