PHYS 1131 Test 1, 2005

Question 1 (23 marks)

a) Investigators at the scene of an accident see that a car has left black rubber marks ("skid marks") that are L = 22 m long on a flat section of road. The car is stationary at one end of the marks, and it is assumed that the car began to skid at the other end. The coefficient of kinetic friction between the rubber and the wheels is $\mu_k = 0.80$ and the skid marks show that all four wheels begin to skid simultaneously. Calculate the speed of the car at the beginning of the skid. Express your answer in kilometres per hour.

b) A bird flies at speed $v_b = 5.0 \text{ m.s}^{-1}$ in a straight line that will pass directly above you, at a height h = 5.0 m above your head. You are eating grapes and it occurs to you that the bird might want one and so you decide to throw it a grape. Of course, you don't want to hurt the bird, so you will throw the grape so that, at some time t, it has the same position, same height and same velocity as the bird. (Hint for 1221: what will be the height and velocity of the grape when the bird takes it?)

You throw the grape from a position very close to your head, with intial speed v_0 and at an angle θ to the horizontal. Air resistance is assumed to be negligible.

i) Should the bird be behind you, or ahead of you when you throw the grape, and by how much? Explain your answer briefly. (3-5 clear sentences should suffice.)

ii) Calculate the required values of v_t and θ.

iii) If air resistance on the grape were *not* negligible, how would that change your answer to (i)? A qualitative but explicit answer is required.

Question 1

a) Normal force N, friction F_f. In the vertical direction, $a_y = 0$, so

$$N = mg$$

In the horizontal direction

$$F_f = \mu_k N = \mu_k mg = ma_x, \text{ so } |a_x| = \mu_k g$$

$$v_f^2 - v_i^2 = 2aL = -2\mu_k gL \quad \text{but } v_f = 0, \text{ so}$$

$$v_i^2 = 2\mu_k gL \qquad v_i = \sqrt{2\mu_k gL} = \sqrt{2*0.80*9.8\text{m.s}^{-2}*22\text{m}} = 18.6 \text{ ms}^{-1} = 67 \text{ kph.}$$

b)

i) There is no air resistance. Neither the bird's nor the grapes horizontal speed changes, so, if they have the same horizontal speed, they always have it. If they ever have the same horizontal position, they must always have it. So you must throw it when the bird is directly overhead.

ii) From (i), $v_{xg} = v_0 \cos\theta = v_b$. (a)

Vertical motion under gravity, measured from y = 0 at the position of the head:

$$v_y^2 = v_{y0}^2 + 2ay$$

At y = h, $v_y = 0$, so

$$0 = v_0^2 \sin^2\theta - 2gh$$

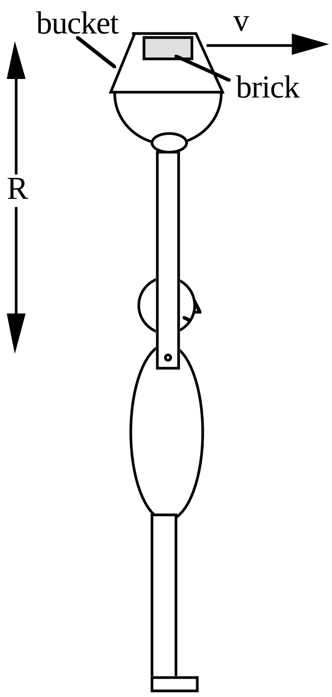
$$v_0 \sin\theta = \sqrt{2gh} \qquad \text{(b)}$$

$$\text{(b)/(a)} \rightarrow \tan\theta = \frac{\sqrt{2gh}}{v_b} \quad \text{so} \quad \theta = \tan^{-1}\frac{\sqrt{2gh}}{v_b} = \tan^{-1}\frac{\sqrt{2*9.8\text{ms}^{-2}*5\text{m}}}{5 \text{ m.s}^{-1}} = 63°$$

$$\text{(a)} \rightarrow \quad v_0 = \frac{v_b}{\cos\theta} = \frac{5 \text{ m.s}^{-1}}{\cos 63°} = 11 \text{ ms}^{-1}.$$

iii) Air resistance would slow the grape during flight. The grape would have greater horizontal velocity until the end of its flight, so it would cover the distance from you to bird faster than the bird would, so you would throw it after it passed overhead.

Question 2 (12 marks)

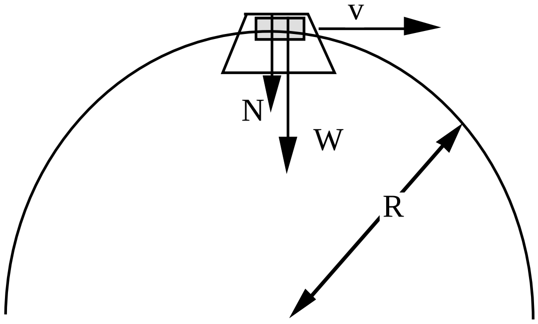


i) A physics lecturer swings a bucket in a vertical circle, about his shoulder, as shown. It executes circular motion with period T. The bucket contains a brick. Derive an expression for the maximum period T that the motion can have in order that that the brick stay in contact with the bucket. Assume that the motion has constant angular velocity.

ii) Put in appropriate values to give a numerical estimate of the period.

iii) Is the assumption of constant angular velocity reasonable? Comment briefly.

Question 2 (marks)



i) If the brick is in contact with the bucket, then both are travelling in a circle with speed v. The centripetal acceleration is

$$a_c = \frac{v^2}{R} \quad \text{down.}$$

Newton's second law for the vertical direction gives

$$N + W = ma_c = mR\omega^2 = \frac{4\pi^2 mR}{T^2}$$

To remain in contact, $N \geq 0$ so

$$\frac{4\pi^2 mR}{T^2} - W \geq 0 \quad \text{so} \quad \frac{4\pi^2 mR}{T^2} \geq mg$$

$$\text{so} \quad T^2 \leq \frac{4\pi^2 R}{g} \quad \text{or} \quad T \leq \sqrt{\frac{4\pi^2 R}{g}}$$

ii) Put R = 0.8 m (*any value between 0.5 and 1 m is okay*) $T \leq \sim 2$ s.

iii) The bucket is likely to slow down while ascending and accelerate while descending. (*So the period should be rather less than this to have a margin of security.*)

Question 3. (19 marks)

i) Assuming the orbit of the Earth about the sun to be a circle with radius R = 1.50 x 10^{11} m, calculate the magnitude of the Earth's centripital acceleration. Neglect the motion of the sun.

ii) State the direction of the centripital acceleration in (i).

iii) The constant of Gravitation is G = 6.67 x 10^{-11} Nm^2kg^{-2}. Use this value and your answer to (i) to determine the mass M of the sun.

iv) The moon has mass m_m 7.36 x 10^{22} kg. The Earth has mass m = 5.98 x 10^{24} kg. The sun has a mass M = 1.99 x 10^{30} kg.

The distance sun-earth = R = 1.50 x 10^{11} m. The distance earth-moon = r = 3.82 x 10^8 m.

At new moon, the moon lies on a line between the Earth and the sun and is at a distance r = 3.82 10^8 m from the Earth. Calculate the total gravitational force on the moon due to the sun and the Earth. (Hint: a diagram may be helpful)

v) State the direction of the force in (iv)

vi) State the magnitude of the acceleration of the moon at new moon, due to the forces exerted by the sun and the earth.

vii) State the direction of the acceleration in (vi).

viii) Compare your answers for (i & ii) and (vi & vii) and comment briefly (about two or three sentences).

Question 3.

i) Let T be the period of the Earth's orbit, ie one year.

$$a_c = R\omega^2 = R\left(\frac{2\pi}{T}\right)^2 = 1.50\ 10^{11}\ \text{m}\left(\frac{2\pi}{365*24*60}\right)^2 = 5.95\ 10^{-3}\ \text{ms}^{-2}$$

ii) towards the sun

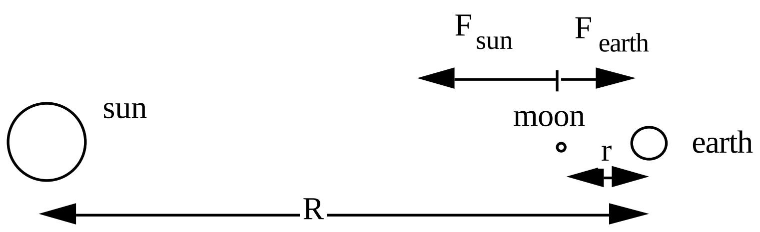
iii) for any body of mass m orbiting the sun in the earth's orbit:

$$ma_c = F_g = \frac{GMm}{R^2}$$

$$a_c = \frac{GM}{R^2}$$

$$M = \frac{R^2 a_c}{G} = 2.01 \times 10^{30}\ \text{kg.}$$

iv)



$$\Sigma F = F_{sun} - F_{earth} = \frac{GMm_m}{(R-r)^2} - \frac{Gmm_m}{r^2} \cong Gm_m\left(\frac{M}{R^2} - \frac{m}{r^2}\right) = \ldots = 2.33 \times 10^{20}\ \text{N.}$$

v) (*$F_{sun} > F_{earth}$, so it is*) towards the sun (*answer only required, not explanation*)

vi) a = $\Sigma F/m_m$ = 3.17 x 10^{-3} ms^{-2}.

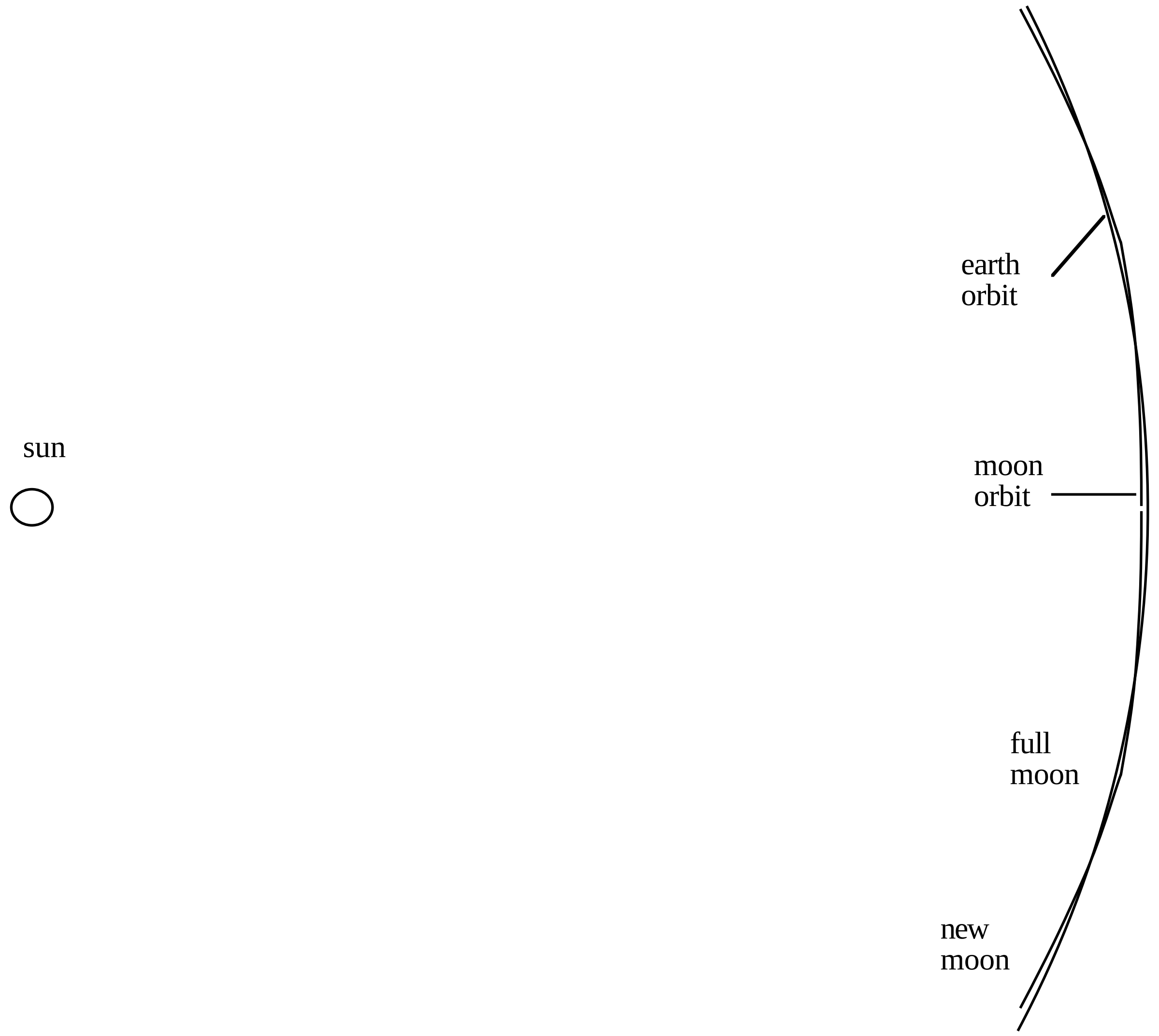
vii) towards the sun

viii) Both Earth and moon accelerate towards the sun. At new moon, the moon accelerates at only about half the rate of the earth's acceleration. When it is closer to the sun, it is not accelerating so rapidly towards the sun as the earth is, because it is beginning to move further from the sun.

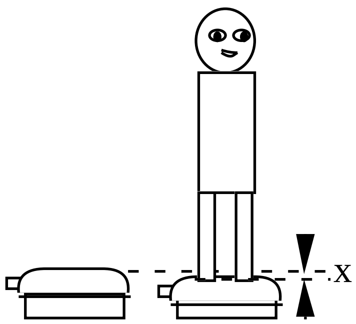
(*or any other reasonable comments*)

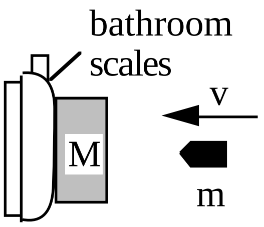
Not for marks: What is interesting, of course, is that at new moon the moon is actually accelerating in the direction away from the Earth and towards the sun—although not as quickly as the Earth is accelerating towards the sun.

The earth travels in a nearly circular path around the sun, with approx constant radius R and approx constant centripital acceleration. The moon's path is not as circular as the Earth's: it is closer to the sun (R-r) at new moon and further from the sun (R+r) at full moon. Because r << R, it's actually hard on this scale to show that the moon's orbit is always concave towards the sun, so that's why a diagram was *not* called for in this question.



Question 4 (13 marks)





Can a bathroom scale (a device usually used for measuring one's weight) be used to measure the speed of a bullet fired from a gun?

A student decides to find out. When she stands on the scale, it accurately reads her mass (60 kg). She observes that, when she stands on the scale, its lid is lowered by 5.0 mm. Assume that the scale behaves like an undamped spring, with spring constant k.

i) Calculate the value of the spring constant k.

(Hint: be careful with units.)

The student then mounts the scale vertically, and fixes a block (M = 10 kg) on its surface. Its mass is considerably greater than that of the scale. In this orientation, and with the block fixed, the scale reads zero. In a preliminary experiment, she discovers that the bulet does not penetrate through the block, and comes to rest inside it.

Her research tells her that a particular model gun fires bullets at a speed of $v = 400\ m.s^{-1}$ (called its muzzle velocity) and that the bullets have a mass m = 6.0 g.

ii) Showing all working, and using the values given, calculate the maximum compression of the scale when a bullet is fired into it at normal incidence (as shown in lower diagram). State any assumptions you make and justify any conservation laws that you use.

iii) Calculate the reading on the scale at this point.

(Under no circumstances should you try to answer this problem experimentally.)

i) The weight of a 60 kg person is 590 N. So 590 N depresses the "spring" by 5 mm, so the spring constant is k = |F|/x = 120 kN/m. (*deduct 2 marks from anyone who uses 50 N as the force.*)

ii) $m_{scale} < M$, so the force to accelerate part of it is small so,

the external horizontal forces acting on the block and bullet are negligible,

so their total momentum will be conserved during their collision.

The block + bullet has mass M = 10 kg + 6 g $\cong$ 4 kg. Let it travel at V, so

$$p_i = p_f$$

$$mv = (M + m)V \cong MV, \text{ so}$$

$$V = mv/M$$

In the compression of the spring in the scale, external forces do negligible work (because it is an undamped spring).

At maximum compression, the block is stationary. Assume that the mechanical energy of the bullet+block is converted into potential energy of the "spring", so

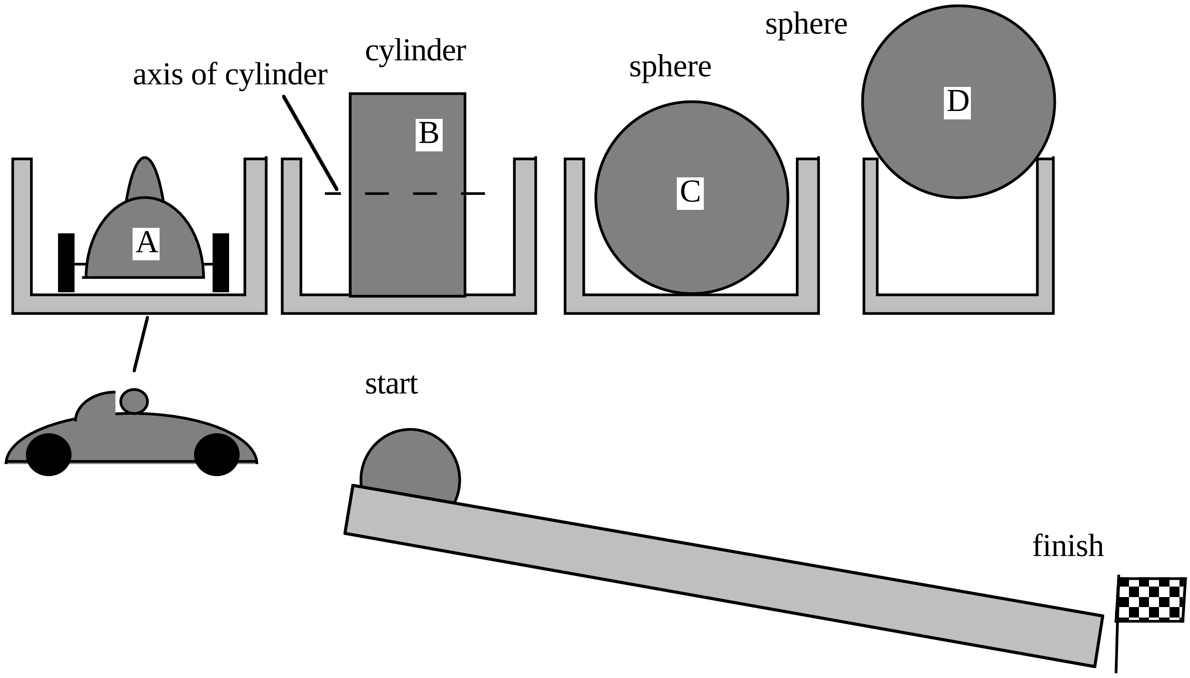
$$\frac{1}{2} MV^2 = \frac{1}{2} kx^2$$

$$x = V\sqrt{\frac{M}{k}} = \frac{mv}{M}\sqrt{\frac{M}{k}} = \frac{mv}{\sqrt{Mk}} = \frac{.006\text{kg}*400\text{m/s}}{\sqrt{10\text{kg}*120\text{kN/m}}} = 2.2 \text{ mm.}$$

iii) for the spring, |F| = kx, so if 60 kg produces a deformation of 5 mm, 2.2 mm will read a "weight" of 27 kg.

Question 5 (12 marks)

The Australian Grand Prix has been cancelled. You decide to offer an alternative event.



The contestants are two identical brass spheres, a brass cylinder (whose axis is horizontal so it can roll), and a toy racing car. All have the same mass. The wheels of the car are light and they turn with negligible friction on the axle. The objects roll down four tracks, which are shown in cross section in the top sketch. The tracks are straight, but inclined downwards (all at the same angle). One of the tracks is narrower than the sphere (D) on it, as shown. The friction between the track and the objects is sufficiently high that the sphere, cylinder and wheels all roll. Air resistance and other losses are negligible.

They race in pairs, and are released from rest at the same time.

You may use without proof $I_{sphere} = \frac{2}{5} mR^2$ and $I_{cylinder} = \frac{1}{2} mR^2$

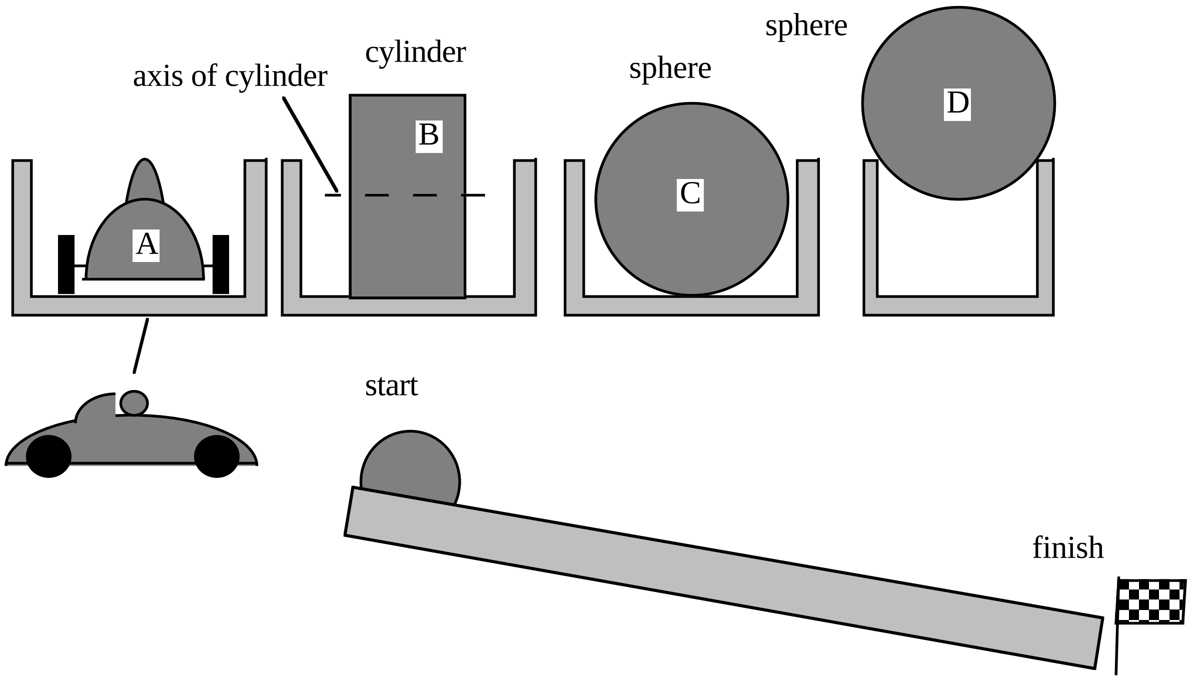
i) In the first race, only A and B compete. Which will win? Explain your answer. (You may use equations if you like, but this is not required. A few clear sentences could be enough.)

Hint: it may be helpful to state some general principles that will be relevant to all of (i), (ii) and (iii).

ii) In the second race, B and C compete. Which will win? Explain your answer. (Here you probably will need an equation or two, plus some explanation.)

iii) In the third race, C and D compete. Which will win? Explain your answer. (You may use equations if you like, but this is not required. A few clear sentences could be enough.)

Question 5 (marks)



In all cases friction acts, but they roll, so there is no relative velocity at the point of contact, so friction does no work. In all cases, they convert the *same* initial amount of gravitational potential energy U_g into kinetic energy. Their kinetic energy includes translational kinetic energy ($K_t = mv^2/2$) and rotational kinetic energy ($K_r = I\omega^2/2$).

i) The wheels of the car have negligible mass and therefore negligible rotational kinetic energy, so all of the U_g is turned into K_t. The cylinder converts the same U_g into both K_t and K_r, so its K_t must be smaller. The car wins.

ii) Initial mechanical energy = final mechanical energy

$$mgh = m\frac{v^2}{2} + I\frac{\omega^2}{2}$$

Rolling on edge, $\therefore$ $v = R\omega$, so

$$mgh = m\frac{v^2}{2} + I\frac{v^2}{2R^2}$$

$I_{sphere} < I_{cylinder}$ $\therefore$ $|v_{sphere}| > |v_{cylinder}|$. Sphere wins.

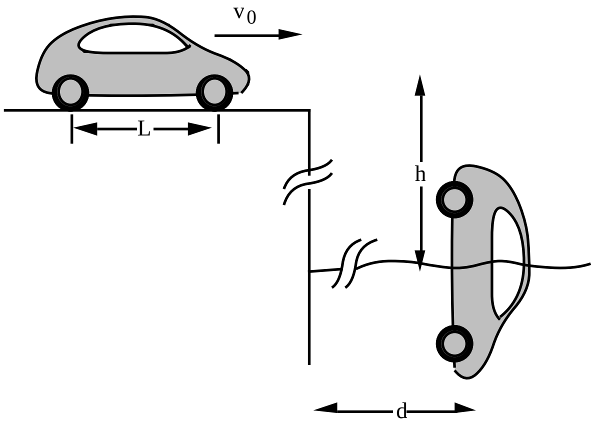
iii) As above, rolling on edge: $mgh = m\frac{v^2}{2} + I\frac{v^2}{2R^2}$

for sphere rolling on r < R $mgh = m\frac{v_D{}^2}{2} + I\frac{v_D{}^2}{2r^2}$ where I is same, C wins

OR, rolling on r <R, same ω gives smaller v, so D loses.

Question 6 (21 marks)

A movie special effects team has consulted you with the following problem. A car (containing only crash test dummies and a radio control unit) is to be driven off a cliff. An underwater camera crew at the base of the cliff will film the action and the team wants the car to enter the water vertically, front first, as shown in the diagram. The car has mass m = 1100 kg, a distance between the wheels L = 2.5 m and a radius of gyration 0.80 m. The centre of mass of the car lies half-way between the front and rear wheels. The cliff height h = 30 m. The crew wants to know the speed v_0 should the car leave the horizontal clifftop. Let's help them. You may make the assumption h >> the dimensions of the car, where necessary.



i) If the car is not rotating as it drives along the clifftop, what will make it rotate? Your answer should include a clear diagram showing the relevant forces acting.

ii) For a fast-moving car, if v_0 is increased, does it make the angular velocity ω of the rotations of the `car during the fall greater or less? Explain your answer in about two clear sentences.

iii) Is there more than one value of v_0 that will achieve the vertical entry? If so why? Explain your answer in one or two clear sentences.

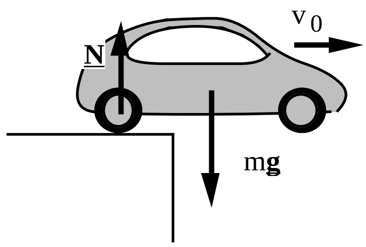
iv) How long will the car take to fall into the ocean from the moment when the wheels leave the cliff? (You may assume that the distance fallen before the rear wheels leave the cliff is negligible.)

v) Showing your working, calculate one of the values of v_0 that satisfies the vertical entry requirement. (You may assume that the angle rotated by the car before the rear wheels leave the cliff is negligible.)

vi) Using your answer to (v), comment briefly but quantitatively on the assumptions in (iv) and (v).

There is no need to verify your answer experimentally, and indeed the School of Physics strongly advises against it.

i) Once the front wheels are no longer supported by the clifftop, the weight exerts an unbalanced torque about the rear axle.



ii) A torque acts on the car while only the rear wheels are in contact with the ground, ie for a time $T = L/v_0$. The longer this torque acts, the greater the final ω, so greater v_0 means smaller ω.

iii) Yes. During the descent to the water, the car could rotate $\frac{1}{4}$ turn, $1\frac{1}{4}$ turns, $2\frac{1}{4}$ turns etc. Each requires a successively greater ω, and so a lower v_0.

iv) initial vertical velocity zero, take initial height = 0, height $y = -\frac{1}{2}gt^2$. We want $-h = -\frac{1}{2}gt^2$,

so $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2*30\text{m}}{9.8\text{ms}^{-2}}} = 2.5$ s.

v) Neglecting the angle rotated of the car between the moments when the front and rear wheels leave the cliff, the torque τ about the rear wheels is mgL/2.

While τ acts, rotational acceleration $\alpha = \frac{\tau}{I}$

τ acts for time $T = \frac{v_0}{L}$, and $I \equiv mk^2$, so

$$\omega = \omega_0 + \alpha T = 0 + \frac{\tau}{I}\frac{L}{v_0} = \frac{mgL}{2mk^2}\frac{L}{v_0} = \frac{gL^2}{2k^2v_0}$$

During the fall, α = 0, so the angle rotated θ = ωt.

θ = (2n+1)π/2, n an even integer (*n odd gives a rear entry, any even n is correct, most will chose n = 0*)

$$(2n+1)\frac{\pi}{2} = \frac{gL^2}{2k^2v_0}t \quad \text{so} \quad v_0 = \frac{gL^2}{k^2(2n+1)\pi}\sqrt{\frac{2h}{g}} = \frac{75\text{ ms}^{-1}}{2n+1} \quad \left(= \frac{270\text{ kph}}{2n+1}\right)$$

So, for this cliff and car, 270 kph for the quarter turn, (90 kph for the rear entry), 54 kph for the front entry after 1.25 turns, 30 kph for the 2.25 turns, etc. The vertical speed is 25 ms^{-1} = 90 kph. So we need to go rather slower than this so that we get a clean entry rather than bouncing off the water.

vi) Let's take one of the slower speeds, eg n = 4 → 8.3 m.s^{-1}.

L = 2.5 m, so the torque acts over 0.30 s, which is 1/8 of the falling time. Both the rotation and the falling start from rest, so only a small fraction of the falling or the turning is done in the first 1/8 of the time.