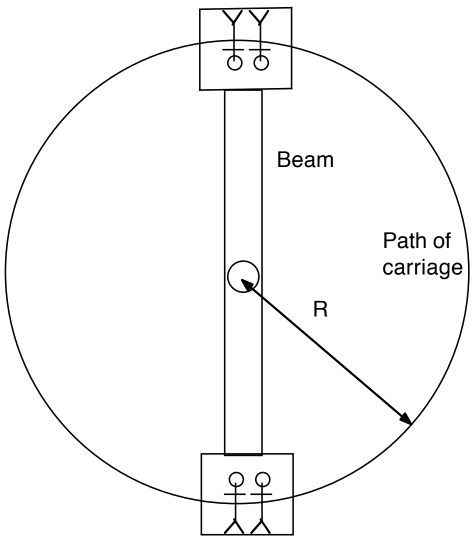
Question 1 (28 marks) **2011 1131 S1**

a) An unmarked police car (a), travelling at the legal speed limit, v_a, on a straight section of highway at constant speed v_b. At time t = 0, the police car is overtaken by a car (b), which is speeding – ie travelling at speed $v_b > v_a$. At t = 0, the police car accelerates at constant acceleration a for time T_1. It then decelerates, at constant acceleration –a for a time T_2. The police car driver judges T_1 and T_2 so that, at time t = $T_1 + T_2$, the police car is alongside the speeding car and travelling at the same speed.

Draw a displacement-time graph to show this situation. Clearly mark the displacements x_a and x_b of the two cars and the time intervals T_1 and T_2. Try to be clear, if not neat (and draw a second one if the first one is messy). Also, think about which is longer, T_1 or T_2.

b) Define an inertial frame of reference.

c) State Newton's second law of motion for a particle, defining carefully each term used.



d) This is a sketch of a circus ride. A beam connects two carriages, in which people ride, secured by seatbelts. The beam rotates about a horizontal axis through its centre, so that the path of the carriages is a circle, radius R, in a vertical plane (as shown). The mass of the carriage (passengers included) is m. The mass of the beam is negligible compared to that of the carriages and the size of the carriages is negligible to that of the length of the beam

The beam rotates at constant angular velocity with period T. Consider the moment when the beam is vertical, as shown

i) Derive one expression for the tension F_{top} in the top half of the beam *and another expression for* F_{bottom} in the bottom half of the beam, in terms of T and the other quantities.

ii) Derive an expression for the largest value of T for which the tension in the beam can be zero.

c) Describe the behaviour of unsecured objects in the upper carriage during condition described in part b). How would they appear to someone inside the carriage?

iv) How would these unsecured objects appear to someone in an inertial frame?

v) Describe a condition in which the tension in the beam could be negative. What does a negative tension mean?

Question 2

A spacecraft (with mass m) is in circular orbit (with radius r) around the earth (mass M), above the equator. With respect to the earth, it is travelling towards the East at speed v.

i) Write an expression for the mechanical energy of the spacecraft in terms of M, m, v, r and G. Specify the reference state for potential energy.

ii) Using Newton's second law, relate v and r to the gravitational force between earth and satellite.

iii) Hence or otherwise derive an expression for the kinetic energy K of the orbit as a function of M, m, r and G.

iv) Using the previous results or otherwise, derive an expression for the mechanical energy of the orbit as a function of M, m, r and G.

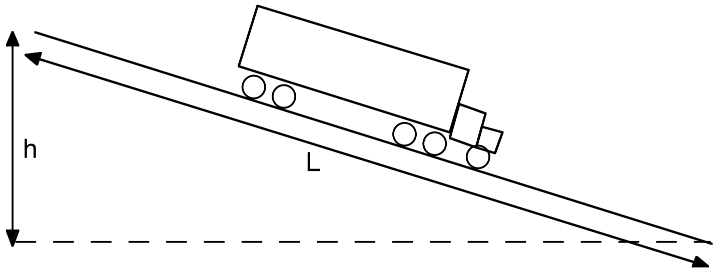
v) Write an expression for v as a function of M, m, r and G.

vi) The spacecraft ignites an engine so that a small mass of exhaust is expelled towards the West (i.e. behind the spacecraft). Explaining each step of your argument, explain what happens to the speed of the spacecraft with respect to the earth.

Question 3

a) State the definition of work as an equation. Define all terms used. Use differential notation: do not assume that the quantities involved are constant.

b)



The sign above is often seen on steep descents. It requires the drivers to use the engine to retard the descent. Let's see why.

A truck with mass 36 tonnes is travelling at the top of a straight descent with constant slope. Over a distance travelled of L = 4.0 km, the altitude of the road decreases by h = 400 m. (This is called a 10% slope and is fairly steep.)

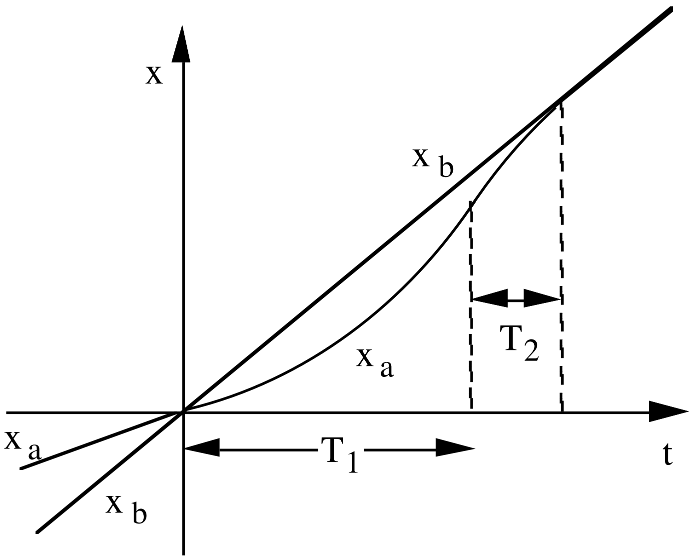
The truck has 10 brake drums, which we'll treat as identical, each with a mass of 30 kg, made from steel with a specific heat of 490 $J.kg^{-1}.K^{-1}$.

i) Suppose that the truck descends the slope at a constant speed of 30 kilometres per hour. Showing your argument and working explicitly, calculate the work done by gravity acting on the truck as it descends this section of road.

ii) If the truck is traveling at 30 kilometres per hour, what is the rate at which gravity is doing work?

iii) If all of the work done by gravity were converted into heat in the brake drums only, and if their temperature were 20°C at the start of the descent, what would be their temperature at the bottom? Show your working.

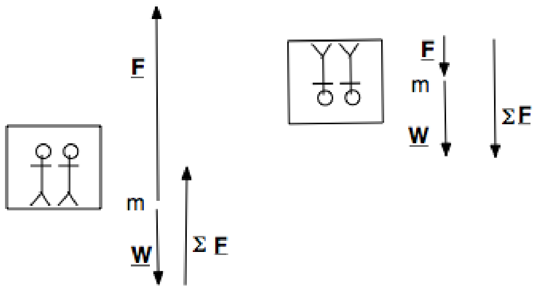
iv) The heat generated by the brakes is ultimately lost to the air (some of it passing via the wheels and axles). As an estimate, let's assume that the air passing near the brakes and wheels is heated by 20 °C. What volume of air must be heated by 20 °C to carry away the heat produced in the brakes in part iv)? Take atmospheric pressure as 100 kPa and treat the air as a diatomic ideal gas with molecular mass 0.029 kg/mol and the temperature as 20 °C.



Question 1

a) *It should look something like this. Not to scale, of course, but T_1 must be bigger than T_2, and x_a and its first derivative should both be continuous.*

b) At top and bottom, free body diagrams look like this.



i) Applying Newton's 2nd law to the carriage at the bottom and top:

$F_{bottom} - mg = ma_{bottom}$ $\qquad$ $F_{top} + mg = ma_{top}$

uniform circular motion, so centripetal acceleration = $R\omega^2$.

$F_{bottom} - mg = mR\omega^2 = mR(2\pi/T)^2.$ $\qquad$ $F_{top} + mg = mR(2\pi/T)^2.$

$F_{bottom} = m(R4\pi^2/T^2 + g)$ $\qquad$ $F_{top} = m(R4\pi^2/T^2 - g)$

ii) As angular velocity decreases and T increases, F_{top} becomes smaller. The top is the first place where F would become zero. This happens when

$$F_{top} = 0 = m(R4\pi^2/T^2 - g)$$

$$R4\pi^2/T^2 = g$$

$$T = 2\pi(R/g)^{1/2}.$$

iii) In this condition, centripetal acceleration = g downwards ($R\omega^2 = R4\pi^2/T^2 = g$) so objects are in free fall. They would appear to drift freely inside the carriage, with little motion relative to the carriage.

iv) With respect to an inertial frame, these unsecured objects are travelling in a parabola with downwards acceleration g, like any normal projectile.

v) If the beam were stationary or rotating very slowly ($R\omega^2 < g$), the tension in the beam would be negative at the top of the beam when vertical. A negative tension means that the beam is under compression. (The beam is supporting the upper carriage rather than pulling it downwards.)

Question 2

i) $E = K + U = \frac{1}{2}mv^2 - GMm/r$ where $U = 0$ at r = infinity

(not for marks: v is the speed with respect to an inertial frame moving with the earth. Viewed from the surface of the earth, its apparent speed is slower because of the rotation of the earth.)

ii) Circular motion: $|\underline{F}| = ma = mv^2/r = GMm/r^2$

iii) From (ii), $mv^2 = GMm/r$

so $K = \frac{1}{2}mv^2 = GMm/2r$

iv) (i) becomes $E = GMm/2r - GMm/r$

$E = - GMm/2r$

v) From (ii), $K = \frac{1}{2}mv^2 = \frac{1}{2}GMm/r$ $(= -E)$

so $v = (GM/r)^{1/2}$

vi) a) engine does work on spacecraft, so E increases, i.e. becomes less negative

b) from iv), less negative E has smaller v

c) so spacecraft slows down

Alternatively

a) iii) says higher orbits have higher energy, so when the engine does work on the spacecraft, it goes into a higher orbit

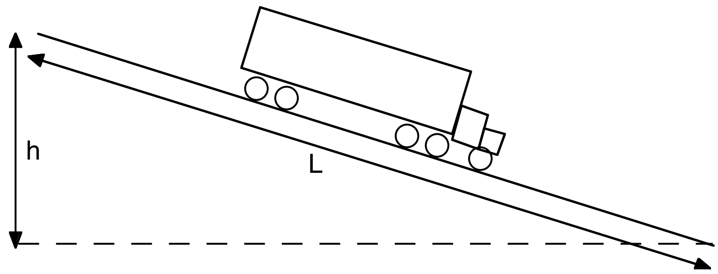
b) iv) says higher orbits are slower so

c) spacecraft slows down

Quesiont 3

a) State the definition of work as an equation. Define all terms used. Use differential notation: do not assume that the quantities involved are constant.

b)



The sign above is often seen on steep descents. It requires the drivers to use the engine to retard the descent. Let's see why.

A truck with mass 36 tonnes is travelling at the top of a straight descent with constant slope. Over a distance travelled of L = 4.0 km, the altitude of the road decreases by h = 400 m. (This is called a 10% slope and is fairly steep.)

The truck has 10 brake drums, which we'll treat as identical, each with a mass of 30 kg, made from steel with a specific heat of 490 $J.kg^{-1}.K^{-1}$.

i) Suppose that the truck descends the slope at a constant speed of 30 kilometres per hour. Showing your argument and working explicitly, calculate the work done by gravity acting on the truck as it descends this section of road.

ii) If the truck is traveling at 30 kilometres per hour, what is the rate at which gravity is doing work?

iii) If all of the work done by gravity were converted into heat in the brake drums only, and if their temperature were 20°C at the start of the descent, what would be their temperature at the bottom? Show your working.

iv) The heat generated by the brakes is ultimately lost to the air (some of it passing via the wheels and axles). As an estimate, let's assume that the air passing near the brakes and wheels is heated by 20 °C. What volume of air must be heated by 20 °C to carry away the heat produced in the brakes in part iv)? Take atmospheric pressure as 100 kPa and treat the air as a diatomic ideal gas with molecular mass 0.029 kg/mol and the temperature as 20 °C.

Question 3

a) The work dW done by a force F whose point of application is displaced by ds is $dW = \underline{F}.\underline{ds}$

b) i) *work done by gravity = – change in gravitational potential energy* Not required

$W = -\ mg\Delta h = -(36\ 10^3\ kg)(9.8\ m.s^{-2})(-400\ m) = 140\ MJ$

Or

$W = F*D = (mg \sin\theta)\ (h/\sin\theta) = (36\ 10^3\ kg)(9.8\ m.s^{-2})(400\ m) = 140\ MJ$

ii) $P = dW/dt = W/(4\ km/30\ kph) = W/(4\ km*3600/(30\ kph)) = 290\ kW$

iii) Definition $Q = cm\Delta T$, and all of W is converted to heat. So

$\Delta T = Q/cm = W/cm = W/(10\ drums*30kg/drum*490\ J.kg^{-1}.K^{-1}) = 950$ or $960\ K$ (*or near – rounding*)

$T = 20° + 960K = 980°C$ (or 970°C) *(not for marks: glowing bright red already)*

iv) specific heat at constant pressure of diatomic gas at ordinary temperatures is (7/2)R $J.mol^{-1}$.

using c per mol, $Q = cn\Delta T$ where n is the number of moles to be heated

$n = Q/c\Delta T = W/(7/2)R\Delta T = 2.4\ 10^5\ mol$

$V = nRT/P = (2.4\ 10^5\ mol)(8.34\ J.mol^{-1}K^{-1})(293\ K)/(1.00\ 10^5\ Pa) = 5.9\ 10^3\ m^3$.

(*not for marks: about 6 Olympic swimming pools*)

Markers please note: although it's superficially a 2 sig fig problem, it's really only 1 sig fig, so don't worry about precision.

Question 4

a) i) Explain what is meant by 'absolute zero'.

Either: In the classical picture it is the temperature at which all movement ceases. This is the coldest possible temperature.
Or: The lowest possible temperature.
Or: any other sensible definition
Just saying 0K or -263.15°C was not enough

ii) Using an equation state the relationship between temperature and molecular translational kinetic energy for a gas.

$$T = \frac{2}{3k_B}(\frac{1}{2}m\overline{v}^2)$$

ie. temperature is $\frac{2}{3k_B} \times$ the average kinetic energy of the particles.

iii) Calculate the temperature of oxygen gas (O_2) consisting of oxygen molecules moving with v_{rms} = 393 m/s. The atomic mass of oxygen atoms is 16.0 g/mol. (3)

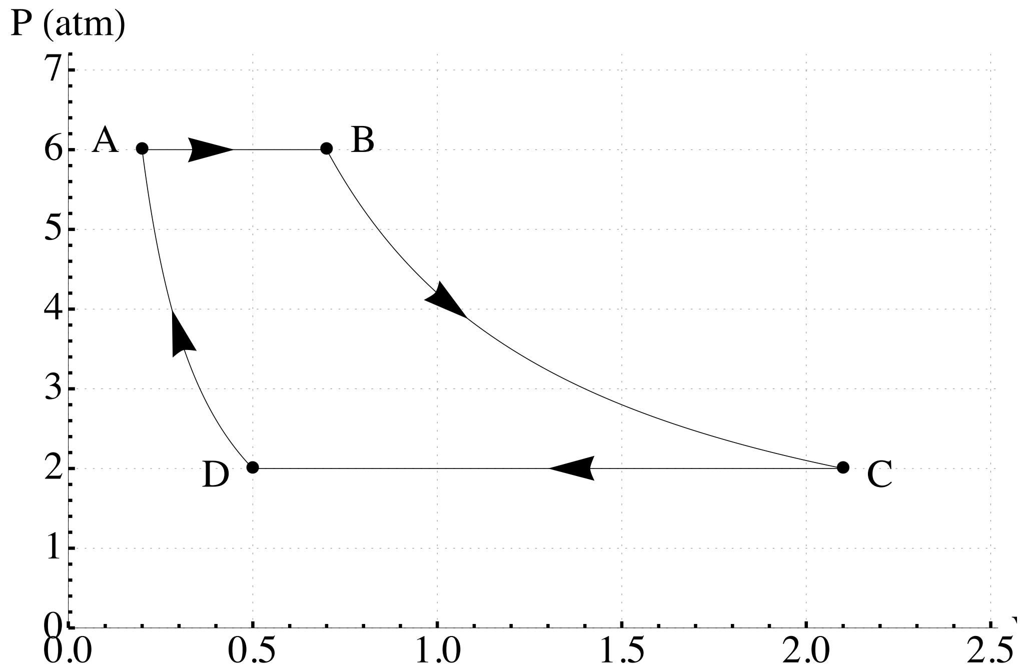
$$m = \frac{16 \times 2}{6.022 \times 10^{23}}$$
$$= 5.31 \times 10^{-26} \text{ kg}$$

$$T = \frac{2}{3k_B}(\frac{1}{2}m\overline{v}^2)$$
$$T = \frac{2}{3 \times 1.381 \times 10^{-23}} \times (\frac{1}{2} \times 5.31 \times 10^{-26} \times 393^2)$$
$$= 197K$$
$$= -75^o\text{C}$$

iv) State the theorem of equipartition of energy, and explain what is meant by a "degree of freedom".

Each degree of freedom contributes ½ k_BT to the energy of the system.
A degree of freedom is a way that the gas can store kinetic energy (move), these include translational (3), rotational and vibrational degrees of freedom.

b)



A sample of ideal gas undergoes the process shown in the diagram above. From A → B the process is isobaric and 100 kJ of heat enters the system. From B → C the process is isothermal. From C → D the process is isobaric and 200 kJ of heat energy leaves the system. From D → A the process is adiabatic.

i) Define the term 'adiabatic process'. Include an equation in your definition.

An adiabatic process is one in which no energy enters or leaves the system via heat
$\Rightarrow \Delta E_{int} = W$
or accept Q $= 0$
or $PV^{\gamma} =$ constant

ii) Calculate the work done on the gas as it goes from state A to state B.

$$
\begin{aligned}
W &= -\int_{V_i}^{V_f} PdV \\
&= -\int_{0.2}^{0.7} 6 \times 1.01 \times 10^5 dV \\
&= -303kJ
\end{aligned}
$$

-300kJ is even better.

iii) What is the change in internal energy of the sample between states A and B.

$\Delta E_{int} = Q + W$
$= (100 - 300)$ kJ
$= -200kJ$
or -203 kJ if they used $W = -303$ kJ

Any answer that was 100kJ less than (ii) was marked as correct.

iv) What is the change in internal energy as the sample moves from state B to state C?

$\Delta E_{int} = 0$
isothermal, so temperature does not change so the kinetic energy
of the particles does not change so the internal energy of the system does not change

v) How much work is done on the sample as it moves from state D to state A? State any assumptions you make.

As ABCD is a cyclic process $\Delta E_{int} = 0$ around the cylcle.
$0 = \Delta E_{intA\to B} + \Delta E_{intB\to C} + \Delta E_{intC\to D} + \Delta E_{intD\to A}$
$\Rightarrow \Delta E_{intD\to A} = -(\Delta E_{intA\to B} + \Delta E_{intB\to C} + \Delta E_{intC\to D})$

$$W_{C\to D} = -\int_{2.1}^{0.5} 2 \times 1.01 \times 10^5 dV$$

$= 323$ kJ
$\Delta E_{intC\to D} = Q_{C\to D} + W_{C\to D}$
$= -200 + 323$ kJ
$= 123$ kJ
$\Rightarrow \Delta E_{intD\to A} = -(-203 + 0 + 123)$ kJ
$\Delta E_{intD\to A} = 80$ kJ
$W_{D\to A} = \Delta E_{intD\to A} - Q_{D\to A}$
Since D $\to$ A is adiabatic, $Q_{D\to A} = 0$.
$\Rightarrow W_{D\to A} = 80\text{kJ}$

vi) Calculate γ for this gas.

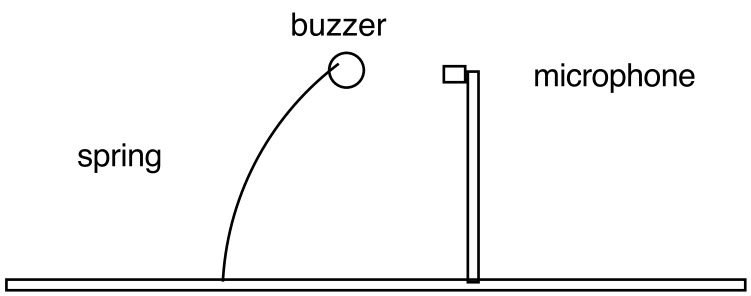
$$P_A V_A^\gamma = P_D V_D^\gamma$$
$$\Rightarrow \ln P_A + \gamma \ln V_A = \ln P_D + \gamma \ln V_D$$
$$\Rightarrow \gamma(\ln V_A - \ln V_D) = \ln P_D - \ln P_A$$
$$\gamma = \frac{\ln P_D - \ln P_A}{\ln V_A - \ln V_D}$$
$$= \frac{\ln 2 - \ln 6}{\ln 0.2 - \ln 0.5}$$
$$= 1.2$$

Question 5

a) A 7.00 kg object is hung from the bottom end of a vertical spring fastened to an overhead beam. When the object is set into a vertical oscillation, it is found to have a period of 2.60 s. Find the force constant of the spring.

$$m = 7.00kg, T = 2.60s$$
$$f = \frac{1}{2.6} = 0.3846s$$
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$
$$\frac{k}{m} = 5.84$$
$$k = 40.9N/m$$

b) Refer to the figure below. A piezo-electric buzzer, emitting a frequency of 3.00 kHz, is mounted on a vertical cantilever spring so that it oscillates horizontally at 10 Hz with simple harmonic motion. The amplitude of this simple harmonic motion is 40 cm. A microphone is mounted on the horizontal line of the buzzer's motion.



(i) Determine the highest and lowest frequencies received by microphone.

(ii) The sound intensity level at the microphone is 60.0 dB (with respect to 1 $pW.m^{-2}$) when the buzzer is closest to it, 0.80 m away. What is the minimum sound level at the microphone?

Assume that the speed of sound is 343 m/s, and ignore reflections from the floor.

i)

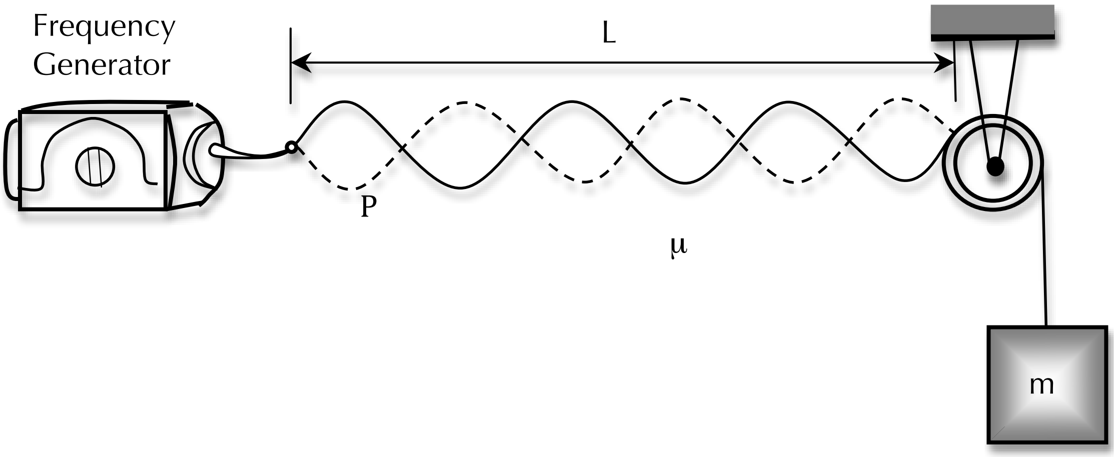
$$f' = f(\frac{c}{c \mp v_s})$$
$$c = 343m/s$$
$$x = A\cos(\omega t + \phi)$$
$$\frac{dx}{dt} = -A\omega \sin(\omega t + \phi)$$
$$= -0.4 \times 2\pi \times 10 \sin(2\pi \times 10 \times t \times d)$$
$$\Rightarrow \text{max and min } v = \pm 25.13ms^{-1}$$
$$\Rightarrow f'_{max} = 3000(\frac{343}{343 - 25.13}) = 3.24kHz$$
$$\Rightarrow f'_{min} = 3000(\frac{343}{343 + 25.13}) = 2.80kHz$$

ii)

$$I = \frac{P}{4\pi r^2}$$
$$\frac{I_{max}}{I_{min}} = \frac{\frac{P}{4\pi(0.8)^2}}{\frac{P}{4\pi(1.6)^2}} = \frac{1.6^2}{0.8^2} = 4$$
$$I_{max} = 60.0dB$$
$$\beta = 10\log_{10}\frac{I}{I_0}$$
$$60.0 = 10\log_{10}(\frac{I_{max}}{I_0})$$
$$\beta = 10\log_{10}(\frac{I_{min}}{I_0})$$
$$6 = \log I_{max} - \log I_0$$
$$\frac{\beta}{10} = \log I_{min} - \log I_0$$
$$6 - \frac{\beta}{10} = \log\frac{I_{max}}{I_{min}}$$
$$60.0 - \beta = 10\log 4$$
$$60.0 - \beta = 6.02$$
$$\beta = 54.0dB$$

6

(a) As shown in the Figure below, an object is hung from a string (with linear mass density μ = 0.00200 kg/m) that passes over a light pulley. The string is connected to a vibrator (of constant frequency *f*), and the length of the string between point *P* and the pulley is *L* = 2.00 m. When the mass *m* of the object is either 16.0 kg or 25.0 kg, standing waves are observed; however, no standing waves are observed with any mass between these values. Note: The amplitude of the vibrator is much smaller than that of the standing waves produced, so you may treat the vibrator as a displacement node.



(i) What is the frequency of the vibrator? (*Note*: The greater the tension in the string, the smaller the number of nodes in the standing wave.)

(ii) What is the largest object mass for which standing waves could be observed?

a i)

$$
\begin{aligned}
&T = mg\\
&\Rightarrow T_1 = 16.0 \times 9.8\\
&= 156.8N\\
&\Rightarrow T_2 = 25.0 \times 9.8\\
&= 245N\\
&v = \sqrt{\frac{T}{\mu}}\\
&v_1 = 280ms^{-1} = f\lambda_1\\
&v_2 = 350ms^{-1} = f\lambda_2\\
&2L = n\lambda_1 = (n-1)\lambda_2\\
&nv_1 = (n-1)v_2 \; f\text{s cancel as they are the same}\\
&\frac{n-1}{n} = \frac{v_1}{v_2} = \frac{280}{350} = \frac{4}{5}\\
&\Rightarrow n = 5\\
&f = \frac{v_1}{\lambda_1} = \frac{280}{0.8} = 350Hz
\end{aligned}
$$

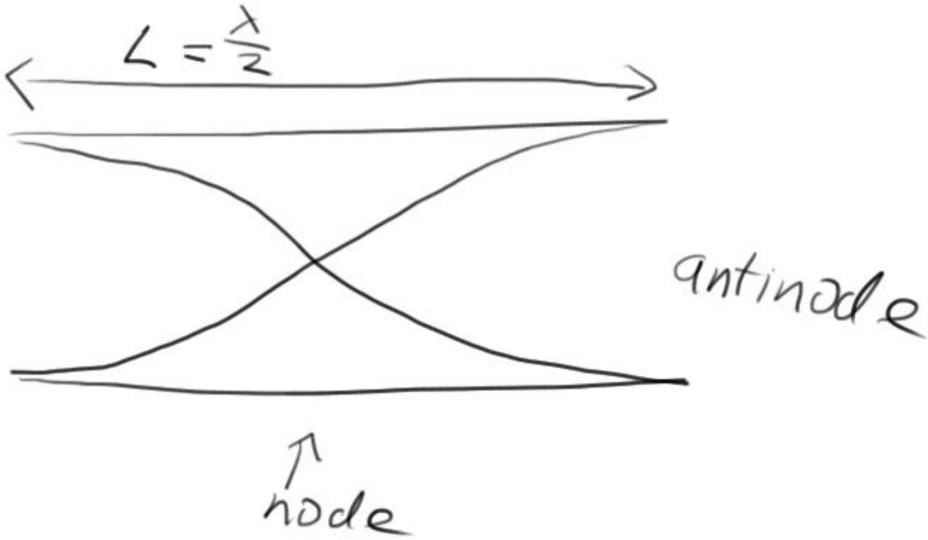
ii)

$$\frac{\lambda}{2} = L \Rightarrow \lambda = 4m$$
$$v = 350 \times 4 = 1400ms^{-1}$$
$$1400 = \sqrt{\frac{T}{0.00200}}$$
$$T = 3920N$$
$$m = 400kg$$

(b) A piccolo is a musical instrument with an overall length of 32.0 cm. The resonating air column can be approximated as a cylindrical pipe, ideally open at both ends.

(i) Find the frequency of the lowest note that a piccolo can play, assuming that the speed of sound in air is 343 m/s.

(ii) Opening holes in the side effectively shortens the length of the resonant column. If the highest note a piccolo can sound is 4,000 Hz, find the distance between adjacent antinodes for this mode of vibration.



b) i)

$$v = 343ms^{-1}$$
$$\frac{\lambda_{max}}{2} = 0.32m$$
$$\lambda_{max} = 0.64m$$
$$v = f_{min}\lambda_{max}$$
$$f_{min}\frac{343}{0.64} = 536Hz$$

ii)

$$f = 4000Hz$$
$$\lambda = \frac{v}{f} = \frac{343}{4000} = 0.08575m$$
$$\frac{\lambda}{2} = 0.042875m$$
$$\Rightarrow \text{ antinodes are 4.29 cm apart}$$