Higher Physics 1A PHYS1131 2006-S1 Answers

1. 12 Marks Total

(a) A neutral atom is one with no net charge. The number of electrons is the same as the number of protons.

(b) A negatively charged atom has one or more excess electrons over the number of protons.

(c)

(i) Charge 1 +q at (0,*a*), Charge 2 +3q at (*2a, a*), Charge 3 -q at (0, -*a*).

Force between charges q_1 and q_2 a distance r apart is given by $F_{12} = k\frac{q_1 q_2}{r^2}$ directed towards each other if the charges have opposite signs, and away from each other if they have the same sign.

Charges 1 and 2 are *2a* apart.

Thus $F_{12} = k\frac{q\ 3q}{(2a)^2} = 0.75k\left(\frac{q}{a}\right)^2$ directed along the positive x-axis.

Charges 1 and 3 are $\sqrt{(2a)^2 + (2a)^2} = \sqrt{8}a$ apart and the line connecting them is at 45° to the axes.

Thus $F_{23} = k\frac{-q\ 3q}{(\sqrt{8}a)^2} = -0.375k\left(\frac{q}{a}\right)^2$; i.e. directed towards each other.

Thus, taking components along the *x*- and *y*-axes:

x-axis: $F_x = k\left(\frac{q}{a}\right)^2\left[0.75 - 0.375\cos(45°)\right] = k\left(\frac{q}{a}\right)^2 0.485$

y-axis: $F_y = k\left(\frac{q}{a}\right)^2\left[-0.375\sin(45°)\right] = -k\left(\frac{q}{a}\right)^2 0.265$

Therefore, the net force on charge *3q* is given by $\underline{F} = k\left(\frac{q}{a}\right)^2\left[0.485\underline{i} - 0.265\underline{j}\right]$.

(ii) Potential at the origin is given by $V = k\sum\frac{q_i}{r_i}$ where charge q_i is at distance r_i from the origin.

Thus $V = k\left[\frac{q}{a} - \frac{q}{a} + \frac{3q}{\sqrt{a^2 + (2a)^2}}\right] = k\frac{3q}{\sqrt{5}a} = 1.34k\frac{q}{a}$.

2. 6 Marks Total

(a) Applying Gauss's Law, the surface must enclose a positive net total charge, since $\frac{q}{\varepsilon_0} = \Phi > 0$.

(b)

(i) Only the charge inside the radius R contributes to the flux. i.e. applying Gauss's law, $\Phi = \frac{q}{\varepsilon_0}$.

(ii) For a sphere of radius $2a$ we must include the total charge from both the ring and the point charge.

Charge on the ring is $2\pi a\lambda$.

Thus, Gauss's law gives $\Phi = \frac{q + 2\pi a\lambda}{\varepsilon_0}$.

3. 12 Marks Total

(a) The potential energy increases. When a charge is made to move in the direction of the field it moves to a region of lower electric potential. Then the product of the negative charge times the lower potential gives a higher potential energy.

(b) Use a conductive box which completely surrounds the equipment, so shielding it. Any stray electric fields will cause charges on the surface of the box to re-distribute (since it conducts), and so cancel any strong fields inside the volume it encloses. No electric field can then penetrate the box to disrupt the circuit inside it. [Note that varying magnetic fields are also attenuated by eddy currents, but this is not asked for in the question.]

(c) The units of linear charge density must be C/m (charge per unit length). Thus units of α are [charge per unit length / length] = [charge/length^2] = [Cm^{-2}].

The electric potential is given by $V = \int \frac{k.dq}{r}$.

Consider an element of length dx at distance x from the origin.

Its charge, dq, must be $\lambda\, dx = \alpha\, x\, dx$

and the contribution to the potential at the point $x = -D$ therefore $k\, \alpha\, x\, dx\, /\, (x+D)$.

Thus the total potential at the point $x = -D$ is given by (noting that $r = x+D$):

$$V = \int_0^L \frac{k.dq}{r} = \int_0^L \frac{k\lambda}{r} dx = \int_0^L \frac{k\alpha x}{x+D} dx = k\alpha \int_0^L \frac{x}{x+D} dx.$$

Making use of $\int \frac{xdx}{a+1.x} = \frac{x}{1} - \frac{a}{1^2}\ln(a+1.x) = x - a\ln(a+x)$, we have

$$\int_0^L \frac{x}{x+D} dx = \left[x - D\ln(D+x)\right]_0^L = \left[L - D\ln(D+L) + D\ln(D)\right] = L - D\ln\left(1 + L/D\right).$$

Thus $V = k\alpha\left(L - D\ln[1 + L/D]\right)$.

4. 10 Marks Total

(a) Nothing happens to the charge if the wires are disconnected – the capacitors remain charged.

If the wires are now connected to each other, the charges can move along the wires until the entire conductor is at a single potential, and the capacitor discharged. There is now no net charge on the capacitor.

(b) (i) The potential energy of a capacitor is given by $U=0.5CV^2$.

Thus for the two capacitors, each charged to a potential difference ΔV, we have
$U = 0.5C(\Delta V)^2 + 0.5C(\Delta V)^2 = C(\Delta V)^2$.

(ii) The altered capacitor has capacitance $C' = C/2$.

The potential across each capacitor must be the same, since they are in parallel.

The total charge, Q, is the same as before. Let $\Delta V'$ be the potential difference across the capacitors after the separation.

Hence, since $Q=CV$, we have $Q = C\Delta V + C\Delta V$ (before) $= C\Delta V' + C/2\Delta V'$ (after)

so that $2\Delta V = 3/2\Delta V'$;

i.e. $\Delta V' = 4/3\Delta V$

(iii) The potential energy is given by $U=0.5CV^2$

so that the new potential energy is:
$U' = \frac{1}{2}C(\frac{4\Delta V}{3})^2 + \frac{1}{2}\frac{C}{2}(\frac{4\Delta V}{3})^2 = \frac{16}{9}\left(\frac{1}{2}+\frac{1}{4}\right)C(\Delta V)^2 = \frac{4}{3}C(\Delta V)^2$.

(iv) The energy has increased by $\left(\frac{4}{3}-1\right)C(\Delta V)^2 = \frac{C}{3}(\Delta V)^2$.

The extra energy comes from the work put into the system when the plates of the capacitor are pulled apart. This requires a force to be applied because the oppositely charged plates attract.

5 13 Marks Total

(a) Applying the right hand rule (motion up, field out of page), the proton experiences a force directed from left to right across the page, and so it veers to the right.

It will proceed to follow a circle in the clockwise direction, as it always experiences a force perpendicular to its direction of motion.

If instead the particle were a negatively-charged electron the path would veer to the left and then continue to move in a circle in the anti-clockwise direction.

At the same speed, the electron's circle would have a much smaller radius.
(this comes from $\frac{mv^2}{r} = qvB$, hence $r = \frac{mv}{qB}$, but they don't need to prove this)

(b) For each segment we have I = 5.00 A and **B** = 0.020 T **j**.

The force on a current carrying wire is given by **F** = I **l x B**, where **l** is the vector denoting the length and direction of the wire.

Resolve **l** into components along each section of the wire.

(i) For segment *ab* **l** = -0.40 m **j**. Hence **F** = 5.00 x -0.40 x 0.020 **j x j** N = 0 N.

(ii) For segment *bc* **l** = +0.40 m **k.** Hence **F** = 5.00 x 0.40 x 0.020 **k x j** N = -0.040 **i** N.

(iii) For segment *cd* **l** = -0.40 m **i** + 0.40 m **j**. Hence **F** = 5.00 x 0.40 x 0.020 (**-i x j** + **j x j)** N = **-**0.040 **k** N.

(iv) For segment *da* **l** = +0.40 m **i** - 0.40 m **k**. Hence **F** = 5.00 x 0.40 x 0.020 **(i x j - k x j)** N = 0.040 **(k + i)** N.

(c) Since **F =** *q* **v x B**, then the acceleration produced by a magnetic field on a charged particle of mass *m* and charge *q* is **a** = *q/m* **(v x B)**.

This is perpendicular to the direction of the motion.

For the acceleration to change the speed, a component of the acceleration must be along the direction of the velocity.

It is not, so that the constant magnetic force changes the direction of the particle, but not its speed.

6. 9 Marks Total

(a) Ampere's law states that $\oint \underline{B}.d\underline{s} = \mu_0 I$ where the integral is around a closed path and I is the total steady current passing though any surface bounded by the path.

Gauss's law states that $\oint \underline{E}.d\underline{A} = q/\varepsilon_0$ where the integral is over a closed surface and q is the charge enclosed by that surface.

Both laws use the concept of "flux" – the "flow" of field lines through a surface to determine the field strength.

They also relate the integral of the field over a closed geometrical figure to a fundamental constant multiplied by the source of the appropriate field.

The geometrical figure is a surface for Gauss's law and a line for Ampere's law; Ampere is the 2D analogue of Gauss.

(b) From Ampere's law, the magnetic field at point a is given by $B_a = \dfrac{\mu_0 I_a}{2\pi r_a}$ where I_a is the net current through the area of the circle of radius r_a, which in this case is 1.00A out of the page.

Hence $B_a = \dfrac{(4\pi 10^{-7}\ Tm/A)(1.00\ A)}{2\pi(1.00\times10^{-3}\ m)} = 2.00\ 10^{-4} = 200\ \mu T$ towards the top of the page (direction from right hand rule).

Similarly, at point b: $B_b = \dfrac{\mu_0 I_b}{2\pi r_b}$ where I_b is the net current through the area of the circle of radius r_b, which in this case is 1.00-3.00 A = -2.00 A into the page.

Therefore $B_b = \dfrac{(4\pi 10^{-7}\ Tm/A)(2.00\ A)}{2\pi(3.00\times10^{-3}\ m)} = 133\ \mu T$ towards the bottom of the page.

7. 8 Marks Total

(a) Faraday's Law of Induction states that $\varepsilon = -\frac{d\Phi}{dt}$; i.e. the emf, ε, generated in a circuit is proportional to minus the rate of change of magnetic flux, Φ, through that circuit.

(b) $\varepsilon = -\frac{d\Phi}{dt}$ with $\Phi=BA$, and A the area of the flux linked.

Thus $\varepsilon = -B\frac{dA}{dt}$

where $\frac{dA}{dt}$ is the rate of change of area = av (as only side a is cutting new flux).

So $\varepsilon = -Bav$.

Now $\varepsilon = IR$ for a current I, so that $IR=Bav$, taking the absolute value.

In equilibrium (i.e. constant speed, v), the weight balances the opposing force.

The force on a current carrying conductor is given $F=BIa$,

and from Lenz's law it must be upwards (so as to oppose the change).

Thus $mg = BIa$ with $I = Bav/R$ (from above).

Hence $\frac{B^2 av}{R} = mg$

or $B = \sqrt{\frac{mgR}{a^2 v}} = \sqrt{\frac{0.5\ \ 9.8\ \ 2}{1^2\ \ 8}} = 1.11T$.