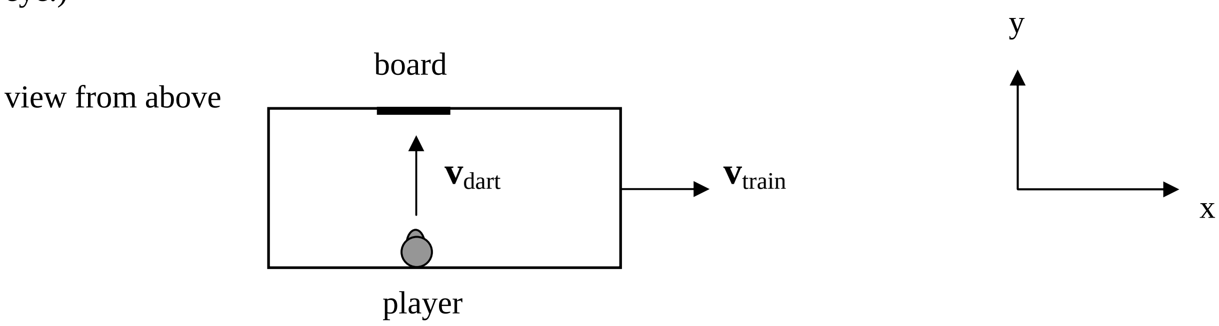
QUESTION 1 (20 marks)

A dart is thrown horizontally with a speed of 12 m/s toward the bull's eye of a dart board. It leaves the player's hand at the height of the bull's eye and lands a distance 25 cm vertically below it.

(a) What is the flight time of the dart?

(b) How far is the player from the board?

The player, dart, and dart board are on a train travelling horizontally at a constant speed of 40 km/h due east (the x-axis). The dart is thrown in a direction perpendicular to the motion of the train, as indicated in the figure. (The dart is thrown in the same manner as above: 12 m/s toward the bull's eye.)



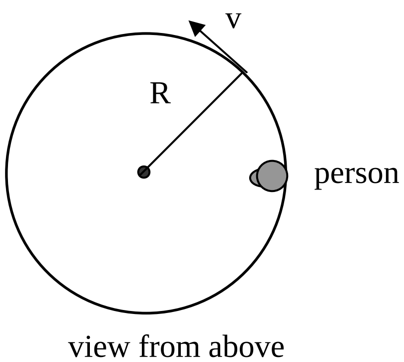
(c) What is the horizontal velocity of the dart according to a stationary ground-based observer? Express your answer in terms of components along x and y axes.

(d) What is the horizontal displacement of the dart during its flight according to the observer on the ground? Express your answer in terms of a magnitude and direction relative to the x-axis.

(e) The train rounds a turn. Again, the dart is thrown horizontally toward the bull's eye at 12 m/s. Will the flight time be different to that found in (a)? Will the dart land on the board at a different position (horizontal or vertical) to that found initially (see beginning of question)? Briefly explain your answers; a diagram may be useful.

QUESTION 2 (20 marks)

A rotor is an amusement park ride. It consists of a hollow cylindrical room that can be set rotating about the central vertical axis. A person enters the room, closes the door, and stands against the wall. The rotor increases in speed. When it reaches a speed v_{min} the floor is taken from underneath the person. The speed v_{min} is the minimum speed of the rotor at which the person will not fall, supported by the static friction between the rotor wall and the person's clothing.



(a) Draw a free-body diagram clearly indicating the forces that act on the person.

(b) What must be the total force acting on the person in the vertical direction so that the person does not fall?

(c) Use Newton's second law in the vertical direction to find an expression (in terms of the person's mass m) for the minimum value of the coefficient of static friction μ_s required to keep the person from falling.

(d) Write down Newton's second law for the force(s) acting on the person in the radial direction.

(e) By combining the expressions from (c) and (d) derive the minimum speed required to keep the person from falling, $v_{min} = \sqrt{gR/\mu_s}$, for a given μ_s. (g is acceleration due to gravity.)

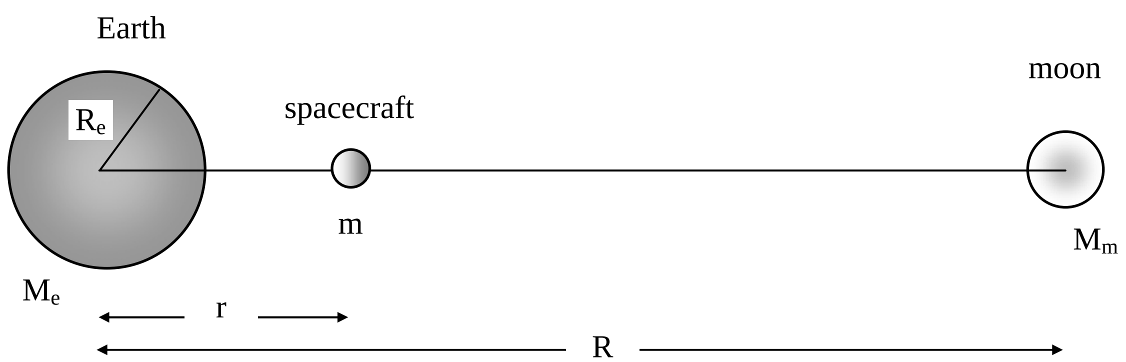
The rotor has a radius R = 2.0m. The floor beneath the person is removed when the rotor reaches a speed 7.0m/s.

(f) What value for μ_s is required to keep the person from falling?

(g) What would happen to the person if the coefficient of static friction between their clothing and the wall were larger than the value found in (f)? What if it were smaller? One or two sentences will suffice.

QUESTION 3 (20 marks)

A spacecraft of mass m is launched from Earth on a mission to the Moon. The mass of the Earth is $M_e = 81M_m$, where M_m is the mass of the Moon; the distance between the centres of the Earth and the Moon is $R = 60R_e$, where R_e is the radius of the Earth.



(a) The spacecraft is located between the Earth and Moon at a distance r from the centre of the Earth. Write an expression for the gravitational force acting on the spacecraft.

(b) Find the point r_0 between the Earth and Moon where the gravitational force acting on the spacecraft is zero. Express your answer in terms of the distance R.

(c) Write an expression for the gravitational potential energy U(r) of the spacecraft as a function of r.

(d) Sketch a graph of the potential energy between the Earth and Moon.

(e) Find the maximum value of the gravitational potential energy between the Earth and Moon. Express your answer as a coefficient times GmM_e/R. (Hint: what is the value of the force at this point?)

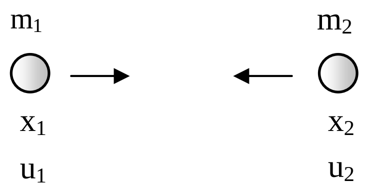
(f) What is the minimum energy required for the spacecraft to reach the Moon?

(g) Find the minimum launch speed of the spacecraft in order for it to reach the Moon. Express your answer in terms of the escape speed from the Earth $v_e = \sqrt{2GM_e / R_e}$.

[G is the gravitational constant.]

QUESTION 4 **(20 marks)**

Two particles move on the x-axis. Particle 1, located at x_1, has mass m_1 and velocity u_1; particle 2, located at x_2, has mass m_2 and velocity u_2.



(a) Write an expression for the centre of mass x_{cm} of the two-body system.

(b) Write an expression for the velocity of the centre of mass v_{cm}.

Particle 1 moves towards particle 2 with velocity $u_1 = u$; particle 2 is at rest. The particles collide elastically.

(c) Derive the expressions:

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u \, , \qquad v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u$$

from conservation of momentum and energy. v_1 is the final velocity of particle 1, v_2 is the final velocity of particle 2.

(d) Find the velocity of the centre of mass of the system before the collision.

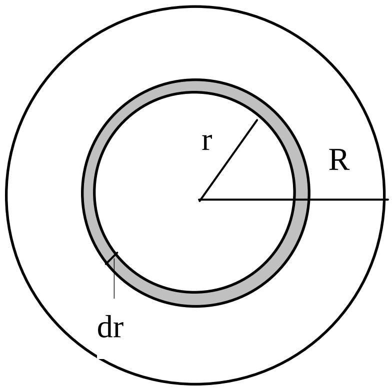
(e) Find the velocity of the centre of mass of the system after the collision.

(f) Compare the results from (d) and (e). Do they agree? Provide a brief explanation.

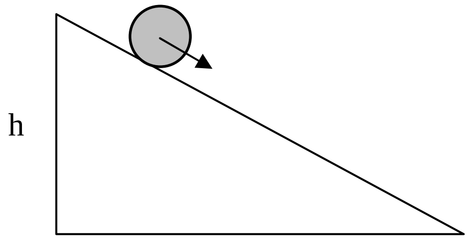
QUESTION 5 (20 marks)

Consider a uniform disc of mass M and radius R rotating about an axis through its centre and perpendicular to its surface.

(a) Find the element of mass dm for a ring of radius r and width dr (see figure) in terms of the total mass M of the disc. (Hint: dm = σdA, where σ is the surface mass density and dA is the element of area.)



(b) Use $I_{cm} = \int_0^R r^2 dm$ to show that the rotational inertia of the disc about the centre of mass is $I_{cm} = MR^2/2$.



The disc rolls without slipping down an inclined plane of height h.

(c) Draw a free-body diagram and clearly indicate the forces that act on the disc. Which of these forces produces a torque about the centre of mass?

(d) Write an expression relating the angular speed and the speed of the centre of mass.

(e) Use conservation of energy to find the speed of the centre of mass of the disc at the bottom of the incline.

(f) If the disc were sliding down the incline (no rotational motion) what would be the speed of the centre of mass at the bottom?

(g) Explain why the speed of the centre of mass found from (e) and (f) differ.