PHYS1131 Higher Physics 1A
2009-T1 Test 2 Solutions
Thermal Physics and Waves

3 (a) Take three from:

(i) large number of molecules, average separation large compared to their dimensions – i.e., point-like

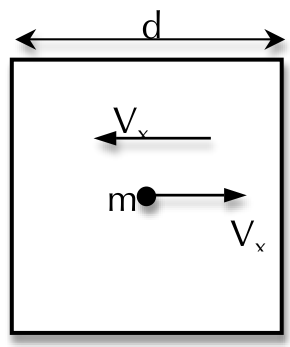
(ii) Don't interact except during collisions

(iii) Elastic collisions with walls

(iv) Obey Newton's 2nd Law; but move randomly

(v) Pure substance (i.e. all particles the same)

(b) Explain elastic collisions or show diagram as below:



Speed v_x, momentum mv_x

Change in momentum on collision of a particle with the wall is

$\Delta p = mv_x - (-mv_x) = 2mv_x$

Average time between collision of particle with the same wall $\tau = \dfrac{2d}{v_x}$

Thus Force exerted, F = rate of change of momentum = N $\dfrac{\Delta p}{\tau}$

i.e. $$F = \frac{N2mv_x}{2d/v_x} = N\frac{mv_x^2}{d}$$

(c) Pressure $P = \dfrac{\text{Force}}{\text{Area}} = \dfrac{F}{d^2} = \dfrac{Nmv_x^2}{d^3} = \dfrac{Nmv_x^2}{V}$

Where $V = d^3$ = volume of box

But $v^2 = v_x^2 + v_y^2 + v_z^2$

and from isotropy $v_x^2 = v_y^2 = v_z^2$ i.e. $v_x^2 = v^2/3$

So that $P = \dfrac{N}{3V}mv^2$

(d) Equation of state for an ideal gas is given by $PV = nRT$

Where P is the pressure

V is the volume

n is the number moles of the gas

R is the gas constant

T is the absolute temperature

Also OK to use $PV = N k T$ where N is the number of particles, k Boltzmann's constant

(e) Kinetic energy $K = N$ x KE per particle $= N$ x $\frac{1}{2} mv^2$

So $K = \dfrac{N3PV}{2N} = 1.5PV = 1.5nRT$ from the ideal gas law

(f) $T = 20^{\circ}C = 293$ K

$M_{O2} = 2 \times 16\ m_p$

From part (e) we have $K = 3/2\ n\ RT = N\ \frac{1}{2}\ mv^2$

i.e. $v^2 = \dfrac{3(n/N)RT}{m}$

But $N = n\ N_A$ where N_A is Avogadro's Number

$$\therefore \quad v^2 = \frac{3RT}{m_{02}N_A} = \frac{3 \times 8.314 \times 293}{32 \times 1.67310^{-27} \times 6.022 \times 10^{23}}\ \mathrm{m^2s^{-2}}$$

$v^2 = 2.267 \times 10^5\ \mathrm{m^2/s^2}$

So that $v = 476.1$ m/s = 480 m/s to 2SF

Q4 (a) Superposition Principle: if two or more travelling waves are passing through a medium then the resultant value of the wave function at any point is given by the algebraic sum of the amplitudes (or wave functions) of the individual waves. i.e. it is a linear super-position.

Key Words - Travelling Waves
Linear superposition of amplitudes (wave-functions)

Interference is the combination of waves in the same region of space.

Two examples:
Constructive Interference: occurs when the displacements are all in the same direction, so as to produce a maximum in the resulting wave.

Destructive Interference: occurs when the pulses add in the opposite direction, so the resulting wave is a minimum.

Other possibilities OK too, e.g. Young's Slit, Diffraction Grating, Standing Waves.

(b)
$$P_1 = P_0 \cos(2\pi ft)$$
$$P_2 = P_0 \cos(2\pi(ft - \alpha))$$

So the Resultant Wave is

$$P = P_1 + P_2 = P_0 \cos(2\pi ft) + P_0 \cos(2\pi(ft - \alpha))$$

Using cos A + cos B = 2 cos $\frac{A+B}{2}$ cos $\frac{A-B}{2}$ then

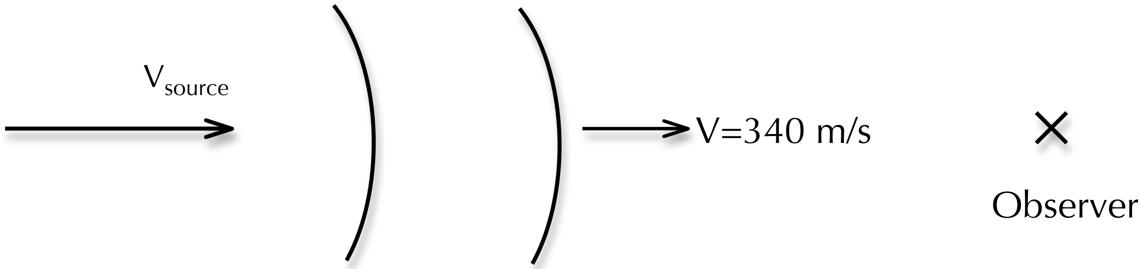
$$P = 2P_0 \cos(\frac{2\pi(2ft - \alpha)}{2})\cos(\frac{2\pi\alpha}{2})$$
$$P = 2P_0 \cos(2\pi(ft - \alpha/2)\cos(\pi\alpha)$$

(c) (i) α=0 so cos($\pi\alpha$)=1 so that $P = 2P_0 \cos(2\pi ft)$
So the amplitude is $2P_0$

(ii) α=1/4 so cos($\pi\alpha$)=cos(π/4)=1/√2 so that
$P = 2P_0 \cos(2\pi(ft - 1/8)).1/\sqrt{2} = \sqrt{2}P_0 \cos(2\pi(ft - 1/8))$
Hence the amplitude is $\sqrt{2}P_0$ = 1.4 P_0

(iii) α=1/3 so cos($\pi\alpha$)=cos(π/3)=1/2 so that
$P = 2P_0 \cos(2\pi(ft - 1/6)).1/2 = P_0 \cos(2\pi(ft - 1/6))$
Hence the amplitude is P_0

Q5 (a). f_{source} = 262.0 Hz, v_{source} = 3 m/s, $v_{sound\ wave}$ = 340 m/s



For a moving musician and stationary observer we have:

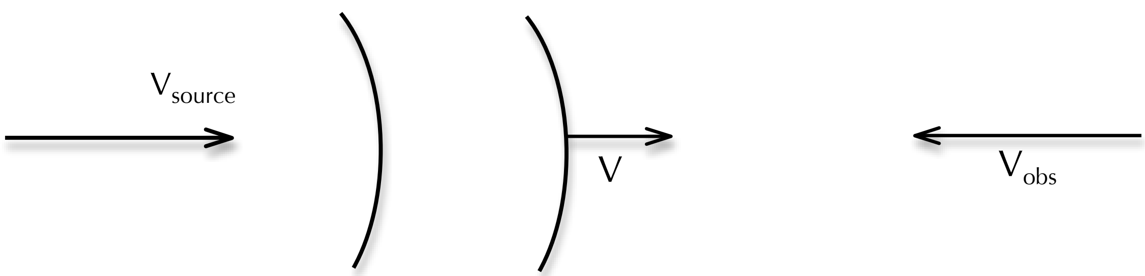
$$f_{observer} = f_{source}\frac{v}{v - v_{source}}$$ using the Doppler formula with $v_{Observer}$ = 0 m/s.

$$= 262\frac{340}{340 - 3} = 264.33 \text{ Hz}$$

So $f_{Observer}$ = 264.33 Hz. But f_{Source} = 262.0 Hz i.e. unchanged.

Thus, the difference in frequencies = 264.3 – 262.0
= 2.3 Hz to 2SF

(b) We now have $v_{observer}$ = 30 m/s moving towards the musician



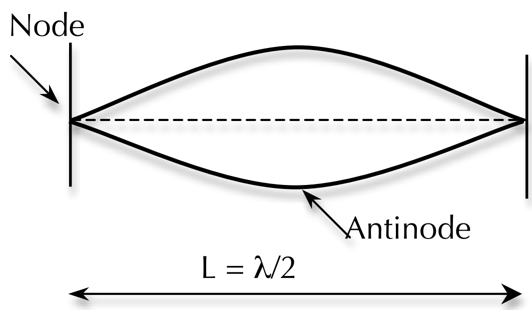
Thus, $f_{observer} = f_{source}\frac{v + v_{Observer}}{v - v_{source}}$ from the Doppler formula

For moving observer, stationary source $f_{Obs,1}$ = 262.(340+30)/340
= 285.12 Hz

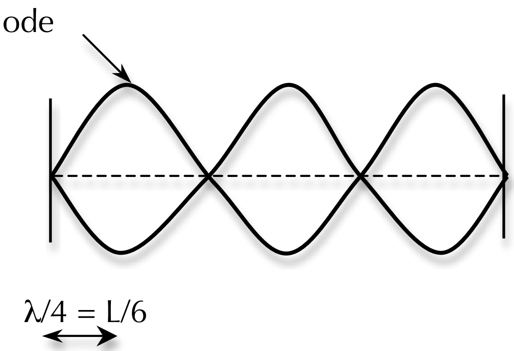
For moving observer, moving source $f_{Obs,2}$ = 262.(340+30)/(340-3)
= 287.66 Hz

Thus, the difference in frequencies Δf = 287.66 – 285.12
= 2.54 Hz
= 2.5 Hz to 2SF

(c) Antinode occurs where the string is plucked.
Node occurs at both fixed ends



Guitar plucked in middle at L/2



Guitar plucked at L/6

For guitar string plucked in middle $\dfrac{\lambda}{2} = L \Rightarrow \lambda = 2L$

For guitar string plucked at *L/6:* $\dfrac{\lambda}{4} = \dfrac{L}{6} \Rightarrow \lambda = \dfrac{2L}{3}$

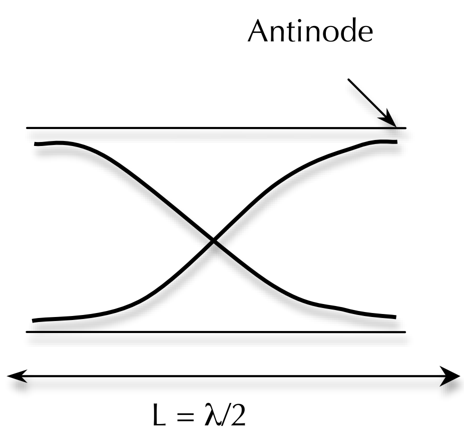
Since $c=f\lambda$ we have $f=c/2L$ in $L/2$ case
$f=3c/2L$ in $L/6$case

Hence, frequency increases by a factor $[3c/2L]$ / $[c/2L]$ = 3 times

(d) Organ pipe open at both ends. Thus antinodes at the ends.

End effects are ignored.

First resonant mode looks like:



$\therefore \lambda/2 = L \quad \Rightarrow \quad \lambda = 2L$

So, $f = v/\lambda = c/2L$

Thus $L = c/2f = 340/2\text{x}262\text{m} = 0.649\text{m}$

Thus, length of pipe = 0.65m to 2SF

(e) Speed of sound in air is proportional to $\sqrt{T}$ where T is measured in Kelvin.
We ignore the contraction of the pipe.

i.e. $\dfrac{v}{v_0} = \sqrt{\dfrac{T}{T_0}}$ where T is measured in Kelvin

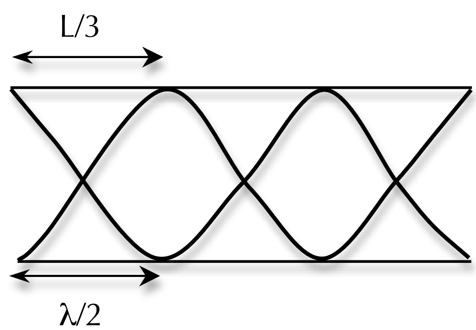
$$f = \frac{v}{L} = \frac{v_0}{2L}\left(\frac{T}{T_0}\right)^{0.5} = f_0\left(\frac{T}{T_0}\right)^{0.5}$$

Thus $f = 262\left(\dfrac{273+10}{273+27}\right)^{0.5}$ Hz

$$= 262\left(\frac{283}{300}\right)^{0.5} = 254.47 \text{ Hz}$$

So f=254.5 Hz to 1 d.p. or 254 Hz to 3SF

(f) With a hole in the pipe there is an antinode at that position, as well as at the ends.



Thus $\dfrac{\lambda}{2} = \dfrac{L}{3} \Rightarrow \lambda = \dfrac{2L}{3}$

Since $c=f\lambda$ we have $f=3c/2L = 3f_0$

i.e. the frequency is tripled to either 763 Hz (cooled case) or 786 Hz (original case).

6. (a) SHM: E=Total Energy $= PE + KE$
$\equiv$ Max value of PE
$\equiv$ Max value of KE

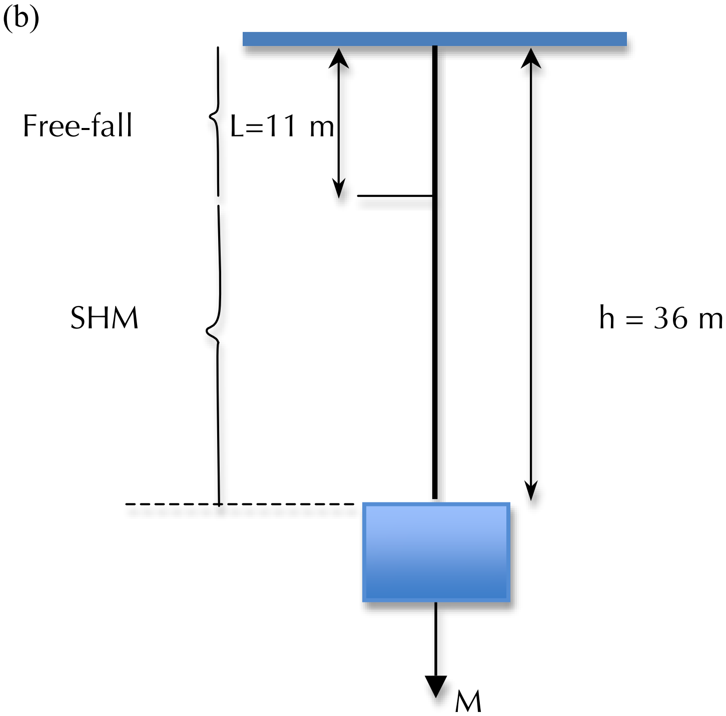
Suppose PE = ½ kx^2

Then $PE_{\max} = \frac{1}{2}kx_0^2 = E$ when x_0 is the maximum extension.

So when x = $x_0/2$ then $PE = \frac{1}{2}k(\frac{x_0}{2})^2 = \frac{1}{4}PE_{\max} = \frac{E}{4}$

Thus, KE when $x = x_o$ is $E\text{-}E/4 = 3E/4$.

(b)



(i) Block is in the free fall when only under the influence of gravity.
i.e., for the first 11.0m of the fall, when the cord does not retard its motion

So using "$s = ut + \frac{1}{2}at^2$" for this portion of the fall we have:

$$t = \left(\frac{2s}{a}\right)^{0.5} \text{ since } u = 0.$$

$$\therefore t = \left(\frac{2\times 11.0}{9.8}\right)^{0.5} \text{s} = 1.498\text{s} = 1.5\text{s to 1d.p.}$$

(ii) Conserving energy, and neglecting air resistance,
PE lost by falling block = PE in cord + KE of falling block.

At maximum distance, h, block no longer moving, so then has no KE,

Thus, $mgh = \frac{1}{2}k(h-L)^2 + 0$

So that $k = \dfrac{2mgh}{(h-L)^2} = \dfrac{2\cdot 65.0\cdot 9.8\cdot 36.0}{(36.0-11.0)^2}$ N/m

Hence k=73.38 N/m = 73 N/m to 2SF.

(iii) Equation of motion is given by N2L; i.e.
Mass x Acceleration = Force = Restoring Force Spring + Weight of Block
i.e. $m\ddot{x} = -kx + mg$

But $mg = kx_0$ for the equilibrium extension of the spring.

$\therefore m\ddot{x} = -k(x - x_0)$
$\therefore m\ddot{x}' = -kx'$
where $x' = x - x_0$ is the extension from equilibrium

This is the equation for SHM hence the motion is SMH.

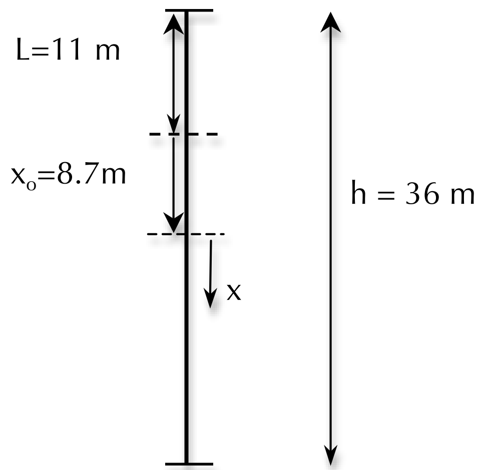
(iv) SHM with $\omega^2 = k/m = (73.4/65.0)^{0.5}$

$\therefore$ ω=1.06 rad/s = 1.1 rad/s to 1 d.p.

(v) Equilibrium extension of the spring given by:

$x_0 = \dfrac{mg}{k} = \dfrac{65.0\cdot 9.8}{73.38}$ m = 8.68m = 8.7m to 2SF

(vi) During the SHM portion of the motion the distance travelled is governed by $x = A\sin(\omega t)$ with x the extension from equilibrium and A the amplitude.



A = Total length – Equilibrium length = 25.0 – 8.7 = 16.3m
$A \equiv (h - L) - x_0$

We have two portions of the motion to consider:
Portion (a) to travel from equilibrium to the maximum extension. This takes $t=T/4$ where T is the period = $2\pi/\omega$ as it is one-quarter of the SHM period.

So $t=2\pi/4\omega = \pi/2\omega = \pi/1.06 = 1.48$s for portion (a).

Portion (b) to travel from x=-8.7m to x=0m.

We have -8.7 = 16.3 sin(ωt)
So $t = \frac{1}{\omega}\sin^{-1}(\frac{8.7}{16.3}) = 0.53s$ (taking the positive value) for portion (b).

Thus total time for chord to stretch 25m is 1.48+0.53=2.01s = 2.0s to 2SF

(vii) Total time for entire drop to h=36.0m is then given by the time to drop 11.0m (i.e. the free-fall) + time to stretch 25m (i.e. the SHM) = 1.498 + 2.01 = 3.508s = 3.5s to 2SF.