2009 Session 1 Test 2   Physics 1a

Question 1

One particle, mass m_1, is fired vertically upwards with an initial velocity v_0 from the ground (height $y = 0$). At the same time ($t = 0$), another particle, mass m_2, is released from rest at a height $y = h$, directly above the first, where $h > 0$. The particles are then in free fall near the earth's surface. Air resistance and the rotation of the earth may be neglected.

a) Write equations for the vertical positions y_1 and y_2 of the particles as functions of time t from their launch, before any collision.

b) Write an equation for the position y_{cm} of the centre of mass of the two particles as a function of time. Simplify your expression by removing any common factors.

c) Derive an expression for the velocity $\underline{\mathbf{v}}_{cm}$ of the centre of mass.

d) Hence derive an expression for the acceleration $\underline{\mathbf{a}}_{cm}$ of the centre of mass.

e) What are the conditions for the two particles to collide before they hit the ground? Derive an inequality relating v_0 to other parameters. (Hint: when do they collide?) [*This part might be 1131 only*]

f) Assume that they collide and that the collision is very brief and completely inelastic. Write an expression for position of the particles after the collision but before they hit the ground.

g) From the results above, derive an expression for the total momentum of the two particles both before and after the collision but before they hit the ground.

h) Is momentum conserved during the flight of the particles? Comment briefly: one clear sentence will be sufficient.

a) $y_1 = v_0 t - \frac{1}{2} g t^2$ $y_2 = h - \frac{1}{2} g t^2$

b) $$y_{cm} = \frac{\sum_i m_i y_i}{\sum_i m_i} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{m_1(v_0 t - \frac{1}{2} g t^2) + m_2(h - \frac{1}{2} g t^2)}{m_1 + m_2}$$

$$= \frac{m_1 v_0 t + m_2 h}{m_1 + m_2} - \frac{1}{2} g t^2$$

c) $\underline{\mathbf{v}}_{cm} = \frac{dy_{cm}}{dt}$ in the y direction

$$= \frac{m_1 v_0}{m_1 + m_2} - gt$$ in the y direction

d) $\underline{\mathbf{a}}_{cm} = \frac{dv_{cm}}{dt}$ in the y direction $= -g$ in the y direction.

e) At collision, $y_1 = y_2$

$v_0 t - \frac{1}{2} g t^2 = h - \frac{1}{2} g t^2$

$t = h/v_0$

so they collide at $y_1 = y_2 = h - \frac{1}{2} g\left(\frac{h}{v_0}\right)^2$ *OR* $v_0\left(\frac{h}{v_0}\right) - \frac{1}{2} g\left(\frac{h}{v_0}\right)^2$

If this height is above ground,

$h - \frac{1}{2} g\left(\frac{h}{v_0}\right)^2 > 0$

$1 > \frac{gh}{2v_0^2}$ so $v_0 > \sqrt{\frac{gh}{2}}$

f) After a completely inelastic collision, there is no motion of component parts with respect to the centre of mass. Hence

$$y_{both} = \frac{m_1 v_0 t + m_2 h}{m_1 + m_2} - \frac{1}{2} g t^2.$$ (i.e. as in part b)

g) The total momentum is $m_{total} \underline{\mathbf{v}}_{cm}$

So, using (c), the total momentum is

$$\mathbf{p} = (m_1 + m_2)\underline{\mathbf{v}}_{cm} = (m_1 + m_2)\left(\frac{m_1 v_0}{m_1 + m_2} - gt\right)$$ in the y direction

$= m_1 v_0 - (m_1 + m_2)gt$ in the y direction

h) No. The momentum increases steadily in the downwards direction because an external force (gravity) acts on the system.

(Not required: *For a sufficiently brief interval, such as the collision, the effect of external forces is negligible. So, during the collision, momentum is approximately conserved.)*

a) $y_1 = v_0 t - \frac{1}{2} gt^2$ $\qquad$ $y_2 = h - \frac{1}{2} gt^2$

b) $$y_{cm} = \frac{\sum_i m_i y_i}{\sum_i m_i} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{m_1(v_0 t - \frac{1}{2} gt^2) + m_2(h - \frac{1}{2} gt^2)}{m_1 + m_2}$$

$$= \frac{m_1 v_0 t + m_2 h}{m_1 + m_2} - \frac{1}{2} gt^2$$

c) $\underline{\mathbf{v}}_{cm} = \frac{dy_{cm}}{dt}$ in the y direction

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Question 2

a) Define the moment of inertia. If your definition is an equation, define the terms used.

b) State the law of conservation of mechanical energy, including any conditions.

c) A ball bearing (solid, uniform, spherical) of mass m and radius r rolls without slipping down a plane inclined at α to the horizontal. The bearing starts from rest at height h.

Derive an expression for its speed at height zero. Be careful to justify any principle you use, and to state it carefully. In particular, mention friction.

a) The moment of inertia of a collection of masses m_i located at minimum distance r_i from the axis of rotation is

$$I = \Sigma m_i r_i^2$$

The moment of inertia of a body is

$$I = \int_{body} dm.r^2$$ (only need one for full marks)

where r is the minimum distance from the axis of rotation.

b) If nonconservative forces do no work, mechanical energy is conserved. [*or equivalent*]

c) Here, friction is non-zero, but because there is no relative velocity at the point of contact, friction does no work. *or equivalent*]

Because nonconservative forces do no work, mechanical energy is conserved.

$U_i + K_i = U_f + K_f$ OR $\Delta U + \Delta K = 0$

$$mgh + 0 = 0 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

If it rolls without slipping, then its speed $v = r\omega$. The moment of inertia of a sphere (from formula sheet) $= \frac{2}{5}mr^2$. Substituting:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2$$ so

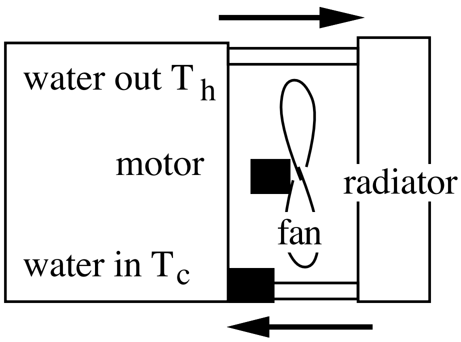
$$gh = \frac{1}{2}v^2 + \frac{1}{5}v^2 = \frac{7}{10}v^2$$

$$v = \sqrt{\frac{10\,gh}{7}}$$

Question N+2

When a car is travelling at a constant speed of v = 100 k.p.h. on a level road, its motor produces P_{level} = 20 kW of mechanical power and we assume that all of this power is delivered to the wheels.

a) Assume that the only drag acting on the car is air resistance, and that this force is proportional to v^2 i.e. $F_{drag} = Kv^2$. Using the data given above, calculate the constant of proportionality K and the drag force acting on the car at 100 k.p.h.



The same car motor has an efficiency of 25%, which we assume to be constant, independent of the speed of the car. So, when it produces 20 kW of power, it produces heat at a rate of 60 kW. We assume that all of this heat is transferred by water in the cooling system to the radiator, where it is transferred to the air passing through the radiator, as indicated in the sketch, where arrows show the direction of water flow.

The car has been travelling long enough for its thermal variables to be in steady state, and the temperature of the water entering the radiator is T_h = 100°C and it is at atmospheric pressure. The rate of evaporation from the cooling system is negligible.

The car, whose mass is 1300 kg, then starts climbing a mountain, with a steady rise of 1.0 m for every 20 m travelled along the road.

b) The driver increases his force on the accelerator so that the car continues travelling at 100 kph. Determine the rate at which the gravitational potential energy of the car is increasing.

c) The pressure in the radiator remains at atmospheric, the distribution of temperature in the radiator is unchanged and so the radiator continues to transfer heat to the air at the same rate. Determine the rate of evaporation of water from the radiator during this climb.

(The latent heat of vaporisation of water is 2.26 MJ.kg^{-1}.)

d) Derive an equation that relates parameters of this problem to the maximum speed v_{max} at which the car can drive up the mountain without boiling away water from the cooling system. State the meaning of any symbols in your equation that are not defined above. *You are not asked to solve this equation for* v_{max}.

a) No acceleration, so $\Sigma\mathbf{F} = 0$, so $F = F_{drag}$ so

$$P_{drag} = Fv = F_{drag}v = Kv^3$$

$$K = (P/v^3) = \frac{20\text{ kW}}{(100*1000\text{m}/3600\text{ s})^3} = 0.93\text{ Ns}^2.\text{m}^{-2}$$

$$F = Kv^2 = 7120\text{ N}.$$

b) The car climbs 0.05 m for every 1 m travelled so it is travelling upwards at 5 kph. It is gaining gravitational potential energy at a rate

$$P_{grav} = mg\frac{dh}{dt} = (1300\text{ kg})(9.8\text{ m.s}^{-2})(\frac{5*1000\text{m}}{3600\text{ s}}) = 18\text{ kW}.$$

c) The motor must now supply 20 kW to move air and 18 kW to climb the mountain, so it is producing 38 kW. At an effiency of 25%, it therefore produces heat at a rate $P_h = 3*38$ kW = 114 kW. At this speed, the radiator dissipates heat at 60 kW to the air, so it will provide 54 kW to boil water at 100°C.

The heat Q required to evaporate a mass M of water (at 100°C and atmospheric pressure) is by defintion LM. So

$$L\frac{dM}{dT} = 54\text{ kW}$$

$$\frac{dM}{dT} = \frac{54\text{ kW}}{2.26\text{ MJ.kg}^{-1}} = 0.024\text{ kg.s}^{-1} \quad (= 24\text{ g.s}^{1} \rightarrow 1.4\text{ litres per minute....})$$

d) The radiator can dissipate 60 kW without boiling. So the car can only produce $P_{max} = 20$ kW of mechanical power without boiling. So

$$P_{max} = P_{drag} + P_{grav}$$

$$P_{max} = Kv_{max}^3 + mg\frac{dh}{dt}$$

$$P_{max} = Kv_{max}^3 + mgsv_{max}$$

where s is the rate of rise of the road, here 1/20.

(*Not required: This could be easily solved with pen and paper:*

Plot Kv^2 *against* v, *and* $P_{max} - mgsv$ *against* v *on the same graph. The intersection is at* v_{max}.

This stuff not used

a) Define the centre of mass. If your definition is an equation, define the terms used.

a) The centre of mass of a collection of masses m_i located at $\mathbf{r}_i$ is a point whose position is

$$\mathbf{r}_{cm} = \frac{\sum_i m_i \mathbf{r}_i}{\sum_i m_i}$$ OR (only need one for full marks)

The centre of mass of a body of mass m is a point whose position is

$$\mathbf{r}_{cm} = \frac{\int_{body} dm\, \mathbf{r}}{m}$$