QUESTION 1 **[Mark 20]**

The components of the position vector of a particle in circular motion are given by

$$x = r\cos\omega t,\ y = r\sin\omega t,\ \text{i.e. } \mathbf{r} = r\cos\omega t\,\mathbf{i} + r\sin\omega t\,\mathbf{j}$$

(a) Calculate the velocity **v**.

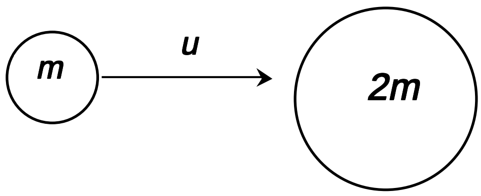
(b) Calculate the scalar (dot) product of velocity **v** and radius-vector **r**. Show that the velocity is perpendicular to the radius-vector.

(c) Calculate the acceleration **a** and show that it is directed toward the centre of circle.

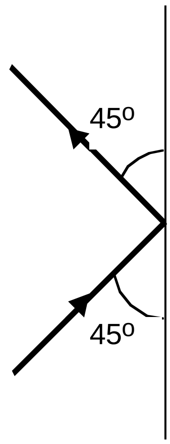
(d) A car rounds a turn on a road. The radius of the turn is *R*, the static friction coefficient is μ. Calculate the maximal allowed speed of the car to avoid slippage.

QUESTION 2 **[Marks 20]**

(a) A one-dimensional problem. A mass $m_1 = m$ moves with velocity u and collides with mass $m_2 = 2m$ which was initially at rest. The collision is elastic. Find the velocity of the first mass after the collision.



(b) A two-dimensional problem. A molecule of mass m strikes a wall and rebounds with the same speed υ and the same angle θ = π/4 radians. Calculate the momentum transferred to the wall.



(c) Now consider a stream of gas striking the wall. N molecules strike the wall during time T (see conditions in part (b)). Calculate the average force exerted by this molecular gas on the wall.

QUESTION 3 **[Marks 20]**

(a) Derive an expression for the speed of a planet orbiting a star of mass M. The radius of a circular orbit is R.

(b) Derive an expression for the escape velocity from a planet of mass m and radius r.

(c) A device is sent from the Earth to another star. An initial velocity of the device (near the Earth) must be large enough to overcome the gravitational attraction of the Earth and Sun. Suggest a method to minimize the initial velocity. Derive a formula for this minimal velocity (no numerical value is needed).