Higher Physics 1A PHYS1131 2010-T1, Solutions

Q3 27 Marks

(a) Work done on the gas $W = -\int_{V_1}^{V_2} PdV$ where pressure and volume are P and V respectively. V_1 is initial volume and V_2 is final volume. NOTE THE MINUS SIGN

(b) (i) When a closed system undergoes a changes in the state the change in internal energy of the system is given by:

$\Delta E_{\text{int}} = Q + W$ where

Q is the heat supplied to the system
W is the work done on the system and
ΔE_{int} is a state function

(ii) For a cyclic system $\Delta E_{\text{int}} = 0$ as it returns to the starting point, so that $Q+W=0$.

(c) (i) Work done $W = -P\Delta V = -P(V_f - V_i)$ where P is a constant and V_i, V_f the initial and final volumes.
For an ideal gas $PV = nRT$ where n is the # moles.

$\therefore V_f = \dfrac{nRT_f}{P}$ and T_f is the final temperature.

For the liquid water $V_i = \dfrac{\text{Mass}}{\text{Density}} = \dfrac{nM_{H_2O}}{\rho_{water}}$ where M_{H2O} is the molar mass of water.

$$\therefore W = -P\left[\frac{nRT_f}{P} - \frac{nM_{H_2O}}{\rho}\right] = -nRT_f + \frac{nPM_{H_2O}}{\rho}$$

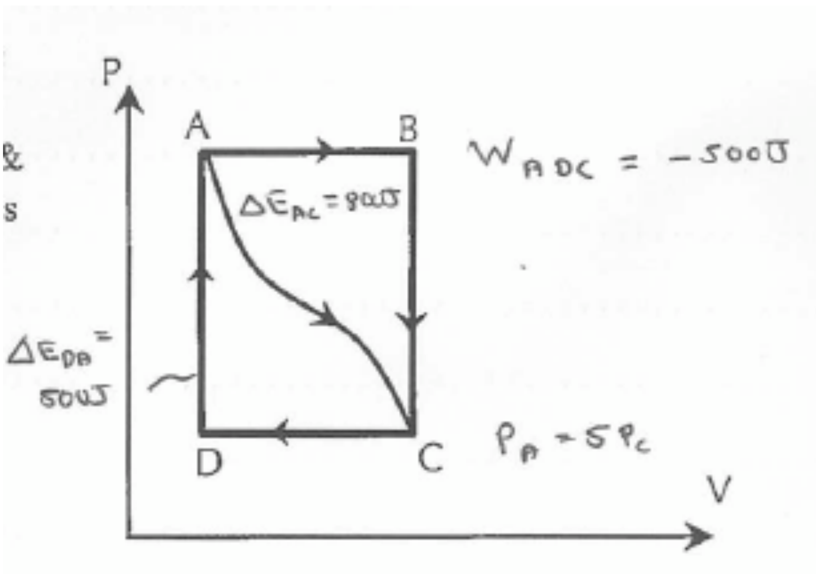
= -[2.00 . 8.314 . (273.16+100)] + [2.00 . 1.013x10^5 . 18/1000]/(1.0 x 10^6) J
=-6204.9 + 3.6 10^{-3} J
=-6204.9 J (i.e. the volume of the liquid water is negligible and can be neglected)
=-6.20 x 10^3 J to 3SF

(ii) Change in internal energy is given by the First Law; i.e.
$\Delta E_{\text{int}} = Q + W$ with Q being heat supplied, W being the work done.

$Q = m_{\text{water}} L_W$ with L_W the latent heat of fusion
$= 2.00 \cdot 18 \cdot 10^{-3} \cdot 2.30 \cdot 10^6\ \text{J}$
$= 82{,}800\ \text{J}$

Thus, $\Delta E = 82{,}800 - 6{,}201J = 76{,}599J = 7.66 \times 10^4\ \text{J to 3SF}$

(d) (i)



We know from the First Law of Thermodynamics that $\Delta E_{\text{int}} = Q + W$.

We have $\Delta E_{AC} = 800\text{J}$, $W_{ABC} = -500J = W_{AB} + W_{BC}$

From conservation of energy $\Delta E_{ABC} = \Delta E_{AC}$ since the same initial and final states.

$= Q_{ABC} + W_{ABC}$ from the 1st Law.

So, $\underline{Q_{ABC} = \Delta E_{AC} - W_{ABC} = 800J - (-500J) = 1{,}300J}$

(ii) We have $P_A = 5P_C, \Delta V_{AB} = -\Delta V_{CD}$ from inspection of the *PV*-diagram.

Now $W_{CD} = -P_D \Delta V_{CD}$ since P_D is fixed along the final change.

We also know that $W_{ABC} = W_{AB} + W_{BC} = -500J$ since $W_{BC} = 0$ (V fixed).

$$\therefore W_{CD} = -P_D \Delta V_{CD}$$
$$\underline{= \frac{+P_A}{5}\Delta V_{AB} = \frac{-W_{AB}}{5} = -(-500/5) = +100J}$$

(iii) From the 1st Law $\Delta E_{\text{int}} = Q + W$

$$\Delta E_{AC} + \Delta E_{CD} + \Delta E_{DA} = 0 \text{ (cyclic)}$$

We know that with $\Delta E_{AC} = +800\text{J}$ (given)

So that $\Delta E_{CDA} = -800J$

Also $W_{CDA} = W_{CD}$ since $W_{DA} = 0$ (Volume fixed)

Thus $Q_{CDA} = \Delta E_{CDA} - W_{CD} = -800J - 100J = -900J$

This is heat taken from the system. <u>Thus +900J is added as heat to the surroundings.</u>

(iv) We know that $\Delta E_{DA} = +500J$

Since also $\Delta E_{DA} + \Delta E_{AC} + \Delta E_{CD} = 0$ (cyclic)
Then $500 + 800 + \Delta E_{CD} = 0$
So that $\Delta E_{CD} = -1{,}300J$

Also, $\Delta E_{CD} = Q_{CD} + W_{CD}$ (1st Law)
$\underline{\therefore Q_{CD} = -1{,}300 - 100J = -1{,}400J}$

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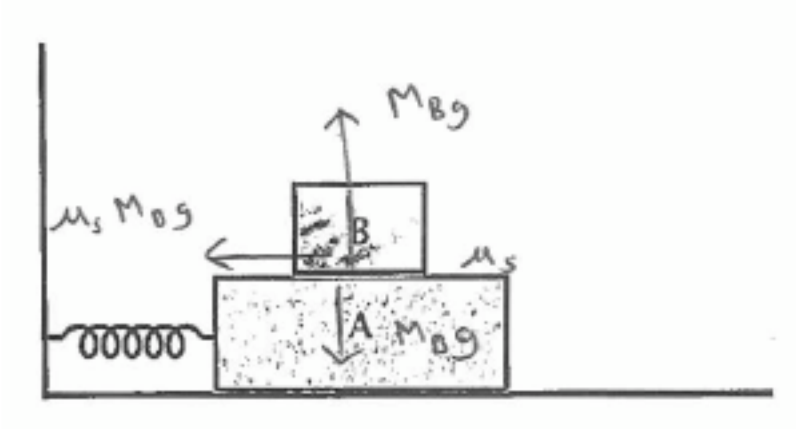
Q4 12 Marks

(a)



Block A executes SHM given by $\ddot{x} = -\omega^2 x$ with $\omega = 2\pi f$ and f=1.50 Hz.

So maximum acceleration is given $\omega^2 x_{max}$

Friction force between A&B given by $F = \mu_s M_B g$.

Thus M_B x Max. Acceleration ≡ Frictional Force before slip occurs

$$\therefore M_B (2\pi f)^2 x_{\max} = \mu_s M_B g$$

$$\text{i.e. } x_{\max} = \frac{\mu_s g}{4\pi^2 f^2} = \frac{0.6 \times 9.8}{4\pi^2 1.5^2} = 0.0663m = \mathbf{6.6cm} \text{ to 2SF}$$

(b) With Block C, μ_s=0.5

Maximum force for friction when $\mu_{sC} M_C g = M_C \omega^2 x$ where x is the displacement from equilibrium, as in part (a).

$$f_{\max}^2 = \frac{\mu_s g}{4\pi^2 x_{\max}}$$

i.e.

$$\therefore f_{\max} = \left(\frac{\mu_s g}{4\pi^2 x_{\max}}\right)^{1/2} = \left(\frac{0.5 \times 9.8}{4\pi^2 0.0663}\right)^{1/2} = \mathbf{1.369s^{-1} = 1.4 \text{ Hz to 2SF}}$$

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Q5 23 Marks

(a) The wave speed of a mechanical wave is related to the <u>elastic</u> properties and the <u>inertial</u> properties of the medium through which it is travelling, and is given by:

$$v = \sqrt{\frac{\text{Elastic Property}}{\text{Inertial Property}}}$$

For a stretched string, of tension T and mass per unit length μ, these combine to give $v = \sqrt{T/\mu}$.

(b) (i) We have a wave of form $y = A\sin(kx - \omega t)$

Amplitude A=0.200mm = 0.000200m

Frequency f=500 Hz with ω=2πf = 1000π rad/s = 3,142 rad/s

Wavespeed $v=196$ m/s$=\omega/k$ so that $k=\omega/v=2\pi f/v=1000\pi/196=16.03$ m^{-1}

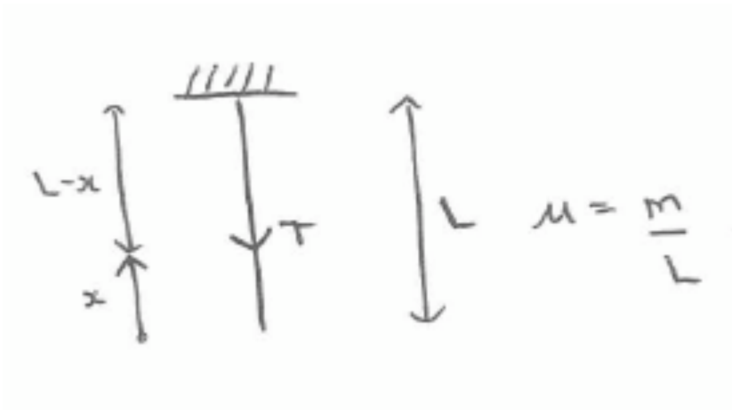
Thus, the wave equation becomes:

$y = 0.00200\sin(16.0x - 3140t)$ m to 3SF.

(ii)We have $v = \sqrt{T/\mu}$ so that

$T = \mu v^2 = 4.10\times10^{-3}\times196^2$ N = 157.5 N = 158 N to 3SF.

(c)



Let the Tension be T at a distance x from the end, as in the diagram.

Then $T = \mu x g$ = Weight of string below x.

Thus, wave speed at x is given by:

$v = \sqrt{T/\mu} = \sqrt{\frac{\mu x g}{\mu}} = \sqrt{xg}$.

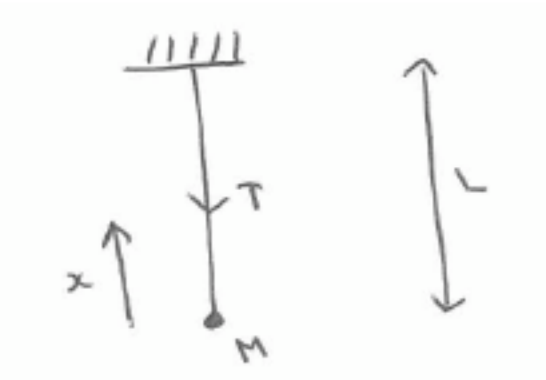
$\therefore \frac{dx}{dt} = \sqrt{gx}$.

So $\int_0^x \frac{\mathrm{dx}}{\sqrt{\mathrm{gx}}} = \int_0^\tau dt$

i.e. $t = 2\sqrt{x/g}$ is the time to traverse a distance *x* along the string, since at *t=0* we have *x=0.*

So when *x=L,* $\tau = 2\sqrt{L/g}$ is the time for the pulse to traverse the length of the string.

(d)



Suppose we add a mass *M* to the bottom of the string, as in the diagram above.

Tension at the point *x* is now given by

T = Weight of Mass + Weight of string above.

i.e. $T = Mg + \mu x g$

$$\frac{dx}{dt} = v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg}{\mu} + xg} = \left(\frac{Mg}{\mu}\right)^{0.5}\left[1 + \frac{\mu x}{M}\right]^{0.5}$$

So that $\int_0^x \frac{dx}{\left[1 + \frac{\mu x}{M}\right]^{0.5}} = \int_0^t \left(\frac{Mg}{\mu}\right)^{0.5} dt$

$$\left[2\left[1 + \frac{\mu x}{M}\right]^{0.5}\left(\frac{M}{\mu}\right)\right]_0^x = \left(\frac{Mg}{\mu}\right)^{0.5} t$$

$\therefore t = 2\left(\frac{M}{\mu g}\right)^{0.5}\left[\left(1 + \frac{\mu x}{M}\right)^{0.5} - 1\right]$

Now $\mu = m/L$ and when $x = L$ we have for the total time τ:

$$\tau = 2\left(\frac{ML}{mg}\right)^{0.5}\left\{\left(1+\frac{m}{M}\right)^{0.5}-1\right\}$$

$$= 2\left(\frac{L}{mg}\right)^{0.5}\left\{\left(M+m\right)^{0.5}-M^{0.5}\right\}$$

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Q6 18 Marks

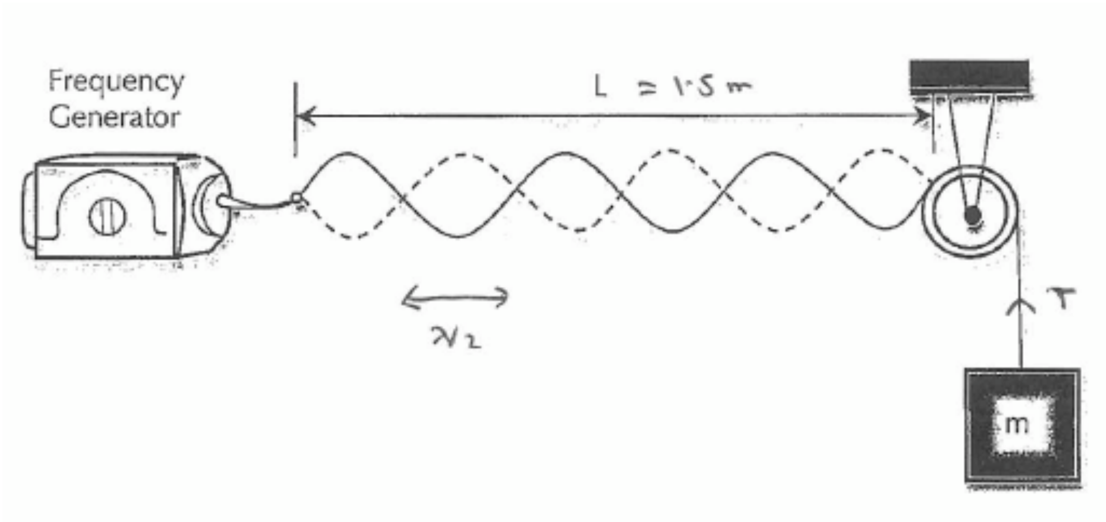
(a) The sinusoidal oscillator is a pure tone generator; i.e. emitting at a single frequency that is desired.

Set the two oscillators in operation, emitting their tones.

If the two oscillators emit at the same frequency you will hear a single pitch. There will be no amplitude modulation in the sound.

If the frequencies are different beats will be heard. Adjust the frequency of one oscillator; as the beat frequency decreases it approaches the frequency of the second oscillator.

(b)



To get standing waves we have to have an integer number of half wavelengths if the string is fixed at both ends.

i.e. $n\left(\frac{\lambda}{2}\right) = L$

Suppose that $f = v/\lambda$ with $v = \sqrt{T/\mu}$.

Take *n* half wavelengths for mass m_n.

Then there will be *n+1* half wavelengths for a mass m_{n+1} with $m_{n+1} < m_n$, since there are fewer nodes when the tension is more (making use of the hint) and we know that there are no standing waves between these values (given in the question).

$$\therefore \sqrt{\frac{T_n}{\mu}} = f\lambda_n = f\,\frac{2L}{n}$$

Similarly, $\sqrt{\frac{T_{n+1}}{\mu}} = f\lambda_{n+1} = f\,\frac{2L}{n+1}$ with the <u>same</u> frequency *f*.

Divide these two equations:

$$\frac{\sqrt{T_n/\mu}}{\sqrt{T_{n+1}/\mu}} = \frac{f\,2L/n}{f\,2L/(n+1)}$$

i.e. $\sqrt{\frac{T_n}{T_{n+1}}} = \frac{n+1}{n}$

But $T_n = 25.0g$ and $T_{n+1} = 16.0g$

So $\frac{n+1}{n} = \sqrt{\frac{25g}{16g}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$

Hence $4(n+1) = 5n$ so that $n=4$. There are therefore 4 half-wavelengths.

So $\frac{2Lf}{n} = \sqrt{\frac{T_n}{\mu}}$

$\therefore f = \frac{n}{2L}\sqrt{\frac{T_n}{\mu}} = \frac{4}{2\times 1.5}\sqrt{\frac{25\times 9.8}{0.003}}$ Hz on substituting.

$\therefore f = 381.03$ Hz = **381 Hz to 3SF**

(c) Largest mass for which standing waves could be observed would have n=1 since the greater the tension the fewer the number of modes (using the hint again).

i.e. $\sqrt{\frac{T_1}{\mu}} = \frac{f\,2L}{1}$

$\therefore T_1 = \mu(2Lf)^2$

So that $m_1 = T_1/g = \frac{\mu}{g}(2Lf)^2 = \frac{0.003}{9.8}\times\left(2\times 1.5\times 381.03\right)^2 = 399.6$ kg

<u>i.e. m_1= 400 kg is the maximum mass possible.</u>