

1121/1131 Question 1 T2 2012

a) i) $s = 4(0)^2 + 1 = 1m$

ii) $v = \frac{ds}{dt} = 8tms^{-1}$

iii) $a = \frac{d^2s}{dt^2} = 8ms^{-2}$

b) Convert the speed to SI units:

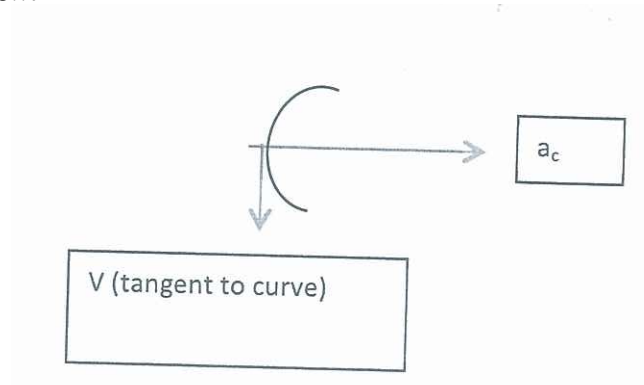
$$50km/hr = \frac{50 \times 10^3}{3600} = 13.9ms^{-1}$$

Constant acceleration:

$$t = \frac{v}{a} = 13.98 = 1.7s$$

c)
$$\begin{aligned} x &= x_0 + v_0t + \frac{1}{2}at^2 \\ &= 1 + 0 \times 8 + \frac{1}{2} \times 8 \times 1.7^2 \\ &= 13m \end{aligned}$$

d) i)



ii) $a_c = \frac{v^2}{r} = \frac{13.9^2}{30} = 6.4ms^{-2}$ towards the center of the curve

e) i) At the same velocity as the car, at a tangent to the curve followed by the car:
 $v = 13.9 ms^{-1}$ (v in direction shown in diagram above)

ii) $y = \frac{1}{2}a_yt^2$
 $\Rightarrow t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2 \times 1.4}{9.8}} = 0.53s$

iii) $x = v_{x0}t = 13.9 \times 0.53 = 7.4m$

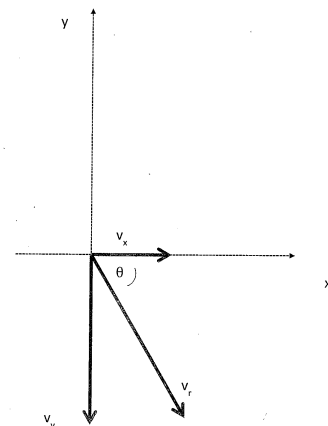
iv) $v_y = a_yt = 9.8 \times 0.53 = 5.2ms^{-1}$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{13.9^2 + 5.2^2} = 15ms^{-2}$$

To work out direction need to calculate θ

$$\tan \theta = \frac{v_y}{v_x} = \frac{5.2}{13.9}$$

$$\theta = 20.5^\circ \text{ below horizontal}$$



Q2

- i) If non-conservative forces do no work, mechanical energy is conserved.
- ii) If external forces are zero (or provide negligible impulse), momentum is conserved.
- iii) At the point of maximal compression (or minimum length) of the spring, its length is instantaneously not changing, so the relative velocity of the two masses is instantaneously zero. Hence the two have the same x component of velocity, call it v_1 .

Here, no external forces in the x direction, so momentum is conserved.

Considering the initial state and that at maximal spring compression:

p_x conserved: $mv_0 = 2mv_1$.

so $v_1 = v_0/2$ (only 1 mark for guessing this)

Here, there are no non-conservative forces so mechanical energy is conserved.

E conserved: $U_{\text{initial}} + K_{\text{initial}} = U_{\text{final}} + K_{\text{final}}$

$$\frac{1}{2}mv_0^2 = \frac{1}{2} \cdot 2mv_1^2 + \frac{1}{2}kx^2 = mv_1^2 + \frac{1}{2}kx^2$$

$$\frac{1}{2}mv_0^2 = m(v_0/2)^2 + \frac{1}{2}kx^2 = mv_0^2/4 + \frac{1}{2}kx^2$$

$$mv_0^2/4 = \frac{1}{2}kx^2$$

$$x = \sqrt{\frac{m}{2k}} v_0$$

- iv) Here, there are no non-conservative forces, and no external forces in the x direction, so both mechanical energy and momentum are conserved.

Considering the states before and after the collision:

p_x conserved: $mv_0 = mV + mv$

E conserved: $U_{\text{initial}} + K_{\text{initial}} = U_{\text{final}} + K_{\text{final}}$

$$0 + \frac{1}{2}mv_0^2 = 0 + \frac{1}{2}mV^2 + \frac{1}{2}mv^2 \quad (2)$$

(1) gives $V = (v_0 - v)$ substitute in (2) gives

$$v_0^2 = (v_0 - v)^2 + v^2$$

$$v_0^2 = v_0^2 - 2vv_0 + 2v^2$$

$$0 = -2vv_0 + 2v^2$$

$$0 = v(v - v_0)$$

So either $v = 0$ or $v = v_0$.

and (respectively) $V = v_0$ or $V = 0$

- v) There are two solutions. If there really is a collision, the velocities will change, so the first solution above ($v = 0$ and $V = v_0$) applies. The second solution ($v = v_0$ and $V = 0$) corresponds to the case where the objects miss each other, so momentum and mechanical energy are both conserved.

vi) If the system comprises N particles with masses m_i at positions \underline{r}_i , the centre of

mass is a point whose position is $\underline{r}_{\text{cm}} = \frac{\sum_{i=1}^N m_i \underline{r}_i}{\sum_{i=1}^N m_i}$ OR $\underline{r}_{\text{cm}} = \frac{\sum_{i=1}^N m_i \underline{r}_i}{\text{total mass}}$.

vii) Taking time derivatives of the expression in (vi) and noting that the masses are constant:

$$\underline{v}_{\text{cm}} = \frac{\sum_{i=1}^N m_i \underline{v}_i}{\sum_{i=1}^N m_i}.$$

Apply this to the condition before the collision and we have

$$\underline{v}_{\text{cm}} = \frac{\sum_{i=1}^2 m_i \underline{v}_i}{\sum_{i=1}^2 m_i} = \frac{mv_0 + 0}{2m} = v_0/2$$

Apply this to the condition of maximum compression gives

$$\underline{v}_{\text{cm}} = \frac{\sum_{i=1}^2 m_i \underline{v}_i}{\sum_{i=1}^2 m_i} = \frac{2mv_1}{2m} = v_1 \text{ and, substituting from part (iii)}$$

$$\underline{v}_{\text{cm}} = v_0/2$$

viii) Newton's second law for an extended object or system is

$\underline{F}_{\text{ext}} = (\text{total mass})\underline{a}_{\text{cm}}$ Here, the external force is zero so the velocity of the centre of mass is constant.

OR

Newton's first law for an extended system is that, if $\underline{F}_{\text{ext}} = 0$, the velocity of the centre of mass is constant.

OR

Some other equivalent statement.

Q3

i) $a = v_e^2/r_e$ its direction is towards the sun

Or centripetal acceleration of v_e^2/r_e or etc.

ii) $F = GM_s M_e / r_e^2$ with direction towards the sun OR
centripetal force of magnitude $GM_s M_e / r_e^2$ or equivalent in vector notation.

iii) Combining the two previous results with Newton's second law gives:

$$F = GM_s M_e / r_e^2 = M_e a = M_e v_e^2 / r_e$$

$$\text{so } GM_s M_e / r_e = M_e v_e^2$$

$$\text{so } K_e = \frac{1}{2} M_e v_e^2 = GM_s M_e / 2r_e$$

iv) $U_e = -GM_s M_e / r_e$

v) hence $E_e = -GM_s M_e / 2r_e$

vi) $\Delta E = E_{\text{now}} - E_{\text{then}}$

$= U_{\text{now}} + K_{\text{now}} - (\text{energy in earth-like orbit}) + \text{energy to get far away from earth but into such an orbit}$

$$= -GM_s m / D + \frac{1}{2} m v^2 + GM_s m / 2r_e + GM_e m / R_e$$

(not including the last term loses only one mark)

vii) head-on collisions (in the astronomical sense) with Jupiter and Saturn (in 1979 and 1980)

Question 4 1131 T2 2012

a) i) 3

ii) $P = \frac{F}{A}$

$$P_f = P_i + \frac{mg}{A} = 1.17 \times 10^5 + \frac{5 \times 9.8}{12.0 \times 10^{-4}} = 1.58 \times 10^5 Pa$$

iii) quickly \Rightarrow adiabatic
 $PV^\gamma = \text{const}$

$$\gamma = \frac{C_P}{C_V} = \frac{R + \frac{1}{2}fR}{\frac{1}{2}fR} = \frac{5}{3}$$

$$P_i V_i^{5/3} = P_f V_f^{5/3}$$

$$1.17 \times 10^5 \times (0.20 \times 12.00 \times 10^{-4})^{5/3} = 1.58 \times 10^5 \times (h_f \times 12 \times 10^{-4})^{5/3}$$

$$(h_f \times 12 \times 10^{-4})^{5/3} = 8.8635 \times 10^{-7}$$

$$h_f \times 12 \times 10^{-4} = 2.004 \times 10^{-4}$$

$$h_f = 16.7 cm$$

iv) $PV = \text{const}$

$$P_i V_i = P_f V_f$$

$$1.17 \times 10^5 \times 0.20 \times 12.00 \times 10^{-4} = 1.58 \times 10^5 \times h_f \times 12 \times 10^{-4}$$

$$h_f = 14.8 cm$$

b) i) $P = kA \left| \frac{dT}{dx} \right|$

P is constant along the rod

$$\Rightarrow P = 385 \times 10 \times 10^{-4} \times \frac{(70 - 45)}{1.00} = 9.63 W$$

ii) $9.625 = 50.2 \times 10.0 \times 10^{-4} \times \frac{(45 - 15)}{L_2}$

$$L_2 = 0.156 m$$

c) i) $\Delta E_{int} = 0$ as isothermal

ii) $\Delta E_{int} = Q + W$

$W = 0$ as volume does not change

$$\Delta E_{int} = -233 kJ$$

iii) $W_{C \rightarrow A} = \Delta E_{int C \rightarrow A} - Q_{C \rightarrow A}$

$Q_{C \rightarrow A} = 0$ a adiabatic

$$\Delta E_{int C \rightarrow A} = -(\Delta E_{int A \rightarrow B} + \Delta E_{int B \rightarrow C})$$

as no change in internal energy over cycle

$$= -(0 - 233)$$

$$= 233 kJ$$

$$W_{C \rightarrow A} = 233 kJ$$

Alternatively could use $W = - \int P dV$ and integrate the expression.

This takes longer.

PHYS1131 Exam 2012-T2**Oscillation question [25] marks****(a)** (i) At the moment of release:

$$\sum F = -kx = ma$$

$$125 \times 0.687 = 5.0a$$

$$a = \frac{85.9}{5.0} = 17.17 = 17.2 \text{ m/s}^2$$

(ii) the period of oscillation for the spring system:

$$T = 2\pi\sqrt{\frac{m}{k}} = 6.28 \times \sqrt{\frac{5.00}{125}} = 1.26 \text{ s}$$

(iii) The work done by the spring between $x = 0.687$ and 0:

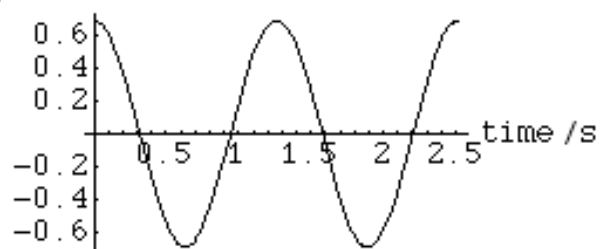
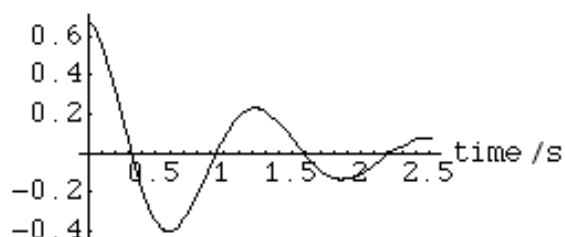
$$W = F \cdot x = \int_0^{0.687} kx dx = \frac{1}{2} kx^2 = 29.5 \text{ J}$$

(iv)

$$f = \frac{1}{T} = 0.794 \text{ Hz}$$

$$2T = 2.52 \text{ s}$$

$$\text{displacement } x = 0.687 \cos(2\pi ft)$$

displacement x/m Small damping ($b = 0.35$ – not for marking)displacement x/m 

- (b) (i) If the path difference results in two periods arrival difference for the wave trains from each source, it must be two wavelengths long (leading to constructive interference)

$$S2P - S1P = 13 - 12 = 1\text{m}$$

$$\text{thus } \lambda = 1/2 = 0.5 \text{ m}$$

$$v = \lambda f$$

$$f = \frac{v}{\lambda} = \frac{350}{0.5} = 700 \text{ Hz}$$

- (ii) For 1.5 periods difference, the new path difference must be 1.5 wavelengths long (which does lead to destructive interference). Both distances S1P and S2P will increase by s1 and s2.

$$(S2P + s2) - (S1P + s1) = 0.75 \quad \text{eqn(1)}$$

from Pythagoras :

$$(S2P + s2)^2 = 25 + (S1P + s1)^2 \quad \text{eqn(2)}$$

$$(S2P + s2) = 12.75 + s1$$

Substitute into eqn(2)

$$(12.75 + s1)^2 = (12 + s1)^2 + 25$$

which can be solved for s1

The answer is then 12 + s1

First find s1:

$$s1 = \frac{-3.2}{-0.75} = 4.29 \approx 4.3 \text{ m}$$

$$\text{new distance from S2: } 13 + s2 = 12.75 + 4.3$$

$$\text{new distance from S2} = 17 \text{ m}$$

- (c) Have to do (ii) before (i) to find the train speed.

- (ii) If the frequency appears lower to the observer, the source and the observer must be moving away from each other: their velocities will be negative

$$f' = \left(\frac{v + v_o}{v - v_s} \right) f$$

$$558 = \left(\frac{340 - 10}{340 + x} \right) \times 600$$

$$x = \frac{600}{558} \times 330 - 340 = 14.8 \approx 15 \text{ m/s}$$

- (i) Train moving away from stationary observer:

$$f' = \left(\frac{v}{v + v_s} \right) f = \left(\frac{340}{355} \right) 600 = 575 \text{ Hz}$$

(iii) Treat the wall as stationary observer: therefore $f = 575 \text{ Hz}$

The cyclist is moving towards the wall, which has now become a source, reflecting sound at 575 Hz .

Frequency received by the cyclist moving towards the wall:

$$f' = \left(\frac{v + v_o}{v} \right) f = \left(\frac{340 + 10}{340} \right) \times 575 = 591.9 \approx 592 \text{ Hz}$$

Beating frequency: $592 - 558 = 34 \text{ Hz}$