TEST 1R

This is the repeat version of TEST 1, which was held during Session.

This repeat test should be attempted by those students who missed Test 1, or who wish to improve their mark in Test 1.

IF YOU ARE ATTEMPTING TEST 1 (Repeat):

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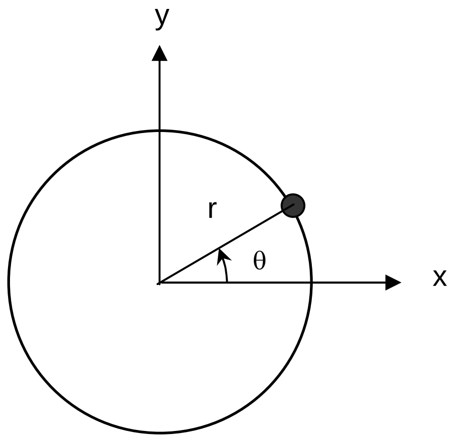
AND INDICATE YES ON THE FRONT (TITLE) PAGE.

QUESTION 5 **[Marks 17]**

A particle executes uniform circular motion with radius r, as shown in the figure.



(a) From the figure, derive an expression for the position vector **r** in terms of r, the angular velocity ω, and the time t by projecting **r** onto the x and y axes. Take the angle θ at t = 0 to be zero.

(b) From (a), derive the velocity **v** and the acceleration **a** of the particle. Express the result for the acceleration in terms of the position vector.

(c) Calculate the scalar product **v** · **r**. What does this say about the relative orientation of **v** and **r**?

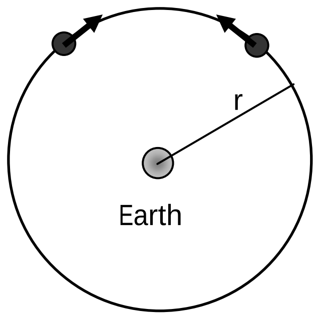
A stone is tied to a string and is swung in a horizontal circle of radius 1.2m at 1.8m above the ground. The string breaks and the stone travels a horizontal distance of 9.0m before striking the ground.

(d) What was the centripetal acceleration of the stone while it was in circular motion?

(e) The shadow of a particle undergoing uniform circular motion is projected onto a screen that is perpendicular to the plane of the circle. Demonstrate that the shadow executes simple harmonic motion.

QUESTION 6 **[Marks 17]**

A satellite of mass m is in a circular orbit of radius r about the Earth with mass M_e and radius R_e.



(a) Derive an expression for the kinetic energy of the satellite in terms of the gravitational constant G, m, M_e, and r.

(b) Hence, derive an expression for the launch speed of the satellite from Earth required to place it in this orbit. Show that your result reduces to the escape speed from the Earth when $r \to \infty$.

Two satellites are in circular orbits about the Earth at radius r and are on a collision course, as shown in the figure.

(c) Derive an expression for the total mechanical energy for the two-satellite-plus-Earth system before the collision in terms of G, m, M_e, and r.

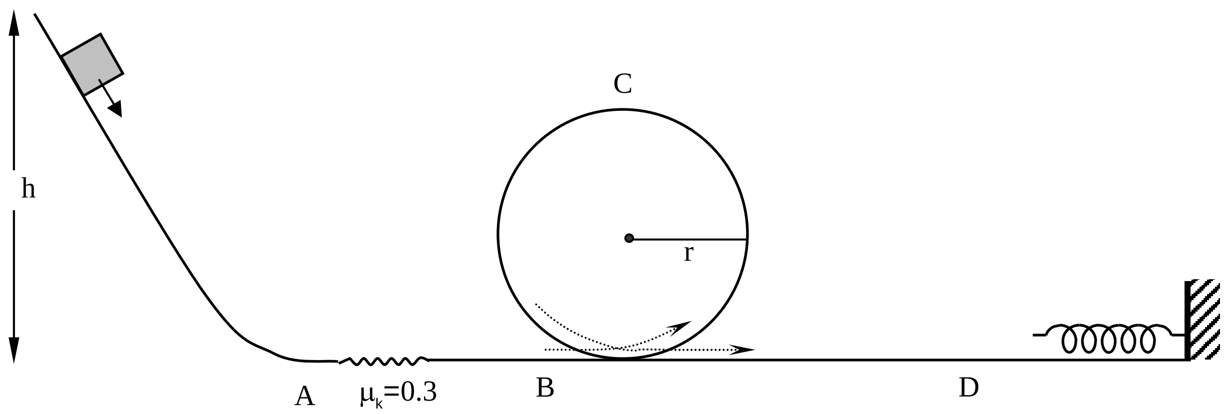
The satellites collide completely inelastically (the wreckage stays in one piece after the collision).

(d) What is the total mechanical energy of the system after the collision?

(e) Describe the subsequent motion of the wreckage.

<u>QUESTION 7</u> **[Marks 16]**

A block is released from a height h on a frictionless incline. At the base of the incline there is a rough surface of length ℓ and the coefficient of kinetic friction between the block and the surface is $\mu_k = 0.3$. The surface becomes frictionless again before a circular loop of radius r which is followed by a spring attached to a wall. The block remains in contact with the surface at all times. The set-up is shown in the figure; different points of the track are labelled A, B, C, D.



(a) What is the speed of the block at point A in terms of the height h?

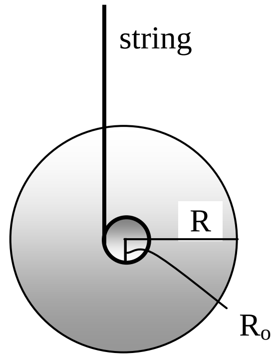
(b) What is the speed of the block at point B in terms of the given parameters?

(c) At point C the block is at the maximum height inside the loop.
  - (i) What is the speed of the block at this point? Express your answer in terms of the given parameters.
  - (ii) Draw a free-body diagram for the forces acting on the block at point C.
  - (iii) Find the minimum height h such that the block maintains contact with the loop at point C. Take ℓ = 3cm and r = 5cm.

(d) The block is brought to rest beyond point D after compressing the spring by 3.0cm. Determine the value for the spring constant k, if ℓ = 3cm, r = 5cm, h = 18cm, and the mass of the block m = 0.025kg.

QUESTION 8 **[Marks 17]**

A yo-yo consists of two discs of radius R connected by a shaft of radius R_0 around which string is wrapped (see figure). The yo-yo has a mass m = 0.020kg, R = 3.0cm, and R_0 = 0.30cm. Assume that the rotational inertia of the yo-yo about the centre of mass is the same as that of a disc, $I = mR^2/2$. The string is held while the yo-yo is released from rest; as the yo-yo falls, the string unwinds without slipping.



(a) Draw a free-body diagram for the yo-yo. Which of the forces produces a torque about the centre of mass?

(b) Write down Newton’s second law for the torque about the centre of mass in terms of the force(s).

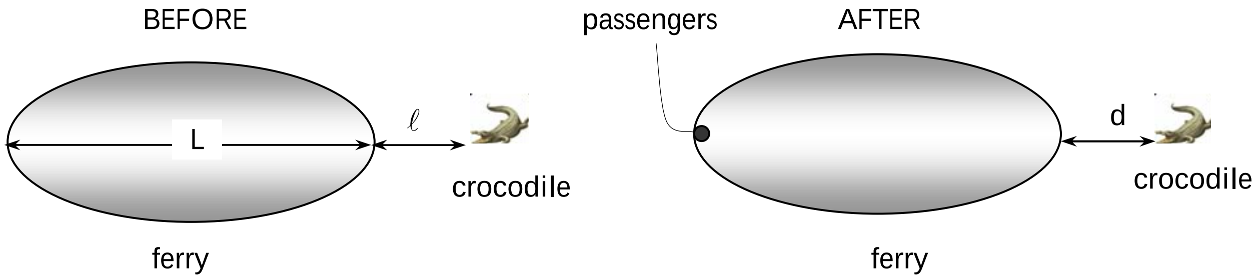
(c) Write down Newton’s second law for the forces acting in the vertical direction.

(d) Hence, find expressions in terms of m, g (acceleration due to Earth's gravity), R, and R_0 for
 (i) the tension in the string T
 (ii) the linear acceleration *a* of the yo-yo.

(e) At the instant when the string is fully unwound, the yo-yo rotates with angular velocity ω_0. What is the minimum value for ω_0 required for the yo-yo to climb a distance l = 0.8m up the string (no slipping)?

QUESTION 9 **[Marks 17]**

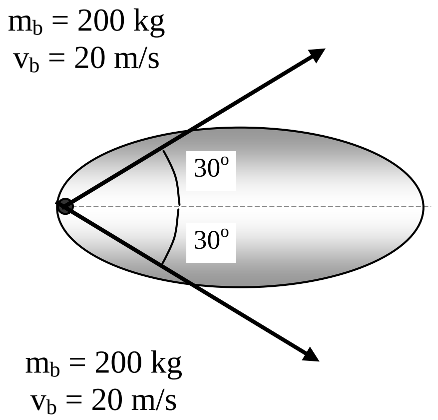
A ferry on a river is at rest relative to the water. A passenger spots a crocodile at rest in the water ℓ = 8.0m from the front of the ferry. In response, the 50 passengers on board run to the far end of the ferry, trying to get as far from the crocodile as possible. The ferry has a mass M =3500kg and length L = 30m; its centre of mass lies at the centre of the ferry. The average mass of a passenger is 75kg. Assume that originally the passengers were evenly distributed around the ferry and after the scare they are all a distance L/2 from the centre. See the figure.



(a) How far from the crocodile is the ferry (d in the figure) after the passengers reach the end? [Hint: Consider the centre of mass of the ferry and passengers.]

(b) What is the speed of the ferry (relative to the water) after the passengers reach the end?

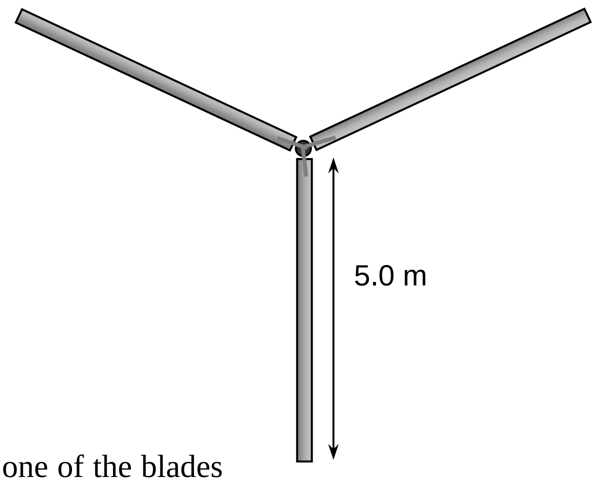
(c) The engine of the ferry will not start. In desperation, the passengers decide to construct a couple of catapults and throw their belongings into the river so that the ferry will recoil. The masses m_b and velocities v_b of the belongings and the angles at which they're thrown are shown in the figure. Calculate the final velocity of the ferry with respect to the water.



(d) A person on the bank watches in amusement. What is the final speed of the ferry according to this observer if the river moves at a speed 3km/h in the same direction as the ferry?

QUESTION 10 **[Marks 16]**

A helicopter propeller consists of three blades, each of length 5.0m and mass 220kg, with the axis of rotation through the centre.



(a) Calculate the rotational inertia of a rod of length L and mass M about an axis at the end of the rod. You may use the formula $I = {}^{M}\!/_{L} \int x^2 dx$.

(b) Hence, calculate the rotational inertia of the propeller.

(c) A block of metal of mass m = 25kg is stuck to one of the blades at a distance l = 2.0m from the centre. Calculate the rotational inertia of the block+propeller system.

(d) A motor provides a constant torque τ = 7000Nm along the axis of the propeller. Calculate the time it takes for the propeller to reach an angular speed of 2000rpm from rest. [rpm ≡ revolutions per minute; remember, 1 revolution= 2π radians.]

(e) What distance has the block travelled in this time?

(f) When the angular speed 2000rpm is reached, the motor is turned off. The metal block shifts 50cm further from the axis. Calculate the new angular speed of the propeller.