QUESTION 1 **[Marks 12]**

(a) The position vector **r** of a particle of mass 5 kg is

$$\mathbf{r} = (2t + t^3)\mathbf{i} + (3 - 3t^2)\mathbf{k} \qquad \text{(SI units)}$$

Determine, at time t = 2 sec.

(i) the kinetic energy of the particle

(ii) the momentum of the particle

(iii) the force acting on the particle

(iv) the angular momentum of the particle, about the origin.

(b) A stone of mass m is projected vertically upwards with speed v_o. Assume a **constant** resistive force f due to air resistance.

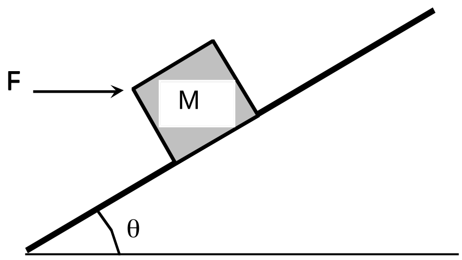
(i) Show that the maximum height reached is $\dfrac{v_o^2}{2g\left(1 + f/mg\right)}$

[Hint: Use the work-energy theorem]

(ii) Derive an expression for the speed of the stone just before it hits the ground.

QUESTION 2 **[Marks 10]**

A block of mass M rests on a rough incline, as shown, and is acted on by a horizontal applied force **F**. The coefficient of static friction is μ.



(a) Draw a diagram showing all forces acting on the body when F is almost large enough to make it move up the plane.

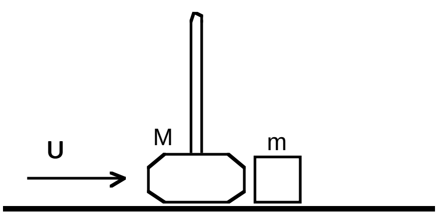
(b) Find the minimum force required to cause the body to start moving up the plane.

(c) Find the minimum force required to prevent the body sliding down the plane.

(d) Hence state the range of values of F for which the block will remain stationary on the plane.

QUESTION 3 **[Marks 10]**

A sledge-hammer of mass M strikes a block of mass m, at rest on a smooth horizontal surface. The velocity of the hammer before impact is **U**. The collision is assumed to be elastic, and M >> m.



(a) State which physical quantities are conserved in this event.

(b) Write down equations which relate the final speeds *v* and V of the block and hammer, respectively, to the other parameters.

(c) Show that the speed of the block after the collision is given by v = 2 MU(M+m). What is the maximum speed that can be given to the block in this way?

(d) Calculate the ratio of the energy of the block to the initial energy of the hammer, in the limit M >> m.

QUESTION 4 **[Marks 12]**

(a) Calculate the angular velocity of a point on the equator of the Earth, assumed to be spherical.

(b) It is proposed to place a communication satellite of mass m in a circular equatorial orbit so that it passes over a given point on the Earth 3 times per day.

(i) Show that, depending on the direction of the orbit, the angular velocity of the satellite must be $2\omega_E$ or $4\omega_E$.

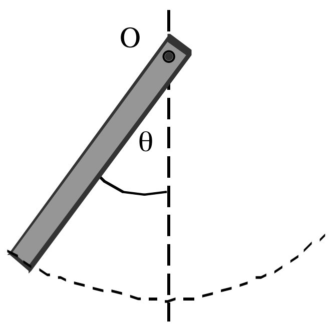
(ii) Calculate the radius of the orbit in each case.

(iii) Calculate the kinetic energy, potential energy, and total energy of the satellite in each case. The mass of the satellite is 100 kg.

QUESTION 5 **[Marks 8]**

(a) Show that the rotational inertia of a thin rod, about an axis through one end and perpendicular to the rod is $\frac{1}{3}Ml^2$ where M, l are the mass and length respectively.

(b)



The rod is allowed to swing in a vertical plane, as shown.

(i) Write down the torque acting on the rod when it makes an angle θ with the vertical.

(ii) Write down Newton's 2nd Law, in rotational form, for the rod and hence show that, for small θ, the motion is simple harmonic.

(iii) Find an expression for the period of the motion.

QUESTION 6 **[Marks 8]**

(a) A star of radius 1.0 x 10^4 km rotates about its axis with a period of 30 days. The star collapses into a neutron star of radius 3 km. Assuming that the neutron star remains spherical and its mass remains unchanged, calculate the period of rotation of the neutron star.
[The rotational inertia of a sphere is $I = \frac{2}{5}MR^2$]

(b) What must the minimum mass of the neutron star in (a) be so that matter on its surface will not be "thrown off" by the rotation?