Question 1 (16 marks)

In general, the velocity of propagation of a wave has the form

$$v = \sqrt{\frac{elastic \quad properety}{inertial \quad property}}$$

(a) Many animals, such as scorpions and ant lions, use the movement of waves through the ground to find their prey. An animal moving along the on ground produces both a transverse travelling wave and a longitudinal travelling wave. The longitudinal waves travel faster than the transverse waves, and a scorpion can tell where an insect is by detecting the difference in arrival time for the two waves.

i. What is the difference between a transverse wave and a longitudinal wave? Illustrate your answer with a diagram that clearly shows the two different types of waves mentioned above travelling through the ground. You must show the direction of propagation of both waves as well as the direction of movement of the "mass elements" of earth in both types of wave.

ii. For a wave travelling through the ground the inertial property is the density of the "mass elements" of earth. This is the same for both the transverse and the longitudinal wave, as they are moving through the same medium. Why then do the two waves have different velocities? Can you suggest how this arises by referring to the properties of the medium?

(b) Using Newton's second law, show that the speed of a wave on a stretched string is given by

$$v = \sqrt{\frac{T}{\mu}}$$

Question 2 (12 marks)

A sinusoidal wave is travelling along a string of length 3 m in the direction of decreasing x. The wave has a frequency of 50 Hz and amplitude of 20 cm. The string has a mass of 75 g and the tension in the string is 3.6 N. At time t=0 seconds, the medium has a displacement of +20cm at the x=0 position. Find:

i. The speed of the wave in the string.
ii. The wavelength
iii. The angular frequency.
iv. Write an equation describing the travelling wave.
v. Write down an expression that describes the ***displacement*** of an oscillating string element from the equilibrium position, ***as a function of time***, at the position x = 0.
vi. Use this expression (from v) to find the ***speed*** of the oscillating element in the string ***as a function of time***, and determine the maximum speed that the element will have.

Question 3 (14 marks)

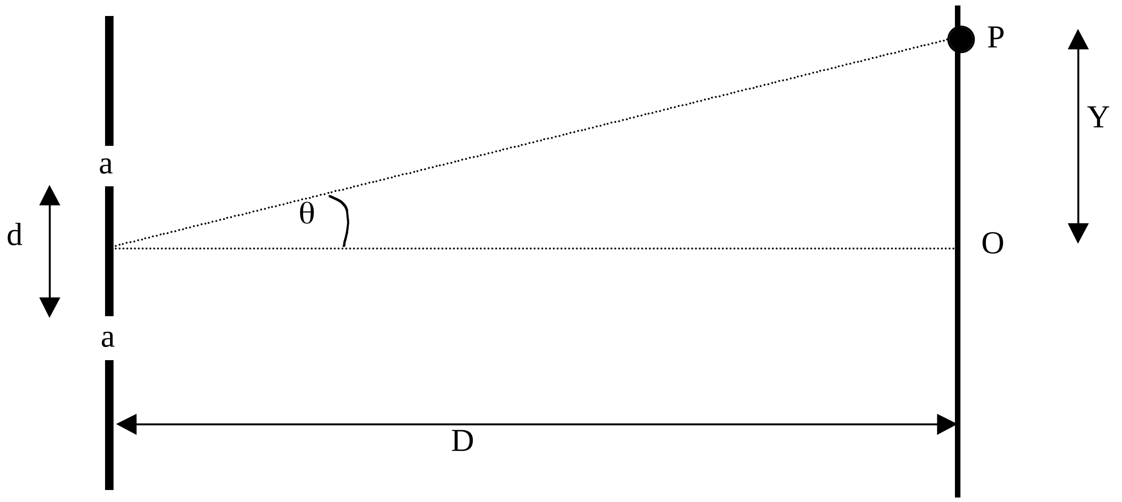
Plane parallel coherent light of wavelength λ is incident on vertically mounted double slit. The separation between the slits is d. The resulting interference pattern is viewed on a distant screen (also vertically mounted), a distance D from the slits. The slits are rectangular and have width a. (The double-slit set-up is illustrated in the figure below).

(a)
i. Draw a clearly labelled diagram of the two slits and the rays from each that are incident on the screen at a point P, an angle θ from the horizontal. Clearly indicate the path difference between the two rays on your diagram.
ii. State the value of the path difference in terms of d and θ.

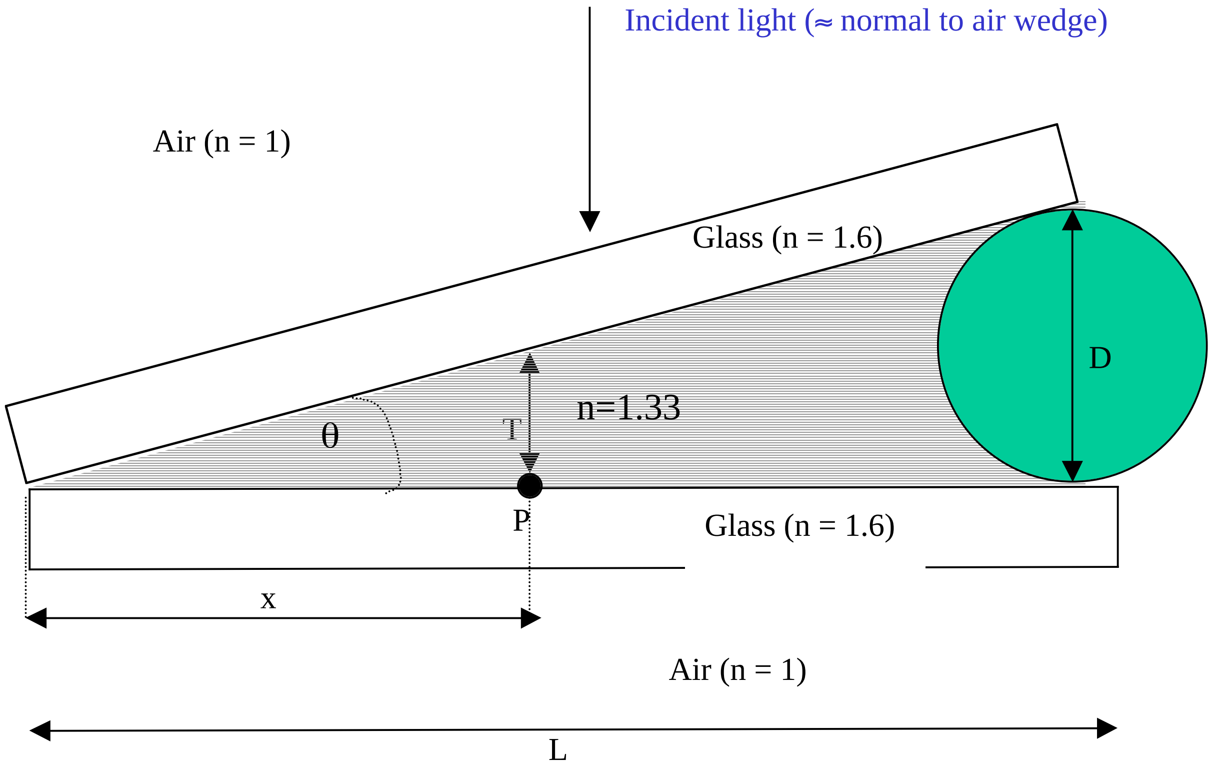
(b) Write down an expression that describes the path difference in terms of d, θ and λ if constructive interference is to result at point P.

(c) From your diagram and the diagram given below, derive an expression that describes the positions of the maxima on the screen in terms of d, D and λ, and Y, the linear height of P above the horizontal position on the screen directly opposite the midpoint of the two slits (point O in the diagram). You must show all steps in your derivation and state clearly any approximations you make in deriving this expression.

(d) You note when examining the double slit interference pattern on the screen that the 6^{th} bright fringe (not including the central fringe) is missing.
i. Sketch the intensity of the interference pattern that you see on the screen as a function of distance from the point O, including the effects of the single slit diffraction pattern due to the slit width a. Clearly mark the double-slit fringes and the single-slit intensity envelope.
ii. Find the width of the slits, a, in terms of their separation, d.



Question 4 (14 marks)



A broad source of light (λ_{air}= 680 nm) illuminates normally two glass plates (n = 1.6), 120 mm long, that touch at one end and are separated by a wire of diameter D at the other, as shown in the figure. The space between the plates is filled with water (n=1.33). One hundred and thirty (130) bright fringes appear over the 120 mm distance, with the last bright fringe being directly above the wire.

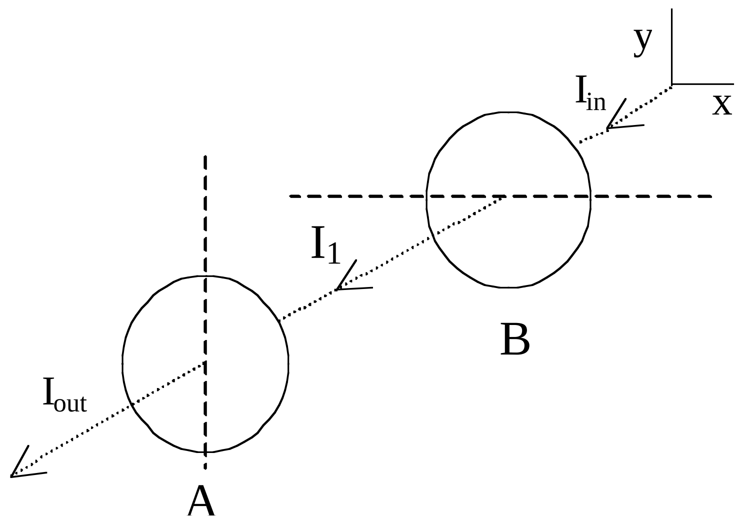
(a) On the diagram above, clearly mark the paths of the reflected rays of light that will produce thin film interference in this situation. The diagram must clearly show which interfaces the reflected rays arise from.

(b) At which interfaces will a phase change in the reflected rays occur? State the magnitude of any phase change at each interface.
(c) Do you expect a bright fringe or a dark fringe where the glass plates touch? Why?
(d) Find the path difference for light reflected from the lower surface compared to that reflected from the upper surface for any point P, a distance x from where the plates touch, in terms of the length of the plates, L, the diameter of the wire, D, the distance x, and the thickness of the thin film, T, at point P.
(e) What wavelength will the incident light have while propagating through the water?
(f) What value should this path difference have, in terms of the wavelength of the incident light in air, to produce a bright fringe?
(g) Find the diameter of the wire.

Question 5 (14 marks)

Light propagates as a transverse wave by means of oscillating electric and magnetic fields.

(a) Draw a diagram showing the relationship between the electric field, the magnetic field and the direction of propagation of the light wave.
(b) Explain what is meant by unpolarized light. You should illustrate your answer with a diagram that shows the direction of propagation of the light wave, and the direction of the electric field vectors. (Don't worry about including the magnetic field vectors.)
(c) Two parallel polarizing sheets are placed with their polarizing axes at right angles as shown in the figure. Unpolarised light (I_{in}) is incident on the polarizing sheet A.



I. What will be the intensity of the light (I_1) passed by the polariser B in terms of I_{in}?
II. Determine the intensity of the light transmitted by both A and B (I_{out}).
III. A third polarizing sheet, C, is inserted between A and B with its polarizing axis at 45^o to both A and B. Draw a labelled diagram showing this new arrangement, with the angles between the polarizing sheets clearly marked.
IV. Determine an expression for I_{out} in terms of I_{in} after this third sheet is inserted.

Given Formulae

$2d \sin\theta = m\lambda$ $\theta_{hw} = \lambda/(Nd)$

$\theta_{hw} = \lambda/(Nd \cos\theta)$ $d \sin\theta = m\lambda$

$\sin\theta = 1.22\, \lambda/d$ Optical path length = nL

$$I_r = 4I_0 \cos^2(\tfrac{1}{2}\phi) \qquad \phi = \frac{2\pi d \sin\theta}{\lambda} \qquad \lambda_n = \frac{\lambda}{n}$$

$$I = I_m\left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad \alpha = \frac{\pi a}{\lambda}\sin\theta$$

$$I = \frac{power}{area} \qquad \overline{P} = \frac{1}{2}\mu v\omega^2 A^2 \qquad \beta = \frac{-\Delta P}{\left(\frac{\Delta V}{V}\right)}$$

$$v = \sqrt{\frac{T}{\mu}} \qquad v = \sqrt{\frac{\beta}{\rho}} \qquad I = \frac{P}{4\pi r^2}$$

$$\frac{\Delta x}{\lambda} = \frac{\Delta t}{\tau} = \frac{\Delta\phi}{2\pi} \qquad f' \approx f\left(1 + \frac{u}{v}\right) \qquad \beta = 10 \log\frac{I}{I_0}$$

$$k = \frac{2\pi}{\lambda} \qquad f' = f\,\frac{v \pm v_D}{v \pm v_s}$$

$I_o = 10^{-12}\ \mathrm{Wm}^{-2}$

$v = f\lambda$

$Y = A \sin(kx - \omega t + \phi)$ $\omega = 2\pi f$

$\sin A + \sin B = 2 \sin \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$

$y_r = 2A \cos(\phi/2) \sin(kx - \omega t + \phi/2)$ $y_r = 2A \sin(kx) \cos(\omega t)$

$S(x,t) = S_m \cos(kx - \omega t)$. $\Delta P_m = (v\, \rho\, \omega)\, S_m$

$I = \frac{1}{2}\rho v \omega^2 S_m^2$

$f_{beat} = f_1 - f_2$ $I = I_o \cos^2\theta$ $\tan\theta_p = n_r / n_i$