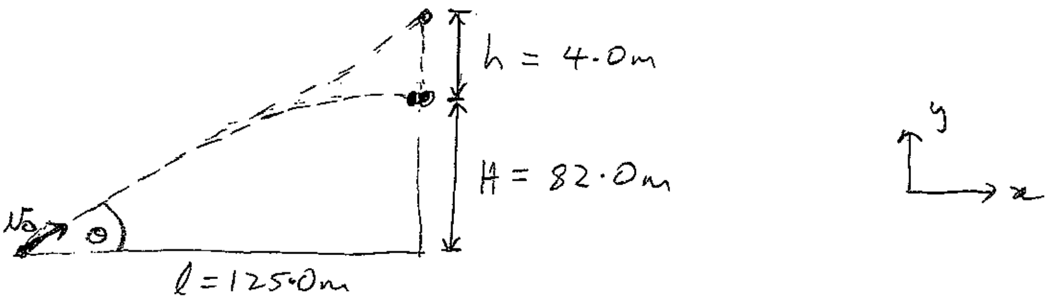
PHYS 1131 TEST 1 S2 2006

1 (a).



(i) Initial speed of cannon-ball $v_0 = \sqrt{v_{0x}^2 + v_{0y}^2}$

$$l = v_{0x} t$$
$$H = v_{0y} t - \tfrac{1}{2} g t^2.$$

Need to find t, time between firing of cannon-ball and collision with target

In time t, target falls a distance h.

$$h = \tfrac{1}{2} g t^2 \qquad \text{(falls from rest)}$$

$$\Rightarrow t = \sqrt{2h/g} = \sqrt{2 \times 4.0/9.8} = 0.904 \text{ s}.$$

$$\Rightarrow \quad v_{0x} = l/t = 125/0.904 = 138.3 \text{ ms}^{-1}$$

$$v_{0y} = \frac{1}{t}\left(H + \tfrac{1}{2} g t^2\right) = \frac{1}{0.904}\left(82.0 + \tfrac{1}{2}\, 9.8 \times (0.904)^2\right)$$
$$= 95.14 \text{ ms}^{-1}.$$

$$\Rightarrow v_0 = \sqrt{(138.3)^2 + (95.14)^2} = 168 \text{ ms}^{-1}.$$

(ii). For cannon to hit target, cannon must have been aimed at target (both cannon-ball and target fall same vertical distance)

So $\tan\theta = h+H/l$. $\Rightarrow \theta = \tan^{-1}(h+H/l)$
$= \tan^{-1}(86.0/125) = 34.5°$

(b) (i) The largest distance the car could travel is L.
(Carriage will shift each time a cannon-ball is fired s.t. the centre of mass of the system stays at the same point. The car will travel a distance L when the total mass of the cannon-balls is much greater than the mass of the carriage)

(ii) After all cannon-balls have reached the wall, the speed of the car is zero.
(Since the centre of mass is at rest and the cannon-balls are at rest, the car must also be at rest.)

(iii) Friction is an external force acting on the system in the horizontal direction. Therefore, it affects the motion of the centre of mass. The friction hinders the motion, so the car will travel a distance less than that when friction is Ignored. After all cannon-balls have reached the wall, the speed of the car may be non-zero: momentum is no longer conserved.

Need only: external force acting horizontally. Therefore, largest distance travelled could be different to L, and speed of car may be non-zero after all cannon-balls have reached wall.

2(a) $m_1 = 2m$
$m_2 = m$

Centre of mass $(m_1 + m_2)\, \underset{\sim}{r}_{cm} = m_1 \underset{\sim}{r}_1 + m_2 \underset{\sim}{r}_2$.

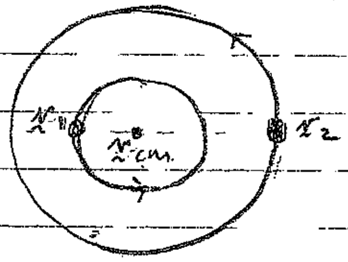
In centre of mass frame, $\underset{\sim}{r}_{cm} = 0$.

$$\Rightarrow \quad 0 = m_1 \underset{\sim}{r}_1 + m_2 \underset{\sim}{r}_2$$
$$\Rightarrow \quad 0 = 2\cancel{m}\, \underset{\sim}{r}_1 + \cancel{m}\, \underset{\sim}{r}_2.$$

$$\Rightarrow \quad \underset{\sim}{r}_1 = -\underset{\sim}{r}_2 / 2.$$

Trajectories:

In general, the particles move in elliptical orbits. Shown case for circular motion



b) $$K = \tfrac{1}{2}\mu v^2 + \tfrac{1}{2}(m_1 + m_2) v_{cm}^2$$

$$= \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(\underset{\sim}{v}_1 - \underset{\sim}{v}_2)^2 + \frac{1}{2}(m_1 + m_2)\, \underset{\sim}{v}_{cm}^2$$

Need centre of mass velocity: $\underset{\sim}{v}_{cm} = \frac{1}{m_1 + m_2}(m_1 \underset{\sim}{v}_1 + m_2 \underset{\sim}{v}_2)$

$$\Rightarrow K = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}\left(v_1^2 + v_2^2 - 2\underset{\sim}{v}_1 \cdot \underset{\sim}{v}_2\right) + \frac{1}{2}(m_1 + m_2)\left(\frac{m_1 \underset{\sim}{v}_1 + m_2 \underset{\sim}{v}_2}{m_1 + m_2}\right)^2$$

$$= \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}\left(v_1^2 + v_2^2 - 2\underset{\sim}{v}_1 \cdot \underset{\sim}{v}_2\right) + \frac{1}{2}\frac{\left(m_1^2 v_1^2 + m_2^2 v_2^2 + 2 m_1 m_2 \underset{\sim}{v}_1 \cdot \underset{\sim}{v}_2\right)}{m_1 + m_2}$$

$$\Rightarrow K = \frac{1}{2} \cdot \frac{m_1 m_2}{m_1+m_2}\left(v_1^2+v_2^2-\cancel{2 v_1 \cdot v_2}\right) + \cancel{\frac{m_1 m_2}{m_1+m_2} v_1 \cdot v_2} + \frac{1}{2}\frac{1}{m_1+m_2}\left(m_1^2 v_1^2 + m_2^2 v_2^2\right)$$

$$= \frac{1}{2}\frac{1}{m_1+m_2} \cdot v_1^2 \cdot \left(m_1 m_2 + m_1^2\right) + \frac{1}{2}\frac{1}{m_1+m_2} v_2^2 \left(m_1 m_2 + m_2^2\right)$$

$$= \frac{1}{2}\cancel{\frac{1}{m_1+m_2}} \cdot v_1^2 \cdot m_1 \cancel{(m_2+m_1)} + \frac{1}{2}\cancel{\frac{1}{m_1+m_2}} \cdot v_2^2 \cdot m_2 \cancel{(m_1+m_2)}$$

$$\Rightarrow K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

This is kinetic energy of two particles

(c) First term is kinetic energy of two particles in centre of mass reference frame. Second term corresponds to motion of whole system (motion of centre of mass).

(d) Change in potential energy

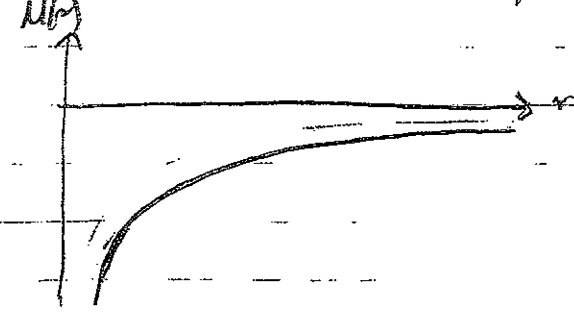
$$\Delta U = U(r) - U(r=\infty) = -\int_\infty^r \underset{\sim}{F} \cdot d\underset{\sim}{s} = +\int_\infty^r \left(+\frac{k q_1 q_2}{r^2}\right) dr$$

$$\Rightarrow U(r) = k q_1 q_2 \int_\infty^r \frac{1}{r^2}\, dr + \underbrace{U(r=\infty)}_{=0}$$

$$= k q_1 q_2 \cdot \left\{-\frac{1}{r}\right\}_\infty^r$$

$$= k q_1 q_2 \left(-\frac{1}{r} + \underbrace{\frac{1}{\infty}}_{=0}\right)$$

$$\Rightarrow U(r) = -\frac{k q_1 q_2}{r}$$



(e). Minimum energy required to separate particles to infinity is energy to give total mechanical energy $E=0$

$\Rightarrow$ minimum energy required $\quad K + U + E_{req} = 0$

$$\Rightarrow E_{req} = -U - K = \frac{kq_1q_2}{r} - \frac{1}{2}\mu v^2$$

(f). Kinetic energy $K = \frac{1}{2}\mu v^2$
Potential energy $U = -\frac{kq_1q_2}{r}$

Need to relate. Do this by expressing v^2 in terms of electrostatic force. Electostatic force provides centripetal force.

$$F = \frac{kq_1q_2}{r^2} = \mu v^2/r$$

μ - reduced mass, v - relative velocity
[in centre of mass reference frame, two-body problem is reduced to one-body problem, see expression in part (b)]

$$\Rightarrow \quad \mu v^2 = \frac{kq_1q_2}{r}$$

$$\Rightarrow \quad K = \frac{1}{2}\mu v^2 = \frac{kq_1q_2}{2r} = -\frac{U}{2}$$

$$\Rightarrow K = -U/2$$

3. (a). Momentum conservation:

$$m\underset{\sim}{v} = m\underset{\sim}{v}/2 + M\underset{\sim}{V}.$$

All move in one direction (immediately before and after collision).

$$\Rightarrow \quad mv = mv/2 + MV.$$

$$\Rightarrow \quad MV = mv/2$$

$$\Rightarrow \quad V = \frac{m}{M} \cdot v/2.$$

(b). Mechanical energy lost is equal to change in kinetic energy

$$E_{lost} = \Delta K = K_f - K_i$$

$$= \frac{1}{2} m \left(\frac{v}{2}\right)^2 + \frac{1}{2} MV^2 - \frac{1}{2} mv^2$$

$$= \frac{1}{8} mv^2 + \frac{1}{2} M \cdot \left(\frac{m}{M} \frac{v}{2}\right)^2 - \frac{1}{2} mv^2$$

$$= -\frac{3}{8} mv^2 + \frac{1}{8} \frac{m^2}{M} v^2.$$

$$= \frac{1}{8} mv^2 \left(-3 + \frac{m}{M}\right)$$

(c). Speed of bob at A: $\frac{1}{2} \cancel{M} V_A^2 + \cancel{M} g (2l) = \frac{1}{2} \cancel{M} V^2$

$$\frac{1}{2} V_A^2 + 2gl = \frac{1}{2} \cdot \left(\frac{m}{M} \frac{v}{2}\right)^2 = \frac{1}{8} \frac{m}{M} v^2$$

$$\Rightarrow \quad V_A = \sqrt{\frac{1}{4} \left(\frac{m}{M} v\right)^2 - 4gl}$$

(d)

T - tension in wire.

(e) Bob will not reach A if $T = 0$.
Bob will swing a complete circle if $T > 0$.

$$mg + T = ma = mV_A^2/l$$

$$\Rightarrow T = mV_A^2/l - mg > 0$$

$$\Rightarrow V_A^2 > gl$$

$$\text{minimum } V_A^2 = gl.$$

$\Rightarrow$ minimum speed v:

$$V_A^2 = \frac{1}{4}\left(\frac{m}{M}\right)^2 v^2 - 4gl = gl.$$

$$\Rightarrow \frac{1}{4}\left(\frac{m}{M}\right)^2 v^2 = 5gl$$

$$\Rightarrow \text{minimum } v = 2\frac{M}{m}\sqrt{5gl}.$$

(f) If bullet lodged in pendulum, then from momentum conservation $mv = (m+M)V$

$$\Rightarrow V = \left(\frac{m}{m+M}\right)v.$$

$\Rightarrow$ speed of bob at A: $\frac{1}{2}V_A^2 + 2gl = \frac{1}{2}\left(\frac{m}{m+M}\right)^2 v^2.$

$$\Rightarrow V_A^2 = \left(\frac{m}{m+M}\right)^2 v^2 - 4gl.$$

Minimum speed corresponds to

$$\left(\frac{m}{m+M}\right)^2 v^2 - 4gl = gl$$

$$\Rightarrow \quad v = \frac{m+M}{m} \sqrt{5gl}$$

~~(g)~~ (Question removed).

4. (a). $\Delta T = 3000 - 300 = 2700\ K$

$\beta = 3.2 \times 10^{-5}\ K^{-1}$

Want to find change in radius $\Delta R / R$

$\Delta V = \beta V \Delta T$

Original volume $V = \frac{4}{3}\pi R^3$

Change in volume $\Delta V = \frac{4}{3}\pi \left[(R + \Delta R)^3 - R^3 \right]$

$= \frac{4}{3}\pi R^3 \left[\left(1 + \frac{\Delta R}{R}\right)^3 - 1 \right]$

$= \frac{4}{3}\pi R^3 \left[\cancel{1} + \frac{3\Delta R}{R} + \dots - \cancel{1} \right]$

$\approx \underbrace{\frac{4}{3}\pi R^3}_{V} \cdot \frac{3\Delta R}{R}$

Can write from beginning if remember that $\beta \approx 3\alpha \Rightarrow \alpha \approx \beta/3$.

$\Rightarrow \Delta R = \alpha R \Delta T$

$\Rightarrow \beta/3 \cdot R \Delta T$

$\Rightarrow \Delta R / R = \beta/3 \ \Delta T$

$\Rightarrow \Delta V = V \cdot \frac{3\Delta R}{R}$

$\Rightarrow \Delta V / V = \frac{3\Delta R}{R} = \beta \Delta T$

$\Rightarrow \Delta R / R = \beta/3 \cdot \Delta T$

$= 3.2 \times 10^{-5} / 3 \cdot (2700)$

$= 2.9 \times 10^{-2}$

$\Rightarrow$ radius has increased by 2.9 %.

(b) Atmospheric pressure $P = 1.01 \times 10^5$ Pa

Pressure $P = F/A$, $F = mg$

Surface area of Earth: $A = 4\pi R_E^2$
$= 4\pi \times (6.37 \times 10^6)^2$
$= 5.10 \times 10^{14}$ m^2

$\Rightarrow$ $P = mg/A$

$\Rightarrow$ $M = PA/g = 1.01 \times 10^5 \times 5.10 \times 10^{14} / 9.80$

$= 5.25 \times 10^{18}$

Mass of Earth $M_E = 5.98 \times 10^{24}$ kg

$\Rightarrow$ mass of atmosphere: $m = 8.8 \times 10^{-7} M_E$

(c) Work done to system is negative of area of loop.

Area of each square $10 \times 10^6 \times 1 \times 10^{-3} = 10^4$ J

Area of half-circle: $\frac{1}{2}\pi R^2$
Area of rectangle (side R by 2R): $2R^2$

$\Rightarrow$ Work added to system $W = \left(\frac{1}{2}\pi \cancel{R^2} / 2\cancel{R^2}\right) \times 4.5 \times 10^4$

$= 3.5 \times 10^4$ J

(Work is positive because path is anti-clockwise ie, more work associated with compression than expansion)

From 1st law of thermodynamics

$$\Delta E_{int} = Q + W$$

In one complete cycle, change in internal energy is zero

$$\Rightarrow \quad 0 = Q + W.$$

$$\Rightarrow \quad Q = -W.$$

$\Rightarrow$ net heat added to system in one cycle is

$$Q = -3.5 \times 10^4 \text{ J}.$$

ie, 3.5×10^4 J of energy is lost by heat