Solution Question 1 (a)
Both 1121 and 1131:

(i) $\mathbf{v} = \frac{d\mathbf{r}}{dt} = \left(\frac{d}{dt}\right)\left(3.00\hat{i} - 6.00t^2\hat{j}\right) = \boxed{-12.0t\hat{j}\ \text{m/s}}$

$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \left(\frac{d}{dt}\right)\left(-12.0t\hat{j}\right) = \boxed{-12.0\hat{j}\ \text{m/s}^2}$

(ii) $\boxed{\mathbf{r} = \left(3.00\hat{i} - 6.00\hat{j}\right)\text{m};\ \mathbf{v} = -12.0\hat{j}\ \text{m/s}}$

Solution Question 1 (b)
Both 1121 and 1131:

$\mathbf{a} = 3.00\hat{j}\ \text{m/s}^2;\ \mathbf{v}_i = 5.00\hat{i}\ \text{m/s};\ \mathbf{r}_i = 0\hat{i} + 0\hat{j}$

(i) $\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2 = \boxed{\left[5.00t\hat{i} + \frac{1}{2}3.00t^2\hat{j}\right]\ \text{m}}$

$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t = \boxed{\left(5.00\hat{i} + 3.00t\hat{j}\right)\ \text{m/s}}$

(ii) $t = 2.00\ \text{s},\ \mathbf{r}_f = 5.00(2.00)\hat{i} + \frac{1}{2}(3.00)(2.00)^2\hat{j} = \left(10.0\hat{i} + 6.00\hat{j}\right)\text{m}$

so $x_f = \boxed{10.0\ \text{m}}$, $y_f = \boxed{6.00\ \text{m}}$

$\mathbf{v}_f = 5.00\hat{i} + 3.00(2.00)\hat{j} = \left(5.00\hat{i} + 6.00\hat{j}\right)\ \text{m/s}$

$v_f = \left|\mathbf{v}_f\right| = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(5.00)^2 + (6.00)^2} = \boxed{7.81\ \text{m/s}}$

Solution Question 1 (c)
PHYS1131 only:

(i) $a_t = \boxed{0.600\ \text{m/s}^2}$

(ii) $a_r = \frac{v^2}{r} = \frac{(4.00\ \text{m/s})^2}{20.0\ \text{m}} = \boxed{0.800\ \text{m/s}^2}$

(iii) $a = \sqrt{a_t^2 + a_r^2} = \boxed{1.00\ \text{m/s}^2}$

$\theta = \tan^{-1}\frac{a_r}{a_t} = \boxed{53.1°\ \text{inward from path}}$

i) If non-conservative forces do no work, mechanical energy is conserved.

ii) If the external forces on a system are zero, the momentum of that system is conserved.

iii) Consider first the swing before the collision. Let the mass have speed v just before the collision. In this phase, air resistance is negligible, so non-conservative forces do no work, therefore mechanical energy is conserved, which we may write as

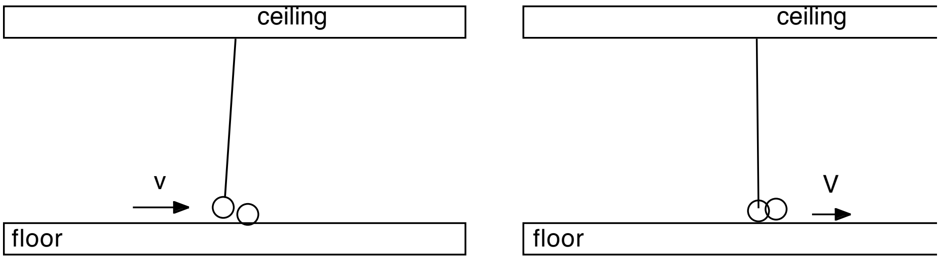
$$\Delta E = \Delta K + \Delta U = 0$$

$$(\tfrac{1}{2} mv^2 - 0) + (0 - mgR) = 0$$

$$v^2 = 2gR$$

$$v = \sqrt{2gR}$$

Next, consider the collision. Let the combined object have speed V after the collision.



1131 The forces between the two objects during the collision will be much greater than the external force (friction) in the horizontal direction. Therefore momentum (in the x direction) is approximately conserved.

1121 In the x direction, the only external force acting during the collision is friction, which we are told is negligible. Therefore momentum in the x direction is conserved.

$$p_{initial} = p_{final}$$

$mv = 2mV$. Rearranging and substituting for v:

$$V = v/2 = \sqrt{gR/2}$$

Finally, consider the rising arc. During this phase, air resistance is still negligible, so non-conservative forces do no work, therefore mechanical energy is conserved, which we may write as

$$\Delta E = \Delta K + \Delta U = 0$$

$(0 - \tfrac{1}{2}(2m)V^2) + (2mgD - 0) = 0.$ Rearranging gives

$D = V^2/2g$ and substitution gives

$= R/4$

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iv) In both states, there is no kinetic energy.

$\Delta E = \Delta K + \Delta U = 0 + \Delta U$ Taking the floor as the zero for gravitational potential energy,

$= 2mgR/4 - mgR = -mgR/2.$

The internal forces during the collision are non-conservative, and they do negative work. (The ball is irreversibly deformed, so work around a closed loop is not zero.) OR: Mechanical energy is lost as heat and in plastic deformation of the ball (and even sound).

Newton's second law applied to *m*:

$$T_2 - mg = ma$$

so $$T_2 = m(a + g) \qquad (1)$$

Let the pulleys turn with angular acceleration α in the clockwise direction. Newton's second law for rotation applied to the disk:

$$\Sigma\tau = I\alpha \qquad \text{so}$$

$$RT_1 - rT_2 = I\alpha \qquad (2)$$

$$\alpha = \frac{RT_1 - rT_2}{I} \text{ clockwise } \left(\text{or } -\frac{RT_1 - rT_2}{I} \text{ anticlockwise}\right)$$

Because the cord doesn't slip, $\alpha = a/r$. Making that substitution and using (1), (2) becomes

$$RT_1 - rm(a + g) = Ia/r$$

$$a(I/r + rm) = RT_1 - rmg$$

$$a = \frac{RT_1 - rmg}{I/r + rm}$$ which is fine for an answer, though we might 'neaten' to

$$a = \frac{RT_1/rm - g}{I/mr^2 + 1}$$

Question 4.

(a) (i) 0 °C as in thermal equilibrium with ice at atmospheric pressure.

(ii)

$$Q = mL$$
$$= 0.00180 \times 3.33 \times 10^5$$
$$= 599J \text{ (3 sig fig)}$$

(iii) Q lost by lead = Q gained by water, ice and copper

$$m_L c_L(255 - T_f) = 599 + (m_c c_c + (m_w + m_i)c_w)(T_f - 0)$$
$$97.5(255 - T_f) = 599 + 716 \times T_f$$
$$T_f = \frac{24263.5}{97.5 + 716}$$
$$T_f = 29.8^o C$$

(b) (i)

$$n = \frac{m}{M} = \frac{3000}{32} = 93.75$$
$$A = 1.00m^2 \Rightarrow V = 1m^3$$
$$PV = nRT$$
$$\Rightarrow P = \frac{nRT}{V}$$
$$= \frac{93.75 \times 8.314 \times (273 - 70)}{1.00}$$
$$= 158225.8 \text{ Pa}$$
$$= 158 \text{ kPa}$$

(ii)

$$m = \frac{M}{N_A} = \frac{32 \times 10^{-3}}{6.022 \times 10^{23}} = 5.3138 \times 10^{-26} \text{ kg}$$
$$\frac{3}{2}k_B T = \frac{1}{2}m\overline{v^2}$$
$$\overline{v^2} = \frac{3 \times 1.381 \times 10^{-23} \times (273 - 70)}{5.3138 \times 10^{-26}}$$
$$= 158271$$
$$\Rightarrow v_{rms} = 398 \text{ m/s}$$

(iii)

1131 only.

$$E_{int} = \frac{f}{2}nRT$$
$f = 5$ (3 translational and 2 rotational at -70° C)
$$E_{int} = \frac{5}{2} \times 5 \times 8.314 \times (273 - 70)$$
$$= 21100J$$

(c) (i) $$W = -\int_1^3 PdV$$
$$= -(3-1) \times 3 \times 1.01 \times 10^5$$
$$= -606 \text{ kJ}$$

(ii) isothermal $\Rightarrow \Delta E_{intD\to A} = 0$

(iii) $\Delta E_{int} = Q + W$
$\Delta E_{intA\to B} = 400 - 606 = -206kJ$

$$W_{C\to D} = -\int_{5.79}^{3.0} 1.01 \times 10^5 dV = 282kJ$$

$\Delta E_{intC\to D} = 407 + 282 = 689kJ$
$\Delta E_{intB\to C} = -(\Delta E_{intC\to D} + \Delta E_{intD\to A} + \Delta E_{intA\to B})$
$= -(689 + 0 - 206)$
$= -483kJ$
Alternatively they could calculate $W = -\int PdV = \Delta E_{int}$
with PV^γ= constant = 1898.
To solve the integral substitute in $P = 1898V^\gamma$.
As it is an ideal monatomic gas $\gamma = 1.67$.

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(iv) $PV^{1.67} = 3 \times 1.01 \times 10^5 \times 3^{1.67} = 1.90 \times 10^6$

$$\gamma = \frac{c_P}{c_V} = \frac{f+2}{f} = \frac{5}{3} = 1.67$$

PV^γ = const can be evaluated at any point eg. $P = 3.0$ atm, $V = 3.0$ m^3

Q5

30 marks 1131 / 25 marks 1121

(a)

(i) For each spring we have $F=-kx$ where F is the restoring force and x the displacement from equilibrium. There are 4 springs, and so the SHM is governed by:

$m_c\ddot{x} = -4kx$ where m_c is the mass of the car plus occupants = $(M+2m)$.

Thus $\omega^2 = 4k/(M+2m)$ governs the angular frequency of the motion.

Then $f = \omega/2\pi = 1/2\pi \bullet \sqrt{4k/m_c} = 1/2\pi \bullet \sqrt{\frac{4 \bullet 8000}{1500+2 \bullet 80}} = 0.6988$ s

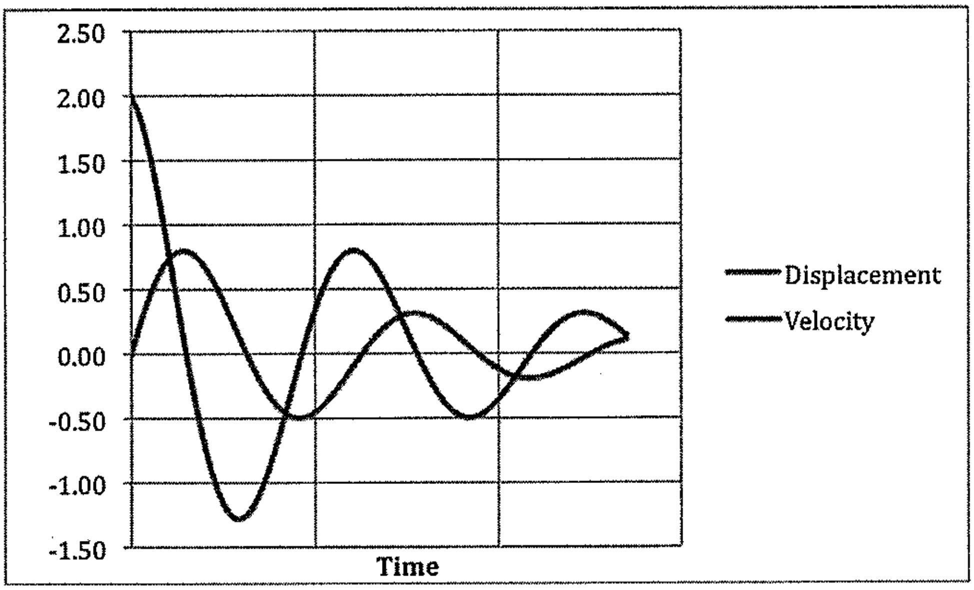
Thus f=0.70s to 2SF

(ii) We simply replace $m_c=M+2m$ by M when there are no occupants in the car.

Thus $f = \omega/2\pi = 1/2\pi \bullet \sqrt{4k/m_c} = 1/2\pi \bullet \sqrt{\frac{4 \bullet 8000}{1500}} = 0.7351$ s

So f=0.74s to 2SF

(iii)



Full derivation not necessary, but answer should show sinusoidally damped displacement and velocity, with velocity at a maximum (either positive or negative) at $t=0$, with displacement 0 at $t=0$. Displacement and velocity are $\pi/2=1/4$ of a cycle out of phase. [strictly speaking this doesn't hold as time increases, but we are in lightly damped case, so the discrepancy is small]. Successive peaks should only be a little smaller than the previous one.

$$x = A\sin(\omega t)e^{-\beta t} \text{ with } x = 0 \text{ at } t = 0$$

Full derivation: $v = \dot{x} = \omega A\cos(\omega t)e^{-\beta t} - \beta A\sin(\omega t)e^{-\beta t}$

$$\therefore v = \omega A\cos(\omega t)e^{-\beta t} - \beta x$$

Lightly damped so that damping constant β is small.

(b)

(i) We have
$$y_1 = A\sin(kx - \omega t)$$
$$y_2 = A\sin(kx + \omega t)$$

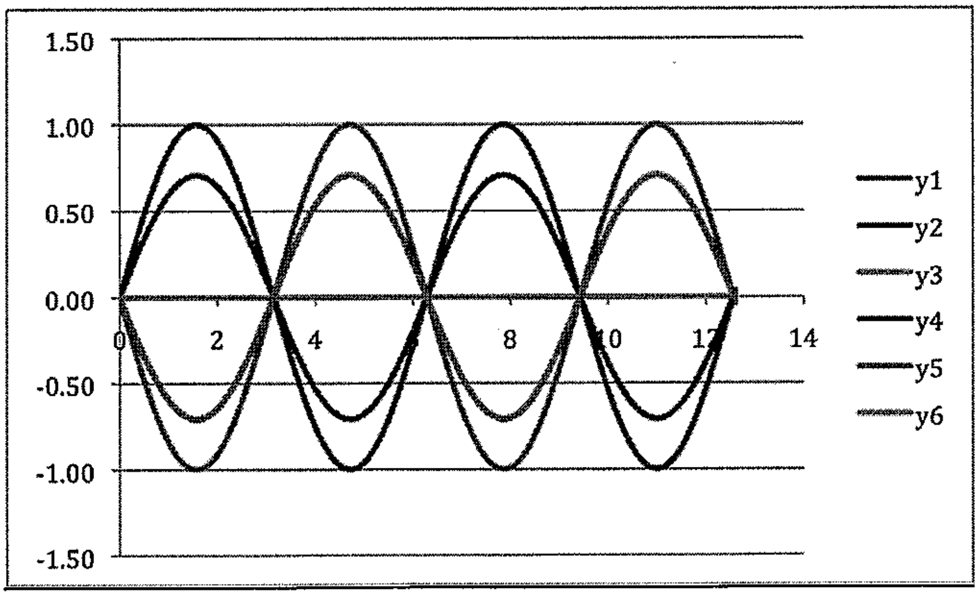
where the – wave travels in the +*x*-direction and the + wave travels in the –*x* direction.

y_1 and y_2 are the displacements from equilibrium.

(ii) Then their sum
$$\begin{aligned} y = y_1 + y &= A\sin(kx - \omega t) + A\sin(kx + \omega t) \\ &= 2A\sin(\frac{kx + kx}{2})\cos(\frac{\omega t + \omega t}{2}) \\ &= 2A\sin(kx)\cos(\omega t) \end{aligned}$$

(iii) This is a standing wave because the spatial and time components are separated. The wave is not travelling along the *x*-direction, but each particle in the medium is simply oscillating about its position in phase (or anti-phase) with every other particle, and with amplitude dependent on its position.

(iv) General form of graph. Axes are distance (*x*-direction) and displacement (*y*-direction), with different plots showing different times. Note that quantitative values are not needed. The plot below actually assumes $A = k = \omega = 1$. Amplitude is $2A$ and half-wavelength is π/k. The period is $2\pi/\omega$.



Note: if the student used $\begin{array}{l} y_1 = A\cos(kx - \omega t) \\ y_2 = A\cos(kx + \omega t) \end{array}$ for the travelling wave their derivation they should then obtain $y = 2A\cos(kx)\cos(\omega t)$, and their graph above should show an anti-node at the origin, x=0.

(c)

(i) The open end is an antinode and the closed end is a node.

First resonance then given by $L_1 = \lambda/4 + \varepsilon$ ε = end effects.

Second resonance is given by $L_2 = 3\lambda/4 + \varepsilon$

In general the n-th resonance is given by $L_n = (2n-1)\lambda/4 + \varepsilon$.

The distance between 2 successive resonances is simply $\Delta L = \lambda/2$

Since $v = f\lambda$ then $v = f2\Delta L = 376 \cdot 2 \cdot [56.2 - 10.2]/100$ m/s $= 345.9$ m/s.

Hence v=346 m/s to 3SF.

(ii) Each resonance is separated by $\Delta L = \lambda/2$ so that the next resonance is at 56.2+[56.2-10.2]=102.2cm

(iii) For a closed pipe at both ends then resonance is given when $n\left(\frac{\lambda}{2}\right) = L_n$ since there are a whole number of half-wavelengths for a node at each end. Given L=102.2cm, then $n = 2L/\lambda$ with $\lambda = v/f = 345.9/376 = 0.920$ m.

Then we find that $n = 2. \cdot 102.2/92.0 = 2.22$.

Thus the closest node has n=2, for which $L = n\lambda/2 = 2/2 \cdot 0.920 = 0.920$ m $= 92$ cm.

Thus the piston needs to be moved 102.2-92.0=10.2cm nearer to the other closed end for the nearest resonance position.