TEST 2

Formulae and data

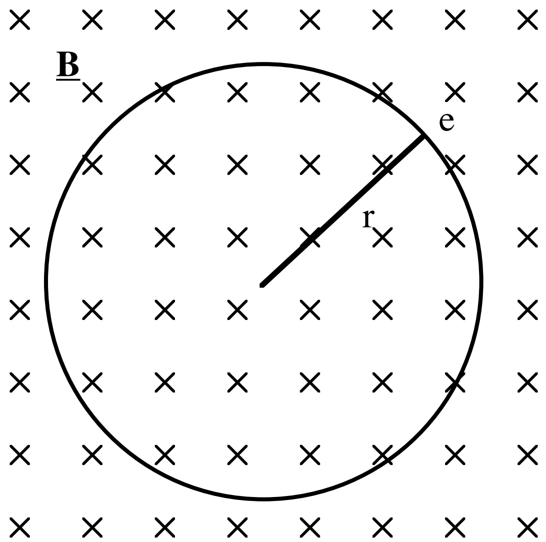
$$E = \frac{hc}{\lambda} \qquad \sigma = ne\mu = \frac{ne^2\tau}{m} \qquad g = \sigma\frac{A}{L} \qquad I = nAve$$

$N_A = 6.023x10^{23}\left(mol^{-1}\right)$

$\mathbf{F} = q\left(\mathbf{v}x\mathbf{B}\right)$

For hydrogen : $E_0 = \dfrac{e^4 m}{32\pi^2\varepsilon_0^2\hbar^2} = \dfrac{e^4 m}{8\pi^2\varepsilon_0^2 h^2} = 13.60ev; \quad a_0 = 0.53x10^{-10}m$

QUESTION 1 **[Marks 12]**



Background to this question (not for marks).

The van Allen belts consist of a plasma trapped in the Earth's magnetic field. One of their effects is to absorb some electromagnetic radiation before it reaches Earth, with important implications for human survival.

In the semi-classical model, as used in the Bohr-Sommerfeld model of hydrogen, electrons are depicted as point charges travelling in circles at non-relativistic speeds.

Here you will begin by making the same assumptions. You may also assume that the potential energy of the electron is zero everywhere and is independent of its motion.

(i) Derive a relation between the radius r of the orbit and the speed v of a single, isolated electron (mass m_e) in a circular orbit about the field axis of a magnetic field **B** that is uniform. (In case you need reminding, the force on a charge q in a magnetic field is q **v** x **B**.)

(ii) Use the de Broglie wavelength to derive an expression for the values of the radius of the circular orbit at which the de Broglie's waves for the electron give constructive interference.

(iii) Derive an expression for the energies E(n) of the allowed orbits in these semi-classical orbits.

(iv) How does the energy of the orbit depend on radius? In one or two sentences compare your answer to this part of the question with that for the Bohr-Sommerfeld model of hydrogen.

(v) In practice, the orbits are not circular: an electron can have a constant component of velocity parallel to the field lines, in addition to the tangential velocity component discussed so far. Consequently the orbits may be in the shape of a helix. Comment on how this affects the quantisation of energy of the orbits.

[m_e = 9.1 x10^{-31} kg   h = 6.63 x10^{-34} Js   e = 1.6 x10^{-19} C]

QUESTION 2 **[Marks 10]**

(i) Sketch a clearly labelled diagram showing the principal elements of an experiment involving light that demonstrates an effect associated with ***waves***. Name the effect, and explain very briefly what happens and why this demonstrates wave-like properties. (Your explanation could be several sentences, ***or*** it could be in point form. In either case, reference to your sketch or to another diagram may be useful.)

(ii) Sketch a clearly labelled diagram showing the principal elements of an experiment involving light that demonstrates an effect associated with ***particles***. Name the effect, and explain very briefly what happens and why this demonstrates properties normally associated with particles. (Your explanation could be several sentences, ***or*** it could be in point form.)

QUESTION 3 **[Marks 18]**

(a) The Fermi-Dirac distribution function for the distribution of electrons of energy E in a solid with Fermi energy E_F is

$$f(E) = \frac{1}{e^{(E - E_f)/kT} + 1}$$

where k is Boltzmann's constant and T the temperature.

(i) Sketch f(E) for the two cases: $T = 0$ and $T > 0$.

(ii) Describe in one or two clear sentences the physical significance of the Fermi energy E_F.

(iii) In the case $E >> E_F$ and $E >> kT$, derive a simplified approximation for f(E)

(iv) For a typical metal, E_F is several eV. Showing your calculation, say whether the conditions described in (iii) are satisfied at room temperature.

(b) (i) Researchers of nanostructures sometimes constrain an electron in a small volume, called a quantum dot, which is actually a potential well. A particular well has physical dimensions of approximately 10 nm. Use Heisenberg's uncertainty principle to estimate the minimum 'height' of the potential (i.e. the minimum magnitude of the energy barrier) required to restrain the electron to remain in the quantum dot. Be careful to explain the steps in your argument. Express your answer in electron volts.

(ii) Is the barrier height likely to be the limiting factor? Discuss briefly in terms of temperature and the energy associated with thermal energy.

(c) (i) Consider an electron constrained to a large cubical 'box' of side L. The electron is 'trapped' in the box, ie its wave function is zero at the walls of the box. What is the longest wavelength λ for an electron having momentum only in the x direction, parallel to a side of the box, that satisfies that condition?

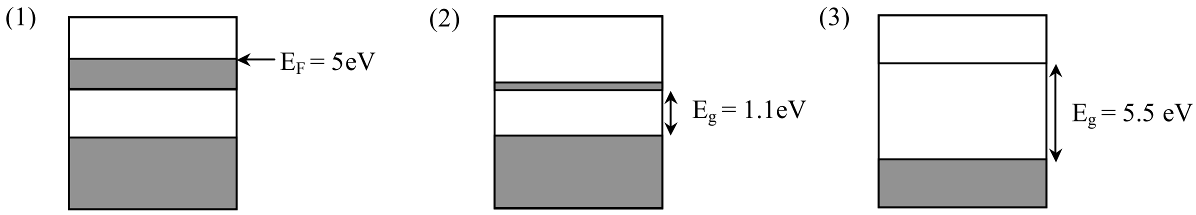
(ii) Write an expression for all wavelengths λ_n that satisfy that condition, again for momentum only in the x direction, parallel to a side of the box.

(iii) Hence write an expression for all values $p_x(n_x)$ of the momentum that satisfy that condition.

(iv) Hence derive an expression for the allowed energy levels E(n) for electrons in the box, no longer constrained in direction, where $n^2 = n_x^2 + n_y^2 + n_z^2$ and where n_x, n_y and n_z are integers.

QUESTION 4 **[Marks 10]**

(a) Three materials have the energy band structures shown schematically in the diagram below representing, (1) a metal, (2) an n-type doped semiconductor and (3) an insulator. The shaded areas indicate occupied (by electrons) energy ranges.



(i) For the metal shown in (1), find the Fermi velocity and the thermal velocity of the electrons at 300K.

(ii) Find the wavelength of EM radiation that will cause a sharp increase in the electrical conductivity of material (2).

(iii) By comparing the energy gap values for materials (2) and (3) state, with your reasoning, whether material (3) is expected to be transparent or opaque to visible light at room temperature. (The visible region of the EM spectrum spans the wavelength range λ=400nm to λ=700nm approx.)

(b) Copper (Cu) metal is monovalent (1 conduction electron per atom) and has density $\rho = 8.9\text{x}10^3$ kg.m^{-3} and atomic mass 63.5. Use this information to estimate the number of occupied electron energy levels in the conduction band of 1 mm^3 of copper at 300K.

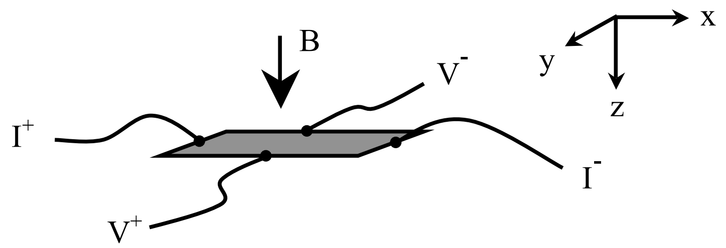
QUESTION 5 **[Marks 14]**

(a) The semiconductor indium antimonide (InSb) has energy gap $E_g = 0.23\,\text{eV}$. The dielectric constant of InSb is $\varepsilon = 17$ and the electron effective mass is $m_e^* = 0.014m_e$. Assuming a simple Bohr model calculate,

(i) the donor ionisation energy,

(ii) the radius of the ground state donor electron orbit.

(b) A Hall effect probe consisting of a thin, rectangular slice of silicon doped at a concentration $n = 1.0\text{x}10^{20}\,\text{m}^{-3}$ is used to measure the total stray magnetic field above an MRI imaging machine. The Hall probe is positioned so that the magnetic induction B is perpendicular to the face and along the thin direction (z-direction) of the slice, as shown below. The slice has dimensions length l (parallel to x-direction), width w (y-direction) and thickness t (z-direction). A constant current of 1.0 mA is passed from I^+ to I^-. The induced Hall voltage, V_H, is measured on wires labelled V^+ and V^-.



(i) Use the condition for equilibrium of the magnetic and electric forces on the charge carriers, and the free electron formulae given at the front of this paper to derive an expression which gives the Hall voltage in terms of the magnetic induction B_Z, the current I_X, the carrier concentration n, charge e and the sample dimensions.

(ii) When the silicon slice carries a constant current 0.1 mA (I^+ to I^-) a Hall voltage of magnitude 2.0 mV is measured. Calculate the value of the stray magnetic field B.

(iii) When the conventional current I^+ to I^- is in the +ve x-direction it is found that probe V^- is positive (with respect to V^+). Is the silicon slice p-type doped or n-type doped? You must provide your reasoning, a simple statement of the answer, p-type or n-type without a supporting argument will not receive marks. Your argument could be a series of dot points connected with "therefore" or a short paragraph of complete sentences.

QUESTION 6 **[Marks 16]**

(a) The current-voltage characteristic for an ideal diode in the forward direction is given by $I = I_0\left(e^{e\Delta V/k_B T} - 1\right)$

(i) Give the meaning of the symbols in this expression.

(ii) Provide a labeled sketch graph of the form of the current-voltage characteristic expected for an ideal diode.

(iii) Sketch a simple circuit diagram showing a forward-biased diode. Your *series* circuit should contain the diode, a battery and a single resistor R only.

(iv) The diode in the circuit of part (iii) is rated at 100mA maximum current. If the battery voltage is $\varepsilon = 1.5\ \text{V}$, calculate the minimum value of R required.

(b) The IR communication port on your PDA (e.g. like the Palm Pilot or pocket PC) operates with an infra-red LED. *Estimate* the band gap of the semiconductor required for the LED in this device.

(c) Light of wavelength $\lambda = 620\ \text{nm}$ $(1\text{nm} = 10^{-9}\text{m})$ and intensity $2.0\ \text{Wm}^{-2}$ is shone onto the surface of a photoconductive detector consisting of a rectangular specimen of semiconductor. The detector has an effective area 1.0x100.0 mm^2. The mobilities μ of the electrons and holes in the semiconductor are $0.001\ \text{m}^2\text{V}^{-1}\text{s}^{-1}$ and $1.0\ \text{m}^2\text{V}^{-1}\text{s}^{-1}$ respectively. Assuming each photon of light striking the semiconductor is absorbed and produces an electron-hole pair, calculate the change in electrical conductance produced by the illumination.

[Hint: the change in conductance is $\Delta g = \Delta\sigma\frac{A}{L}$. The conductivity is $\sigma = ne\mu$ where $n = \frac{\Delta N + \Delta P}{AL}$ for ΔN photoexcited electrons and ΔP photoexcited holes and $AL = V$ is the volume of the semiconductor detector.]

TEST 1R

QUESTION 7 **[Marks 18]**

A baby-bouncer consists of a flat seat of mass 5.0 kg hanging by four elastic cords from a small frame. A 12 kg baby is placed on the seat, and the cords stretch by a further 18 cm as a result.

(a) When the baby starts to cry, its father sets the bouncer in motion by pulling the seat down a further 10 cm and releasing it. Describe the resulting motion.

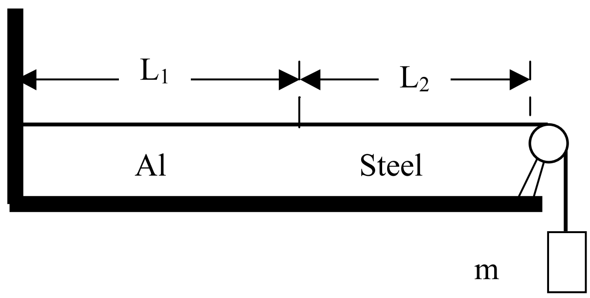
(b) Calculate the vertical oscillation frequency of the bouncer, assuming that the elastic cords are very light compared to the baby and the seat.

(c) What is the maximum velocity experienced by the baby during the oscillation? At what point (or points) during the motion does this maximum velocity occur?

(d) What is the maximum safe amplitude of oscillation, beyond which the baby will no longer remain sitting on the seat?

QUESTION 8 **[Marks 18]**

An aluminium wire of length L_1 = 60.0 cm and of cross-sectional area 1.00 x 10^{-2} cm^2 is connected to a steel wire of the same cross-sectional area. The compound wire; loaded with a block m of mass 10.0 kg is arranged as shown in the diagram so that the distance L_2 from the joint to the supporting pulley is 86.6 cm. Transverse waves are set up in the wire by using an external source of variable frequency.



(a) Find the lowest frequency of excitation for which standing waves are observed such that the joint in the wire is a node.

(b) What is the total number of nodes observed at this frequency, excluding the two at the ends of the wire?

[Note: The density of aluminium is 2.60 g cm^{-3}, and that of steel is 7.80 g cm^{-3}.]

QUESTION 9 **[Marks 18]**

(a) A racing car travels clockwise on a circular track 1 km in diameter. It is moving at constant speed and takes 45.0 seconds to complete each lap. The turbocharger on the car emits a loud whistle at 12 kHz.

An observer is standing outside the track at a point 100 metres from the edge of the track.

(i) What frequency does the observer hear when the car is directly opposite the observer (i.e., at its point of closest approach to the observer)?

(ii) What is the lowest frequency that the observer hears from the turbocharger?

(iii) At what point on the track is the car when the observer hears this frequency? Draw a diagram to illustrate your answer.

(b) The threshold of normal human hearing is about 1.00×10^{-12} W/m^2. Calculate the amplitude of the air vibrations at 4.00 kHz for this sound level. Take the density of air as 1.20 kg/m^3.

Compare this to the size of a nitrogen molecule (about 3×10^{-10} m) and comment.

QUESTION 10 **[Marks 16]**

The E field component of a particular light wave propagating in a block of glass is given by

$$E_y = 50 \sin \pi(4.50 \times 10^6\ x - 9.00 \times 10^{14}\ t)\ \text{V/m}$$

where x is displacement in metres and t is time in seconds.

(a) For this wave, find
  (i) the wavelength in the glass
  (ii) the wave number is the glass
  (iii) the frequency of the light in Hz
  (iv) the electric field amplitude

(b) What is the refractive index of the glass?

QUESTION 11 **[Marks 16]**

A plane wave of visible light shines vertically onto a horizontal glass plate. The plate is covered with a uniform, thin film of oil of thickness 670 nm. The index of refraction of the oil is 1.30, and that of the glass plate 1.45.
At what wavelengths between 400 and 750 nm will there be complete destructive interference of reflected light?

QUESTION 12 **[Marks 14]**

(a) A two-slit interference experiment (Young's experiment) is illuminated by light of wavelength $\lambda = 600$ nm. The slits are each 0.1 mm wide and have a centre-to-centre spacing of 0.5 mm. An interference pattern is observed on a screen 1.0 m away.

(i) Calculate the linear separation of the fringes on the screen.

(ii) Sketch a diagram showing the light intensity as a function of position on the screen, for the first 7 fringes to one side of the central maximum. Label all features of your sketch carefully.

(b) Astronomers observe a 60.0 MHz radio source both directly and by reflection from the sea. As the earth rotates its angle above the horizon changes. If the small receiving dish is 20.0 m above sea level, what is the angle of the radio source above the horizon when the receiver detects a first-order maximum?