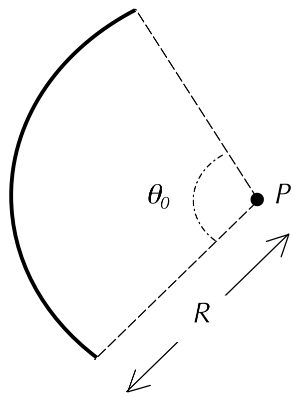
TEST 2

This test is on the final sections of this session's syllabus and should be attempted by all students.

QUESTION 1 **[Marks 23]**

A thin non-conducting rod is bent to form the arc of a circle of radius R and subtends an angle θ_0 at the centre of the circle P as shown in the diagram. A total charge $+q$ is spread uniformly along its length.



(a) Determine the electric field (intensity) $\boldsymbol{E}$ (magnitude and direction) at the point P due to the charge distribution.

(b) Determine the electric potential at the point P (relative to a point at infinity) due to the charge distribution.

(c) Determine the work required (by an external agent) to move a point charge +Q to the point P from a distance initially infinitely far from the above charge distribution.

QUESTION 2 **[Marks 27]**

An electron is accelerated from rest through a potential difference of 1.0 kV and directed into a region between two parallel metal plates separated by 20 mm with a potential difference of 100 V between them. The parallel plates are thin and very large compared to the separation between them and each plate has the same magnitude of surface charge density on it and the plates are in the horizontal plane.

(a) Determine the velocity of the electron before it enters the parallel plates.

(b) Determine the magnitude of the electric field (intensity) $\boldsymbol{E}$ between the parallel plates.

The electron now enters the region between the parallel plates moving perpendicular to the electric field between them.

(c) Draw a clear diagram showing the parallel plates, the direction of the electric field and the velocity vector of the electron on entering the plates.

A magnetic field is also applied between the parallel plates.

(d) Determine the magnetic field (induction) $\boldsymbol{B}$ (magnitude and direction) which is perpendicular to both the electron path and the electric field, such that the electron will travel in a straight line. Neglect the gravitational force on the electron.

(e) Show the direction of $\boldsymbol{B}$ from part (d) on your diagram in part (c).

(f) Use the Gauss Law to determine the numerical value of the surface charge density on the parallel plates. You must show all working including the Gaussian surface that you choose.

(g) Determine the electric field (intensity) $\boldsymbol{E}$
   (i) in the region above the parallel plate system due to the charge distribution on them,
   (ii) in the region below the parallel plate system due to the charge distribution on them. You must show all working and reasoning.

QUESTION 3 **[Marks 23]**

(a) (i) Sketch (i.e. not to scale) a diagram of the electric field (intensity) $\boldsymbol{E}$ pattern of two point charges $-q$ and $+q$ that are at a fixed separation d.

(ii) Write down the magnitude of the electric dipole moment of the charge configuration in (i).

(iii) Show on your diagram the direction of the electric dipole moment vector.

The two charges from (i) are now placed in an (external) uniform electric field (intensity) of magnitude E_1 such that the axis of the two charges is at an angle θ with respect to the direction of $\boldsymbol{E}_1$.

(iv) Draw a diagram of the orientation of the two charges in the electric field.

(v) Determine the forces (magnitude and direction) acting on each charge due to $\boldsymbol{E}_1$, in terms of the given quantities and show their directions on your diagram in part (iv). You must show all working.

(vi) When the charges (still at a fixed separation d) are free to move under the action of the forces in part (v) above, describe in words and with the aid of a precise diagram, the resulting motion of the two charges and show their equilibrium position.

(b) (i) Sketch (i.e. not to scale) a diagram of the magnetic field (induction) $\boldsymbol{B}$ pattern of a square loop of wire of side length a carrying a current I.

(ii) Write down the magnitude of the magnetic dipole moment of the current configuration in (b) (i).

(iii) Show on your diagram the direction of the magnetic dipole moment vector.

The current loop from (b) (i) is now placed in an (external) uniform magnetic field (induction) of magnitude B_1 such that the normal to the plane of the loop is at an angle θ with respect to the direction of $\boldsymbol{B}_1$.

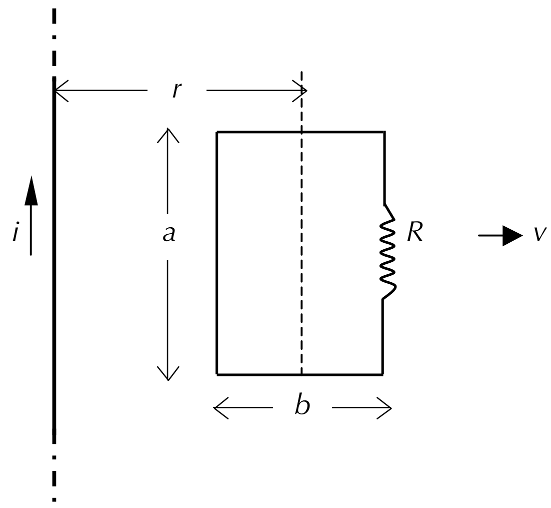
(iv) Draw a diagram of the orientation of the current loop in the magnetic field.

(v) Determine the forces (magnitude and direction) acting on each side of the current loop due to $\boldsymbol{B}_1$, in terms of the given quantities and show their directions on your diagram in part (b) (iv). You must show all working.

(vi) When the current loop is free to move under the action of the forces in part (b) (v) above, describe in words and with the aid of a precise diagram, the resulting motion of the current loop and show its equilibrium position.

QUESTION 4 **[Marks 27]**

A rectangular loop of wire with length a , width b , has a resistance R is placed near an infinitely long wire carrying current i as shown on the diagram.



The distance from the long wire to the centre of the loop is r.

(a) Use the Ampere Law to derive an expression for the magnetic field (induction) $\boldsymbol{B}$ at a perpendicular distance r from the long straight wire carrying current i . You must show all working including the Amperian loop that you choose.

In terms of the given symbols:

(b) Determine the magnitude of the magnetic flux through the loop.

(c) Determine the induced emf in the loop as it moves away from the long wire with speed v .
(Hint: From the chain rule $\frac{df}{dt} = \frac{df}{dr} \times \frac{dr}{dt}$.)

(d) Determine the magnitude of the induced current in the loop.

(e) Determine whether the current flows clockwise or anticlockwise as viewed in the diagram above. You must give full reasons for your answer.

TEST 1R

This is the repeat version of TEST 1, which was held during Session.

This repeat test should be attempted by those students who missed Test 1, or who wish to improve their mark in Test 1.

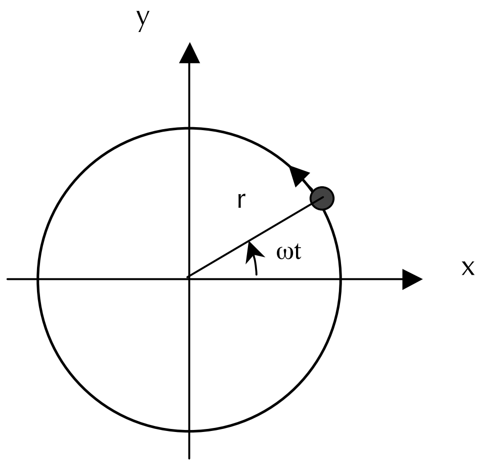
IF YOU ARE ATTEMPTING TEST 1 (Repeat):

CROSS THIS BOX ☐

AND INDICATE <u>YES</u> ON THE FRONT (TITLE) PAGE.

QUESTION 5 **[Marks 25]**

(a) A particle executes uniform circular motion with radius *r*, as shown in the figure.



(i) From the figure, derive an expression for the position vector $\vec{r}$ in terms of the radius *r*, the angular velocity ω, the time *t*, and unit vectors $\vec{i}$ and $\vec{j}$. Take the angle at time $t = 0$ to be zero.

(ii) From (i), derive the velocity $\vec{v}$ and acceleration $\vec{a}$ of the particle. Express the result for the acceleration in terms of the position vector.

(iii) Calculate the scalar product of the velocity and position vectors, $\vec{v} \cdot \vec{r}$. What does this say about the relative orientation of $\vec{v}$ and $\vec{r}$?

(b) A stone is tied to a string and swings with uniform motion in a horizontal circle. The string breaks and at a time t_1 later, the stone is displaced $\vec{r}_1 - \vec{r} = (5.5\vec{i} + 7.9\vec{j} - 3.1\vec{k})$ metres. (The positive z-axis is vertically up.)

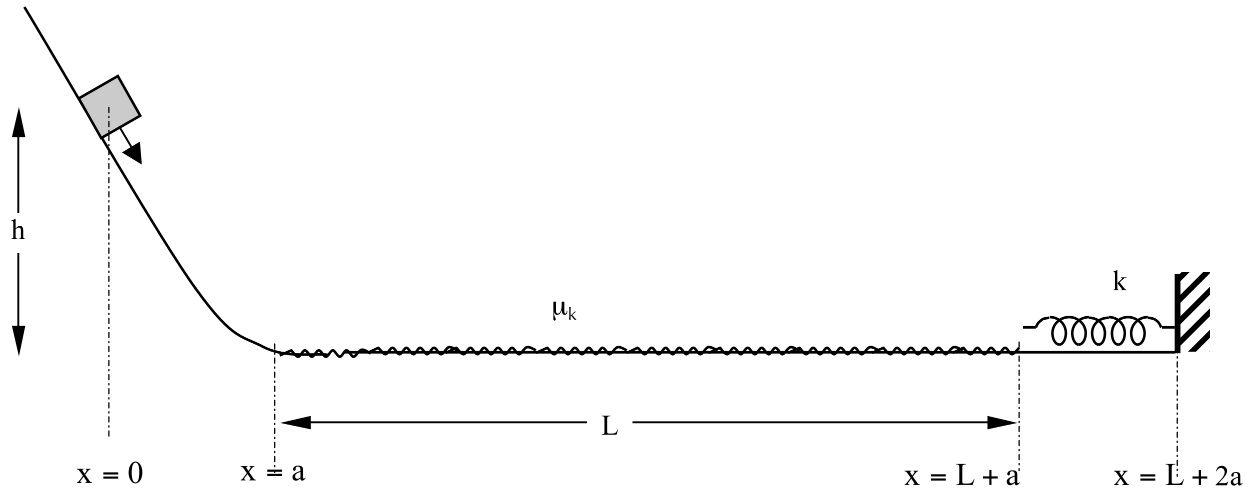
(i) Find time t_1.

(ii) What was the speed of the stone while in circular motion?

(iii) At what angle (in degrees) was the string with the *x*-axis when the string broke?

QUESTION 6 **[Marks 25]**

A small block of mass m=254g slides along a track as shown in the figure. The left side of the track is elevated and has a frictionless surface. From x=a=35.0cm, the surface is rough and flat, with a length L=2.37m. At the far right, at x=L+a, is a spring with spring constant k=4.36N/cm above a frictionless surface of length a=35.0cm. In traversing the flat part of the track (through a distance L), 728mJ of mechanical energy is lost due to friction. The block is released from a height h=95.0cm above the flat part of the track.



(a) For the first return trip to the incline, calculate:

(i) the speed of the block at x=a (on first approach to the rough surface);

(ii) the maximum compression of the spring;

(iii) the maximum height on return to the incline.

(b) Find the position where the block comes to rest on the rough flat surface.

(c) What is the coefficient of kinetic friction μ_k between the block and surface along the rough part of the track?

(d) If a block made of a different material were used instead, at what position would the block come to rest if the coefficient of kinetic friction between this block and the rough surface were half that considered above?

QUESTION 7 **[Marks 22]**

(a) The Sun, mass m=2.0×10^{30} kg, revolves around the centre of the Milky Way galaxy, a distance R=2.2×10^{20} m away. The Sun completes one revolution every 2.5×10^{8} years.

(i) Calculate the tangential speed v of the Sun about the galactic centre.

(ii) From Newton's second law and Newton's law of gravitation, derive an expression for the mass of the galaxy M in terms of the Sun's tangential speed v, the gravitational constant G, and the distance R. (Assume that the mass of the galaxy is distributed with spherical symmetry about its centre and that the Sun is at the galactic edge.)

(iii) Hence, estimate the number of stars in the Milky Way. (Assume the mass of the galaxy M is comprised entirely of stars and that the mass of the Sun is typical for a star.)

(b) Two stars move in circular orbits about the galactic centre with radii R_1 and R_2, where $R_2 > R_1$. They are measured to have tangential speeds v_1 and v_2, respectively.

(i) Calculate the mass of matter in the spherical shell between radii R_1 and R_2 in terms of the parameters given and the gravitational constant G. (Assume all mass is distributed with spherical symmetry about the galactic centre.)

(ii) All visible mass is concentrated near the galactic centre. The two stars considered lie further from the galactic centre than the Sun, where stars are very sparse. The result from (b)(i) gives the mass of dark matter between the orbits of the stars, the mass of visible matter being negligibly small here. If there were no dark matter, what would be the relation between the tangential speeds v_1 and v_2?

QUESTION 8 **[Marks 28]**

(a) A 104g copper ring has a diameter of 2.54000cm at its temperature of 0 degrees Celsius. An aluminium sphere has a diameter of 2.54533cm at its temperature of 100 degrees Celsius. The sphere is placed on top of the ring, and the two are allowed to come to thermal equilibrium, no heat being lost to the surroundings. The sphere just passes through the ring at the equilibrium temperature. What is the mass of the sphere?

Coefficients for linear expansion -
Copper: $\alpha = 17 \times 10^{-6}\ K^{-1}$; Aluminium: $\alpha = 23 \times 10^{-6}\ K^{-1}$
Specific heat capacities -
Copper: c = 387 J/(kg.K); Aluminium: c = 900 J/(kg.K)

(b) A quantity of ideal gas occupies an initial volume V_0 at a pressure P_0 and temperature T_0. The gas expands to volume V_1. [Cautionary note: in answering questions (ii)-(iv), pay attention to signs. E.g., -5 is less than -4.]

(i) On a *PV* diagram, graph the expansion for three cases:
(1) constant pressure (isobaric expansion),
(2) constant temperature (isothermal expansion),
(3) no heat exchange (adiabatic expansion).

(ii) In which case is the work done on the gas the greatest? The least? Present all reasoning/working.

(iii) In which case is the heat added to the gas the greatest? The least? Present all reasoning/working.

(iv) In which case is the change in internal energy ΔE_{int} the greatest? The least? Present all reasoning/working.