## Answer 1

Let primed quantities indicate the values after the switch is in the right-most position for the first time. Double primed quantities are after the switch is in the right-most position for the second time.

(a) Initially, the charge on capacitor $C_1$ is $Q_1 = VC_1$, and the charge on the other two capacitors is zero. By conservation of charge, after the switch is set to the right, the final total charge remains the same. The equivalent capacitance is the parallel combination of all three, so the final charge is $Q_1 = V'(C_1 + C_2 + C_3)$. Therefore

$$V' = \frac{VC_1}{C_1 + C_2 + C_3}.$$

(b) The charges on the three capacitors are simply calculated by multiplying each capacitance by the final voltage, found in part (a), i.e.,

$$Q'_1 = \frac{VC_1^2}{C_1 + C_2 + C_3}$$
$$Q'_2 = \frac{VC_1C_2}{C_1 + C_2 + C_3}$$
$$Q'_3 = \frac{VC_1C_3}{C_1 + C_2 + C_3}$$

(c) $C_1$ will now transfer an addition $VC_1$ coulombs of charge to the other two capacitors. The new total charge will therefore be charge on $C_1$ and $C_2$.

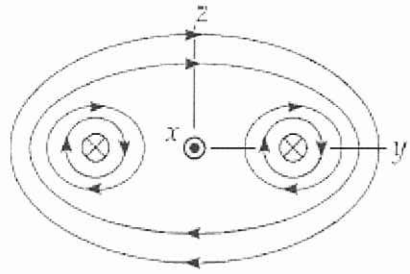
charge on $C_1$ $\rightarrow$ $$VC_1 + V'C_2 + V'C_3 = \frac{V(C_1 + 2C_2 + 2C_3)}{C_1 + C_2 + C_3}$$ sub in $V'$ from part (a)

and the new total voltage is simply this divided by the total capacitance, i.e.,

$$V'' = \frac{V(C_1 + 2C_2 + 2C_3)\,C_1}{(C_1 + C_2 + C_3)^2}$$

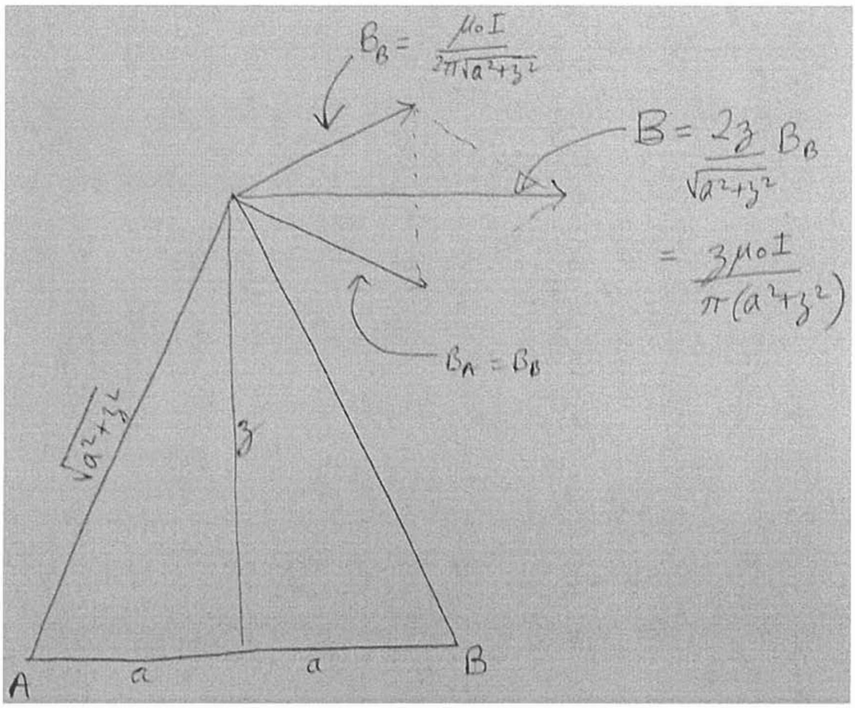
## Answer 2

(a) Note: it is important that students show at least one line of magnetic flux around each wire, and around both wires




(b) 0

(c) The magnetic field from a single wire is $B = \mu_0 i/2\pi R$. In this case we have two wires, and we have to add their contributions vectorially. See the Figure below. $B_B$ is the magnetic field due to wire B. $B_A$ is the magnetic field due to wire A, and is equal in magnitude to $B_B$. The total magnetic field is found by vectorially adding $B_A$ and $B_B$. By geometry we can see that the direction of the final magnetic flux $B$ is as shown, parallel to the $y$ axis. The component of $B_B$ in this direction is $B_B \times z/\sqrt{a^2+z^2}$, where the term to the right of the "×" comes from looking at similar triangles.



The final result is

$$B(z) = \frac{z\mu_0 I}{\pi(a^2 + z^2)}$$

and substituting the numerical values we get

$$B(z) = \frac{3.78 \times 10^{-6} z}{1.502 \times 10^{-3} + z^2}$$

in the $+y$ direction.

(d) To find the maximum value we first evaluate $dB/dz$ and set it to zero.

$$\begin{aligned} \frac{dB}{dz} &= \frac{\mu_0 I}{\pi}\left[\frac{1}{a^2 + z^2} - \frac{2z^2}{(a^2 + z^2)^2}\right] \\ &= \frac{\mu_0 I}{\pi}\left[\frac{a^2 - z^2}{(a^2 + z^2)^2}\right]. \end{aligned}$$

which is clearly zero when $z = a$, i.e., $z = 3.88$ cm.

(e) Substituting the value $z = 3.88$ cm into the equation found in part (c) gives $B = 48.8\mu$T in the $+y$ direction

## Answer 3

(a) As the loop drops, the magnetic flux through it is increasing. By Lenz's Law a current will be induced that will oppose the change in flux, i.e., will produce a magnetic field pointing *into* the page. By the right-hand rule this requires a CW (clockwise) current. [4 marks]

(b) The magnetic flux is given by $B$ times the area. $B$ is constant, but the area is increasing by $lv$ m$^2$/s. The rate of change of magnetic flux is therefore

$$\frac{d\phi_B}{dt} = Blw$$

(c) By Faraday's Law, the emf $\epsilon$ generated in the loop is simply the rate of change of magnetic flux. This emf drives a current $I = \epsilon/R = Blv/R$ around the loop. The power dissipated is the product of the emf and the current,

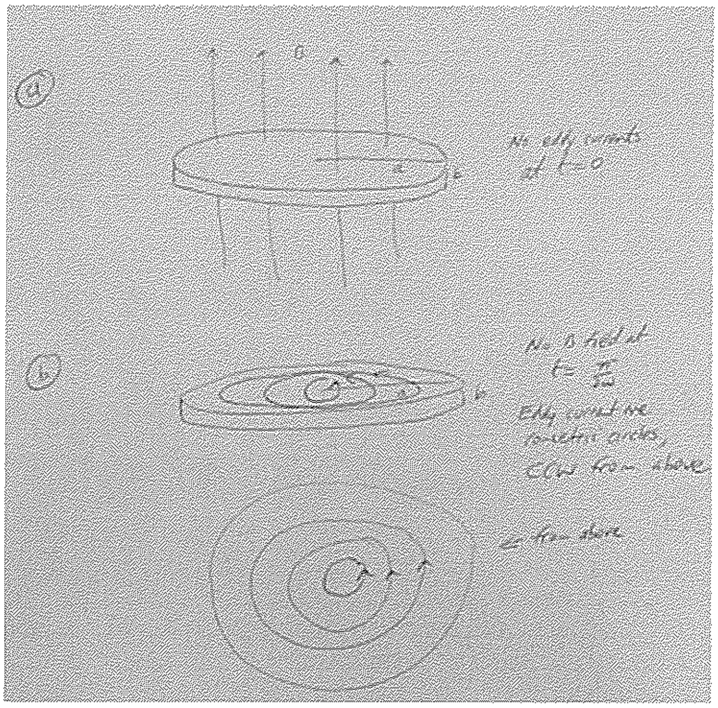
$$P = \frac{(Blw)^2}{R}$$

(d) The magnetic force acting on the loop is simply the force on the horizontal side within the magnetic field, and is $BIl = B^2l^2v/R$. At the terminal speed, this equals the gravitational force $Mg$, so

$$v_T = \frac{RMg}{B^2 w} \; 2$$

(e) If a loop of $N$ turns was used, the emf generated would be $N$ times as large, the resistance of the coil would be $N$ times larger, and so the current $I$ would be the same. The magnetic force would be $N$ times larger since there are now $N$ parallel wires each with the same current moving through the uniform $B$ field. The gravitational force would also be $N$ times larger, since the mass has now gone up by a factor of $N$. So the net result is no change in the terminal velocity

## Answer 4




(a) as above

(b) as above

(c) Divide the pan up into rings of radial thickness $dr$ and height $b$. The resistance of each ring is (as given in the hint)

$$R = \frac{2\pi r \rho}{b dr}$$

the power dissipated in each ring is $\epsilon^2/R$ where $\epsilon$ is the enduced emf, given by Faraday's Law

$$\epsilon = -\frac{d\phi_B}{dt} = \pi r^2 \omega B_{\max} \sin \omega t$$

so the power is

$$dP = \frac{b\pi^2\omega^2 B_{\max}^2 \sin^2 \omega t}{2\pi\rho} r^3 dr$$

To average over time we need to find the average of $\sin^2 \omega t$, which is just $1/2$. We now need to integrate over the disk, from $r = 0$ to $r = a$, which gives

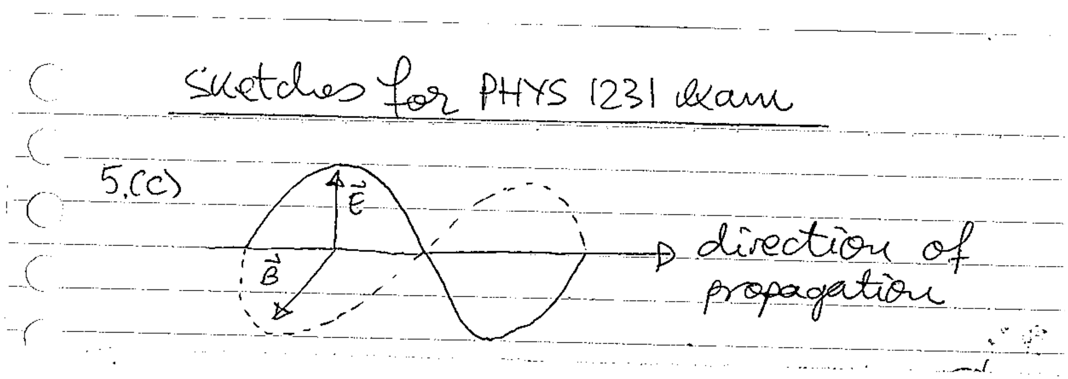
$$P = \pi B_{\max}^2 a^4 b \frac{\omega^2}{16\rho}$$

**Solutions to 1231 Final Exam**

5. (a) The speed of light is read off as $1/\sqrt{\mu_0\varepsilon_0}$ . Its numerical value is 3 x $10^8$ m/s (299792458 also accepted) .

(b) $E_0$ cos(kx - ωt) since it has to be $E_0$ at x=0, t =0 . [$E_0$ sin(kx - ωt) is not correct].

(c) An electromagnetic wave consists of an oscillating electric field and an oscillating magnetic field which are perpendicular to each other and perpendicular to the direction of propagation. Sketch attached.




(d) Energy flows along z.

(e) The sail is perfectly reflecting, so the force is (2/c) $dT_{ER}$/dt .

$$F = \frac{2}{c}\frac{dT_{ER}}{dt} = \frac{2}{c} \times I \times A = \frac{2}{c} \times 1400\,Wm^{-2} \times 9 \times 10^4\,m^2 = 0.84\,N$$

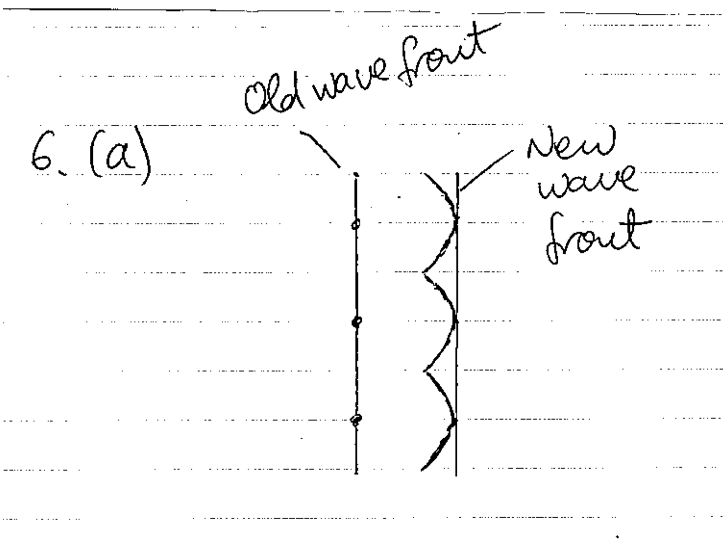
The acceleration is found by

$$a = \frac{F}{m} = \frac{0.84}{900} = 9.33 \times 10^{-4}\, ms^{-2}$$

(5% error tolerance on all the numbers)

(f) x = 1/2 at$^2$ (1), so $t = \sqrt{\frac{2x}{a}} = 4.63 \times 10^5 s$ .

6. (a) Each point on the wave front acts as a secondary source of spherical (circular) waves. Sketch attached.




(b) λ/n = 436.2 nm (5% tolerance)

(c) Sketch attached .

6. (c)

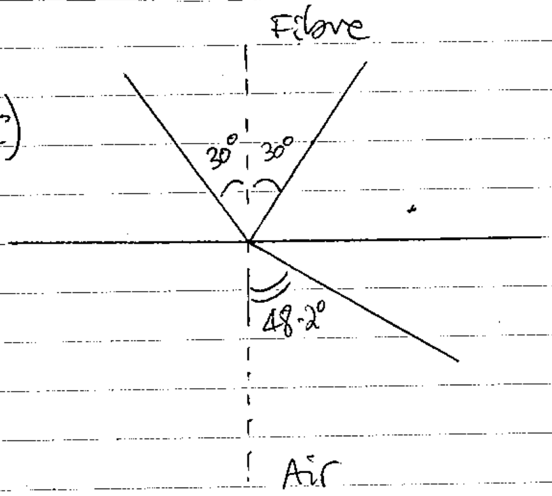



Figure OK as long as the angle of refraction is shown to be larger than the angle of incidence

Angle of reflection 30°.

For the angle of refraction:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$n_1 \sin 30° = \sin\theta_2$$

$$0.745 = \sin\theta_2$$

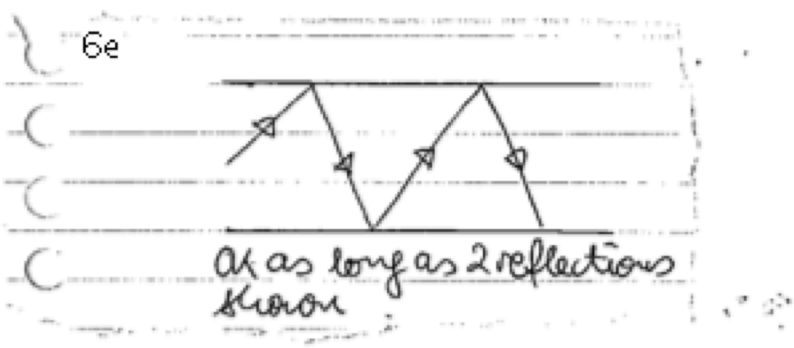
$$\theta_2 = 48.2°$$

(d) The critical angle is

$$\sin\theta_c = \frac{1}{1.49}$$

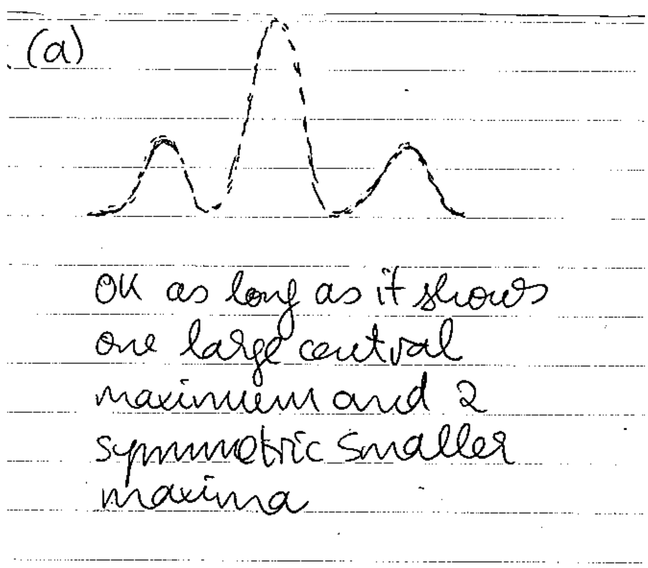
$$\theta_c = 42.2°$$

(e) The light is totally internally reflected inside the fibre .
Sketch attached.




(f) The critical angle for blue light is 41.5 degrees.

7. (a) Sketch attached .



(b) Need to mention constructive interference: peak + peak, trough + trough , and destructive interference (peak + trough) .

(c) n λ = 2 d sin θ, where n is an integer.

(d) As in the example of NaCl discussed in class:

$$a^3 = \frac{(4\times 24 + 4\times 16)g}{3.58gcm^{-3}\times 6\times 10^{23}} = 7.45\times 10^{-23}\,cm^3$$
$$a = 4.21\times 10^{-10}\,m$$

(e) d = a/2 = 2.11 x $10^{-10}$ m

(f) Assuming the angle θ is small, the angular separation of the maxima n and n+1 is given by

$$\sin\theta \approx \theta$$
$$\theta_{n+1} - \theta_n \approx \frac{\lambda}{2d} = \frac{1.5\times 10^{-10}}{4.21\times 10^{-10}} = 0.356\ rad$$

Or 20.4° also accepted.

(g) The spatial separation is

$$\Delta y = L\tan(\theta_{n+1} - \theta_n) \approx L(\theta_{n+1} - \theta_n) = 1\times 0.356 = 0.356m$$

8. (a) (1/2) $kx^2$

(b)

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \frac{1}{2}kx^2\psi = \varepsilon\psi$$

$\varepsilon$ is the energy.

(c)

$$\psi = ae^{-bx^2}$$

$$\frac{d\psi}{dx} = -2abxe^{-bx^2}$$

$$\frac{d^2\psi}{dx^2} = (-2ab)(1-2x^2b)e^{-bx^2}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \frac{1}{2}kx^2\psi = \frac{\hbar^2 ab}{m}(1-2x^2b)e^{-bx^2} + \frac{1}{2}kx^2ae^{-bx^2}$$

$$= [\frac{\hbar^2 b}{m}(1-2x^2b) + \frac{1}{2}kx^2]ae^{-bx^2}$$

This satisfies the Schrodinger equation if

$$\frac{\hbar^2 b}{m}(2x^2b) = \frac{1}{2}kx^2$$

$$b^2 = \frac{km}{4\hbar^2} \Rightarrow b = \frac{1}{2\hbar}\sqrt{km}$$

This result is acceptable. One can also remember that $\omega = \sqrt{\frac{k}{m}}$ and write

$$b = \frac{m\omega}{2\hbar}$$

(1 mark)

$$\varepsilon = \frac{\hbar\omega}{2}$$

(d) Same method as above. Show it satisfies the Schrodinger equation.

$$d = b$$

$$\varepsilon = \frac{3}{2}\hbar\omega$$

(e) The wave function of part (c) is for the ground state, that of part (d) for the first excited state.

(f)

$$\varepsilon = (n + \frac{1}{2})\hbar\omega$$