PHYS 1131 T1, 2007 UNSW

Question 1

a) Your car is stopped on the side of a straight highway. The street is busy: many cars are going past, all of them at 100 km per hour. It's a sunny day and the solar cells are clean, so assume that your car accelerates at a constant rate $a = 1.9\ \mathrm{m.s^{-2}}$ from zero until it reaches a final speed $v_f = 100$ km per hour. You need a long gap between cars to be able to accelerate to 100 km per hour to join the stream of the traffic. In this question you will work out how long.

i) Sketch a displacement - time or x(t) graph showing the position of a car accelerating from rest to v_f at constant acceleration a, and then continuing at constant speed v_f.

ii) On the same graph, show the displacement of a following car. This is a car, which travels at a constant speed v_f at all times and which, when your car has finished accelerating, is a safe distance L behind yours. Show L clearly on the graph.

iii) Below or above your displacement graph, and using the same scale for the time axis, sketch a velocity - time (v(t)) graph for the two cars. Show v_f on the graph.

iv) Some authorities judge that, in good conditions, the safe distance L between cars on a highway is the distance travelled by a car in 2 seconds*. What is L for this case?

v) Assume that you start accelerating when the car ahead is a distance L in front of you and finish accelerating when the car behind (which travels at constant speed v_F) is a distance L behind you. (It is not required, but it may help to draw the x(t) for the car ahead of yours, as well.)

Showing your working, calculate the minimum necessary distance between the car in front of you and the car behind you.

b) With respect to the ground, the wind is blowing from the North East at speed $v_w = 15$ km per hour. You are bicycling South at speed v (with respect to the ground).

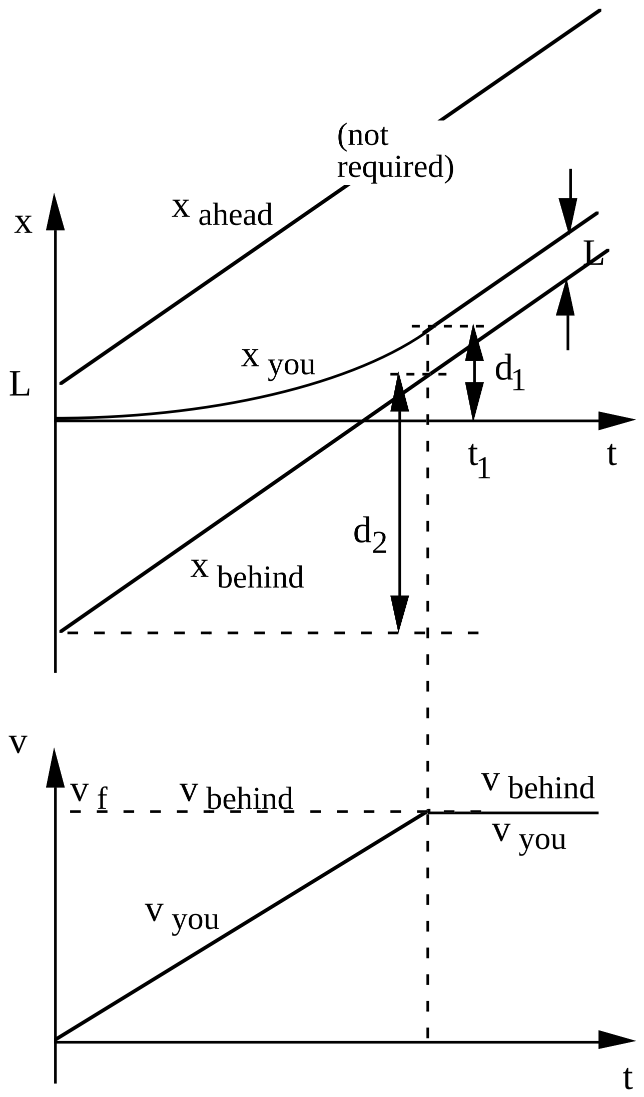
i) Relative to you, the wind is coming directly from the East. Determine your speed v.

ii) Relative to a second cyclist, also travelling South, the wind is coming directly from the SouthEast. Determine her speed (with respect to the ground).

* A comment for street safety but not for marks. The safe distance is a minimum for good conditions. In poor visibility, leave larger gaps. The timing requires judgment and the acceleration $1.9\ \mathrm{m.s^{-2}}$ over this distance is not always achievable. Consider this calculation as an *under*estimate.

Question 1

i-iii)



iv) $v_f = 100 \text{ km/hr} = \dfrac{10^5 \text{ m}}{3600 \text{ s}}$

$= 28 \text{ m.s}^{-1}$.

$L = (2\text{s}).v_f = 56$ m.

v) Distance d_1 travelled by you while accelerating:

$2ad_1 = v_f^2 - 0^2$

$d_1 = v_f^2/2a$

Time taken to reach v_f:

$t_1 = v_f/a$

Distance travelled by following car in that time

$d_2 = t_1 v_f = v_f^2/a$

Minimum initial separation between you and following car

$= d_2 - d_1 + L$

Add L for car in front: minimum gap $= d_2 - d_1 + 2L$

$= v_f^2/2a + 2L$

$= 310$ m.

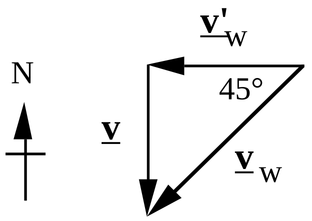
(*which is why there are speed matching lanes on freeway entrances*)

Attention marker: some candidates may just write somethink like

gap = d_1 + 2L.

This gives the correct numerical value but loses marks as indicated.

b)

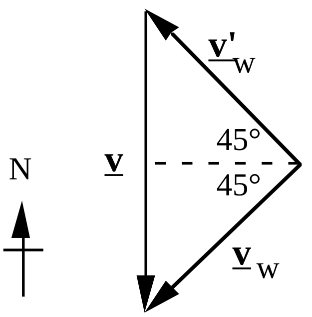


Let the relative wind velocity be $\underline{\mathbf{v}}'_w$

$\underline{\mathbf{v}}_w = \underline{\mathbf{v}} + \underline{\mathbf{v}}'_w$ as shown

$v = v_w \sin 45° = 15$ km per hour * sin 45°

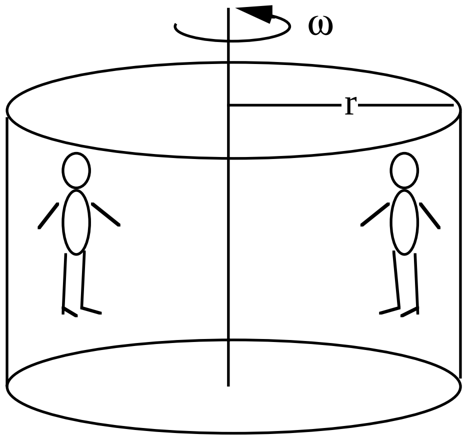
$= 11$ km per hour.



As before, $\underline{\mathbf{v}}_w = \underline{\mathbf{v}} + \underline{\mathbf{v}}'_w$

$v = 2\, v_w \sin 45° = 21$ km per hour.

Question 2



The sketch represents a fairground ride called the Gravitron. Jack and Jill stand against the inside, vertical wall of a cylindrical chamber, radius r, that is initially stationary.

When the chamber rotates about its axis, Jack and Jill discover that they need not touch the floor: the vertical wall alone stops them from falling.

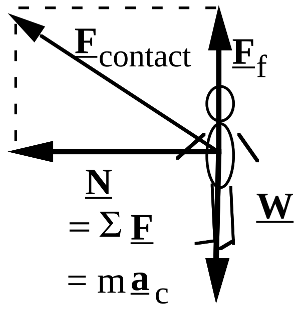
i) In several clear sentences and with the aid of at least one vector diagram, explain the origin of the force that stops Jack and Jill from falling.

ii) On your vector diagram, show the contact force that the wall exerts on Jack.

iii) Derive an expression for the minimum angular frequency ω that will suffice to stop them from falling. The coefficients of kinetic and static friction between the wall and their clothes are μ_k and μ_s.

iv) If $r = 3.0$ m, $\mu_s = 0.28$ and $\mu_k = 0.21$, calculate the minimum number of revolutions per minute that the chamber must make to prevent them from falling.

Question 2



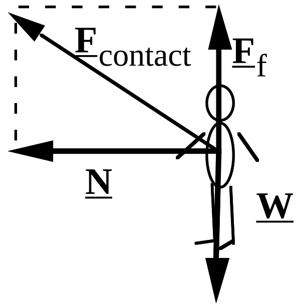
i) (*Fairly complete answer*.)

When the chamber rotates at constant angular velocity ω, Jack is travelling in uniform circular motion and therefore is accelerating with centripetal acceleration

$$a_c = r\omega^2$$

Because the axis is vertical, this centripetal vector is horizontal. Therefore the total force on him is centripetal and horizontal. The contact force is conceptually divided into a normal component $\underline{\mathbf{N}}$ and a friction component $\underline{\mathbf{F}}_f$.

The friction force must balance the weight, ie $\underline{\mathbf{F}}_f = -\underline{\mathbf{W}}$, as shown. Thus the total force is just the normal force, as shown and $\underline{\mathbf{N}} = mr\omega^2\hat{\underline{\mathbf{r}}}$. If ω is sufficiently large, then $\underline{\mathbf{N}}$ is large and so the friction ($\leq \mu_s N$) may be large enough to balance his weight.



i) (*Minimal answer for full marks*)

Because it is rotating, centripetal force is required on Jack, and this is provided by the normal force from the wall. If the normal force is big enough, friction can be big enough to support his weight.

iii) For no relative motion, $F_f \leq \mu_s N$

$$W \leq \mu_s N = \leq \mu_s \Sigma F$$

$$mg \leq \mu_s m a_c = \mu_s m r\omega^2.$$

$$\omega_{min}^2 = \frac{mg}{\mu_s m r}$$

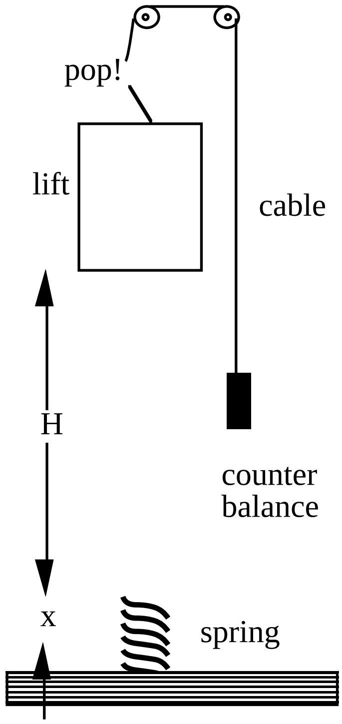
$$\omega_{min} = \sqrt{\frac{g}{\mu_s r}}$$

iv) If r = 3.0 m, $\mu_s = 0.28$ and $\mu_k = 0.21$, calculate the minimum number of revolutions per minute that the chamber must make to prevent them from falling.

$$f_{min} = \frac{1}{2\pi}\,\omega_{min} = \frac{1}{2\pi}\sqrt{\frac{g}{\mu_s r}} = 0.54 \text{ Hz}$$

$$= 33 \text{ r.p.m.}$$

Question 3



As a safety precaution, you decide to install a large spring in the bottom of a lift shaft. (A lift is the same thing as an elevator. The shaft is the volume in which it travels.) The mass of the spring is negligible compared to the mass M of the lift (and, happily, there are no passengers in the lift for this problem). Assume that the spring constant is k, and the spring obeys Hooke's law for the range considered in this problem.

Suppose that bottom of the lift is a distance H above the spring when the cable breaks, while the lift is travelling downwards at speed v_i. It then falls and hits the spring. Air resistance and friction are negligible.

i) Explaining your reasoning, derive an expression for the maximum compression x_m of the spring. To simplify the algebra, you may assume that $H >> x_m$ and that $d > x_m$.

ii) Briefly explain why the dimensions (or units) in your equation are correct.

iii) When will the acceleration of the lift be greatest? Explain your answer briefly. You may assume that $kx_m > 2\ Mg$.

iv) Derive an expression for the greatest acceleration a_{max} in terms of the parameters of the problem.

v) Consider the case of a lift falling from rest from height H above the spring. How long must d be so that $a_{max} \leq 5g$? (Hint: use your results for (i) and (iv))

Question 3

i) During both the fall and during the compression of the spring, non-conservative forces are negligible, so mechanical energy is conserved.

$K_i + U_i = K_f + U_f$

Let's take the point of maximum compression of the spring as the zero for gravitational potential energy. Maximum compression of the spring occurs when the lift is instantaneously stationary, so

$\frac{1}{2} Mv_i^2 + Mg(H+x_m) = 0 + \frac{1}{2} kx_m^2$ but $H >> x_m$, so

$\frac{1}{2} Mv_i^2 + MgH \cong 0 + \frac{1}{2} kx_m^2$

$kx_m^2 = Mv_i^2 + 2MgH$

$$x_m = \sqrt{\frac{M}{k}(v_i^2 + 2gH)}$$

ii) Mv^2 and MgH both have dimensions of energy (or units of Joules), which is force times distance (Newtons times metres). The spring constant k has dimensions of force per unit length, (units of Newtons per metre), so the argument of the square root is

$$\frac{\text{Force x distance}}{\text{Force / distance}} = \text{distance}^2 \qquad \left(\text{has units of } \frac{\text{N*m}}{\text{N.m}^{-1}} = \text{m}^2\right),$$

so the RHS has dimensions of distance (units of m), which is the same as the LHS.

iii) When the spring is compressed, the total upwards force on the lift will be

$\Sigma F = F_{spring} - W = kx - Mg$. At maximum compression, the force is

$\Sigma F = kx_m - Mg$. (*In practice, $Mg << kx_m$. so only penalise 1 mark lost if* –*Mg omitted.*)

(*FYI but not required: We are given $kx_m > 2$ Mg. Unless H is extremely small or unless the spring is very weak and also very long, kx_m will be > 2 Mg. We are told that $H >> x_{max}$, so the acceleration is greatest when the spring is most compressed.*)

iv) $$a_{max} = \frac{F_m}{M} = \frac{k\sqrt{\frac{M}{k}(v_i^2 + 2gH)} - Mg}{M} = \sqrt{\frac{k}{M}(v_i^2 + 2gH)} - g.$$

v) For $v_i = 0$, $a_{max} \leq 5g$ implies that

$$\sqrt{\frac{2kgH}{M}} \leq 6g \qquad \text{so rearranging gives} \qquad k \leq 18\frac{Mg}{H}$$

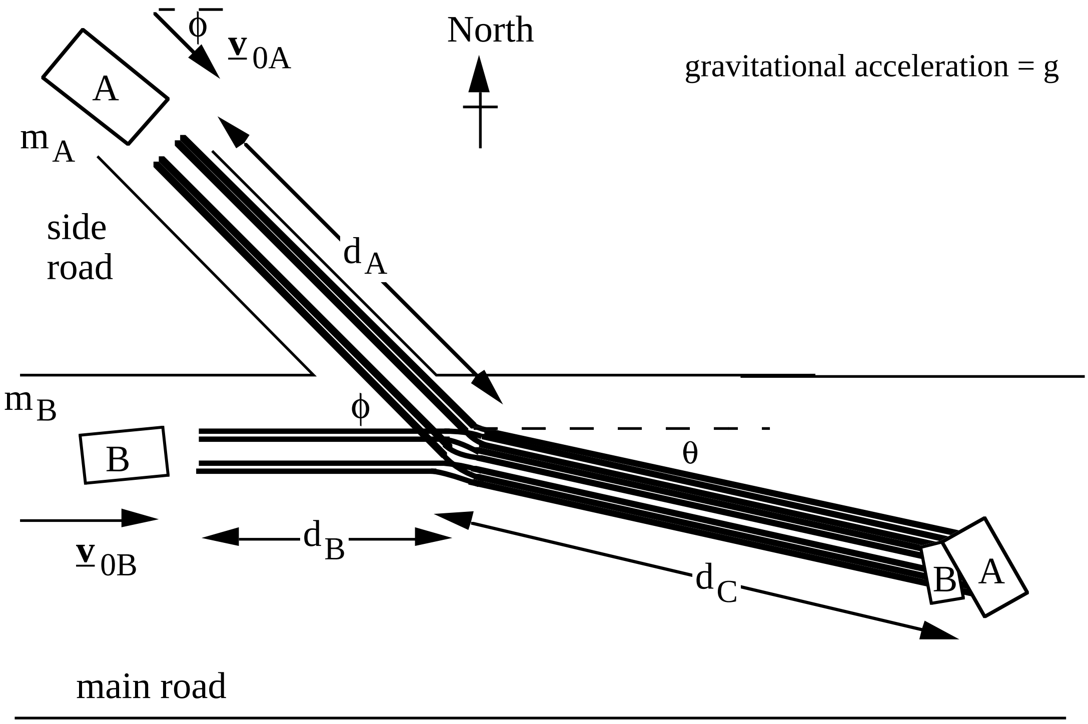
For maximum compression x_m, using parts (i) and (iv) and $v_i = 0$,

$$x_m = \sqrt{\frac{M}{k}2gH} = \sqrt{\frac{MH}{18Mg}2gH} = \frac{H}{3}$$

By the way, and not for marks, most lifts have emergency brakes, that are more practical than springs. However, the lift in the physics building has a pair of springs under it. (Two similar springs in parallel act like one spring of half the k, so the analysis above applies.) Just looking at it, I estimate that x_m *~ 40 cm*

Question 4

A traffic accident investigator finds a scene represented by the following schematic. She makes the following observations and deductions:



Car A has skidded a distance d_A along a side street before colliding with car B. Vehicle B has skidded a distance d_B along a main street before colliding with car A. Squashed together during the collision, the two wrecked cars have skidded together, sideways but without rotating. They have skidded a distance d_C, at an angle θ to the main street and have come to rest together as shown. From the black marks left on the street, we know that all four wheels on both vehicles skidded both before and after the collision. Neither street has any slope.

The investigator assumes that the initial velocities $\underline{\mathbf{v}}_{0A}$ and $\underline{\mathbf{v}}_{0B}$, before either vehicle started skidding, are in the directions shown. For legal reasons, she wishes to calculate their magnitudes, v_{0A} and v_{0B}. The masses of the cars A and B are m_A and m_B, respectively. The coefficients of static and kinetic friction between the rubber and the street are μ_s and μ_k, respectively.

i) Explaining any assumptions and reasoning, derive an expression for the momentum $\underline{\mathbf{p}}_c$ of the two cars together, after the collision, in terms of m_A, m_B, d_C, g and the appropriate μ.

ii) Between the time when either of the cars begins to skid and the time when they come to rest, is there any stage where conservation of mechanical energy is an appropriate approximation? If so, explain when and why. If not, explain why not.

iii) Between the time of the cars beginning to skid and the time when they come to rest, is there any stage where conservation of momentum is an appropriate approximation? If so, explain when and why. If not, explain why not.

iv) Explaining any assumptions and reasoning, derive an expression for the speed v_{0A} of car A, before it started skidding, in terms of parameters given in the sketch.

v) Explain in one clear sentence why there are skid marks before the collision?

(*In practice, the wreckage usually rotates about a vertical axis, giving rise to loops and sometimes discontinuities in the skid marks, but the principles are similar.*)

Question 4

i) During the 8 wheel skid, there has been no vertical acceleration, so the normal force N equals the combined weight $(m_A + m_B)g$. Because it is skidding, kinetic friction applies, so

$$F_{fC} = \mu_k N = \mu_k(m_A + m_B)g$$

The frictional force is the only horizontal force, so the acceleration in the direction of the skid is

$$a_C = -\frac{F_{fC}}{m_A + m_B} = -\mu_k g,$$ which is constant.

Under constant acceleration, distance travelled d satisfies $v_f^2 - v_i^2 = 2ad$, so

$$0 - v_{Ci}{}^2 = -2a_C d_C,$$ so the combined velocity v_C after the collision is

$$v_{Ci} = \sqrt{2a_C d_C} = \sqrt{2\mu_k g d_C}\ .$$ Therefore

$$\underline{\mathbf{p}}_c = (m_A + m_B)\sqrt{2\mu_k g d_C}$$ at a direction of θ to the main street, as shown.

ii) No. Conservation of energy applies if non-conservative forces do no work. During the skids, friction does negative work. During the collision, the forces that the work required to deform the cars are non-conservative.

iii) Yes. During the collision, very large forces act between the cars (large enough to bend metal et c). In comparison with these forces, and over the very short time of the collision, external forces are negligible and so the momentum of the two car system is conserved.

iv) The collision occurs over a short time, during which the effects of external forces (weight, friction etc) are negligible in comparison with the internal forces. Therefore, the total momentum of the two cars is conserved.

$$\underline{\mathbf{p}}_A + \underline{\mathbf{p}}_B = \underline{\mathbf{p}}_c$$

We take components in the North direction (North positive), and note that car B has no initial momentum in the North direction.

$$-p_A \sin\phi = -p_c \sin\theta$$

$$m_A v_{fA} = p_c \frac{\sin\theta}{\sin\phi} \quad \text{so} \quad v_{fA} = \frac{p_c}{m_A}\frac{\sin\theta}{\sin\phi}$$

While car A skids alone, the only horizontal force on it is friction and, by an argument like that in part (i)

$$a_A = -\frac{F_{fA}}{m_A} = -\frac{\mu_k N}{m_A} = -\mu_k g,$$ which is constant, so

$$v_{fA}{}^2 - v_{0A}{}^2 = 2a_A d_A$$

$$v_{0A}{}^2 = v_{fA}{}^2 - 2a_A d_A = \left(\frac{p_c}{m_A}\frac{\sin\theta}{\sin\phi}\right)^2 + 2\mu_k g d_A$$

using part (i) for p_c, $$v_{0A} = \sqrt{2\mu_k g d_C \frac{(m_A + m_B)^2 \sin^2\theta}{m_A{}^2 \sin^2\phi} + 2\mu_k g d_A}$$

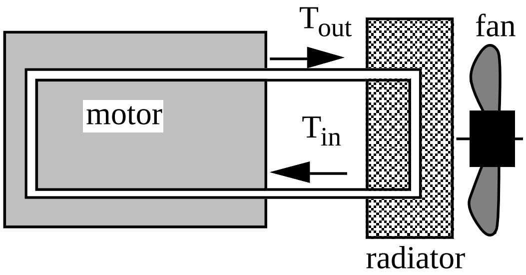
$$= \sqrt{2\mu_k g\left(d_C \frac{(m_A + m_B)^2 \sin^2\theta}{m_A{}^2 \sin^2\phi} + d_A\right)}$$ *simplification unnecessary.*

v) The drivers have pushed too hard on the brakes and locked up the wheels.

(*They would have decelerated at a greater rate if the wheels had been rolling, so that μ_s rather than μ_k applied. Anti-lock braking systems would have helped*.)

Question 5

a)



An internal combustion engine (the motor in a car) generates heat at a rate of 20 kW. It is cooled by water that flows in a continuous circuit through a radiator and the motor, as shown in the very simplified sketch. (The radiator dissipates the heat because cool air is forced through the radiator by a fan. This detail is not relevant to our problem.)

The density of water is 1000 kg.m^{-3}, its specific heat capacity is 4.2 kJ.kg^{-1}.K^{-1}, its latent heat of vaporisation is 2.3 MJ.kg^{-1} and it boils at 100°C.

i) If the temperature of the water going into the motor is 50°C and the temperature of the water coming out of the motor is 85°C, what is the rate of flow of water through the motor? (Express your answer in litres per minute.)

ii) Due to a malfunction, flow of the cooling system ceases. The motor comprises 110 kg of a metal whose specific heat capacity is 0.43 kJ.kg^{-1}.K^{-1}, and it contains 3 litres of cooling water. Assume that the motor and the water are all at the same temperature (this is a severe oversimplification). The motor continues to produce heat at 20 kW, and loses it at a negligible rate. Determine long does it take the temperature to rise from 85°C to 100°C.

iii) From the time when the water starts to boil, how long is it before the 3 litres of water is all boiled away?

iv) Once the water is boiled away, what is the rate of temperature rise in the motor? Express your answer in °C per minute.

b) i) For a gas, which is greater: the specific heat at constant volume c_V or the specific heat at constant pressure c_P? Explain your answer in about four or five clear sentences.

ii) What is the difference between the internal energy of an ideal gas and the internal energy of a non-ideal gas (for instance, a gas at high density)? Explain in about two clear sentences.

Question 5

a) i) Definition of specific heat: $Q = cm\Delta T$

$$\frac{m}{t} = \frac{1}{c\Delta T}\frac{Q}{t} = \frac{1}{c\Delta T}P = \frac{20\ \text{kW}}{4.2\ \text{kJ.kg}^{-1}.\text{K}^{-1} * (85 - 50)\ \text{K}}$$

$= 0.14\ \text{kg.s}^{-1}$ 1 litre of water has a mass of 1 kg, so

$$\text{flow rate} = 0.14\ \text{kg.s}^{-1} * \left(\frac{60\ \text{s}}{1\ \text{minute}}\right) = 8.2\ \text{litres per minute}$$

ii) $Q = (c_w m_w + c_m m_m)\Delta T$

$P = \frac{Q}{t}$ so

$$t = \frac{Q}{P} = \frac{(4.2\ \text{kJ.kg}^{-1}.\text{K}^{-1} * 3\text{kg} + 0.43\ \text{kJ.kg}^{-1}.\text{K}^{-1} * 110\ \text{kg})*15\ \text{K}}{20\ \text{kW}}$$

$= 45$ seconds

iii) Definition of latent heat: $Q = mL$

$$t = \frac{Q}{P} = \frac{mL}{P} = \frac{3\ \text{kg} * 2.3\ \text{MJ.kg}^{-1}}{20\ \text{kW}} = 350\ \text{s} \quad (= 5.8\ \text{minutes})$$

iv) $P = \frac{Q}{\Delta t} = \frac{c_m m_w * \Delta T}{\Delta t}$ so

$$\frac{\Delta T}{\Delta t} = \frac{P}{c_m m_m} = 0.42\ \text{K.s}^{-1} = 25°\text{C per minute.}$$

b) i) $c_P > c_V$. At constant volume, no work is done by the gas, so all the heat added goes into the internal energy U of the gas, and U is proportional to T. At constant pressure, the gas expands when its temperature rises and so does work (pdV). Because of the first law of thermodynamics (dQ = dU + dW), some of the heat added goes into work. So, to raise the temperature and therefore U by the same amount, more heat must be added.

ii) In an ideal gas, the intermolecular forces are negligible, except during collisions, so no energy is stored as potential energy, and all internal energy is kinetic. In a non-ideal gas (where, at high density, the molecules are on average closer to each other), the intermolecular forces and energies are non-negligible and some internal energy is stored in that potential energy.