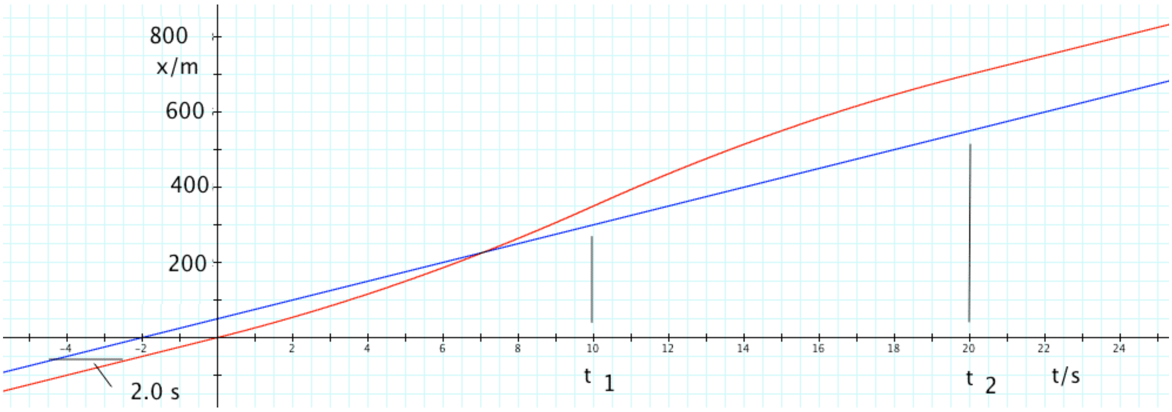
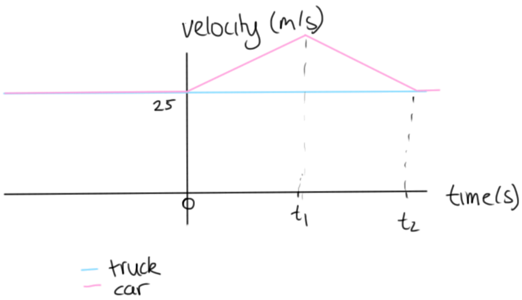
Question 1

i) (a sketch similar to this)



ii)



iii) v_0 = 90 k.p.h. = (90,000 m /3600 s) = 25 m.s^{-1}

Car starts at x = 0, truck starts at x = d

where d = (2 s * v_0) = 2 s * 25 m.s^{-1} = 50 m.

iv) $x_{truck} = v_0 t + d$

$x_{car} = v_0 t + ½at^2$ while acceleration is positive

car is 50 m ahead when

$(v_0 t + ½at^2) – (v_0 t + d) = 50$ m

$½at_1^2 = 100$ m

$t_1 = \sqrt{2 * 100m / 2.0m.s} = 10$ s

v) $x_1 = v_0 t_1 + ½at_1^2 = 25*10 + ½*2*10^2 = 350$ m

vi) and vii) See graph in part (i) and (ii) working for calculating t_2 and x_2 is below.

For the truck: $x_{2,truck} = v_0 (t_2 + 2.0 \text{ s})$ (i.e. it arrives 2 seconds later)

For the car: $t_2 = 2t_1 = 20$ s (as it accelerates and decelerates with same magnitude so time spent accelerating is same as time spent decelerating.

There are other ways to calculate this as well such as finding velocity at t_1 and then working out time for car to get back to 25m/s)

$x_{2,car} = x_1 + v_0 (t_2 - t_1) + \frac{1}{2}a(t_2 - t_1)^2$

$= 350 + 45*10 - 0.5*2*100 = 700$ m

$x_{2,truck} = 25*22 = 550$ m

viii) $v_{max} = v_0 + at_1 = 25 \text{ m.s}^{-1} + (2 \text{ m.s}^{-2})(10 \text{ s}) = 45 \text{ m.s}^{-1}$ (= 162 k.p.h.)

(*and lots of points off the licence*)

Question 2

a)

i) $\boldsymbol{v}_W = -v_W \cos\theta\,\mathbf{i} - v_W \sin\theta\,\mathbf{j}$

ii) $\boldsymbol{v}_W = \boldsymbol{v} + \boldsymbol{v}'$ where $\boldsymbol{v}'$ is the apparent wind velocity

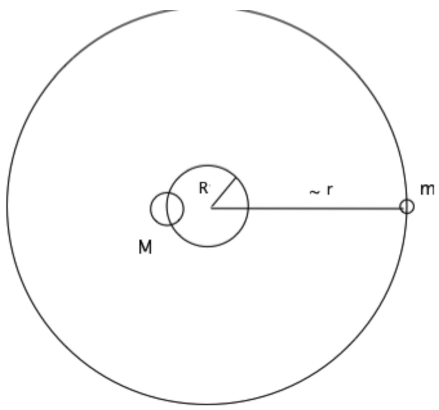
$\boldsymbol{v}' = \boldsymbol{v}_W - \boldsymbol{v} = -v_W \cos\theta\,\mathbf{i} - v_W \sin\theta\,\mathbf{j} - v\,\mathbf{j}$

$\boldsymbol{v}' = -v_W \cos\theta\,\mathbf{i} - (v + v_W \sin\theta)\,\mathbf{j}$

iii) $v' = (v_W^2 \cos^2\theta + (v^2 + 2vv_W \sin\theta + v_W^2 \sin^2\theta))^{1/2}$

$= (v_W^2 + v^2 + 2vv_W \sin\theta)^{1/2}$

b)



i)

Note that the radius of the planet's orbit is approximately a circle with radius r, because r >> R

ii) Kepler's law of periods

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

Rearranging $r = \sqrt[3]{\dfrac{GMT^2}{4\pi^2}}$ *The longer ways of getting here also score.*

iii) Either:

Newton's 3rd law:

|Force on star| = |force on planet|

$Ma_{star} = ma_{planet}$

$MR\omega^2 = mr\omega^2$

$m = MR/r$

or

Newton's 2nd law and law of gravitation

$Ma_{star} = |F_{grav}|$

$MR\omega^2 = GMm/r^2$

$m = r^2R\omega^2/G = 4\pi^2r^2R/GT^2$

or anything else correct.

Question 3

i) Before the collision, there are no non-conservative forces acting, so mechanical energy is conserved.

$U_i + K_i = U_f + K_f$ or just $E_i = F_f$

Immediately before the collision, mass m is travelling horizontally with speed v. So

$mgR + 0 = 0 + ½mv^2$

so $v = \sqrt{2gR}$

During the collision, no external horizontal forces act, therefore momentum is conserved. Take the speed of the combined object as V. So

$mv = (m + M)V$.

$V = mv/(m + M)$

$= m\sqrt{2gR}/(m + M)$ or $= \sqrt{2gR}/(1 + M/m)$

ii) During the collision there is no vertical motion so the normal force between the two masses equals the weight of m, i.e. $N = mg$

So, while there is relative motion between m and M, the friction between them is $\mu_k N = \mu_k mg$

Work energy theorem or conservation of energy:

Work done by non-conservative forces = change in mechanical energy = $\Delta U + \Delta K$. Let d be the distance of travel between m and M.

$-\mu_k mg\, d = \Delta U + \Delta K$

$-\mu_k mg\, d = ½(m+M)V^2 - ½mv^2$ OR $= ½(m+M)V^2 - mgR$

$\mu_k mg\, d = ½mv^2 - ½(m+M)(m\sqrt{2gR}/(m + M))^2$

$\mu_k mg\, d = ½m2gR - (m^2gR/(m + M))$

$\mu_k\, d = R - (mR/(m + M))$

$d = R(M/(m + M)\mu_k)$

4 a) i)
|Heat energy gained by water and bucket| - |heat energy lost by axe head| = 0

$$
\begin{aligned}
& m_w c_w (T_f - T_{wi}) + m_b c_s (T_f - T_{wi}) + m_{ah} c_s (T_f - T_{ahi}) = 0 \\
& 12.5 \times 4186 \times (T_f - 24.3) + 0.500 \times 456 \times (T_f - 24.3) + \\
& + 0.755 \times 456 \times (T_f - 337) = 0 \\
& \Rightarrow T_f (344.28 + 52325 + 228) = 1406830 \\
& T_f = \frac{1406830}{52553} = 26.60 \\
& T_f = 26.6^o\text{C (3 sig. fig)}
\end{aligned}
$$

ii)
In this case the total heat loss is given by:

$$
\begin{aligned}
& Q = 0.755 \times 456 \times (25.4 - 337) + 4186 \times 12.5 \times (25.4 - 24.3) + \\
& + 0.500 \times 456 \times (25.4 - 24.3) \\
& = -49469
\end{aligned}
$$

This heat goes into raising some of the water from 25.4 to 100 ^{o}C and then converting this into steam.

$$
\begin{aligned}
& Q = m_s L_{vap} + m_s c_w \Delta T \\
& 49469 = m_s (2.26 \times 10^6 + 4186 \times (100 - 25.4)) \\
& \Rightarrow m_s = 0.0192 kg \\
& = 19.2 g
\end{aligned}
$$

b) i) This is a diatomic gas at approximately room temperature. It has 5 degrees of freedom. 3 of these degrees of freedom correspond to the translational movement of the particles (one in x, one in y and one in z direction) and 2 correspond to the rotational movement of the molecules (abut the two axes perpendicular to the line joining the two atoms).

ii) This gas is kept at a constant volume by the air tight seals so:

$$Q = nC_V \Delta T$$

Need to calculate *n* using the ideal gas law:

$$
\begin{aligned}
n &= \frac{PV}{RT} \\
&= \frac{1.01 \times 10^5 \times 5.00 \times 5.00 \times 2.00}{8.314 \times 273.15} \\
&= 2223.72
\end{aligned}
$$

Alternatively use the fact that at 0.00 ^{o}C 1 mol of ideal gas takes up 2.241×10^{-2} m^3 to calculate *n*, this gives *n* = 2 231 mols.

$$
\begin{aligned}
Q &= 2223 \times (\tfrac{1}{2} f R) \times 24.5 = 1132 kJ \\
&= 1130\text{kJ (3 sig. fig)}
\end{aligned}
$$

Or if using *n* calculated the second way you get Q = 1136 kJ = 1140 kJ (3 sig fig) which is also correct.

c) i) $W = -\int P.dV$

which is just the area under the rectangle

$W = -3.00 \times 1.01 \times 10^5 \times (2.000 - 0.6347) \times 10^{-3}$

$= -413.685J = -414\text{J (3 sig fig)}$

ii) $\Delta E_{intB\to C} = Q_{B\to C} + W_{B\to C}$

$= 1033 - 414 = 619J$

iii) $\Delta E_{intC\to D} = 0$ isothermal process

$W_{C\to D} = \Delta E_{intC\to D} - Q_{C\to D}$

$= 0 - 664 = -664J$

Alternatively could integrate

$$W_{C\to D} = -\int P.dV$$

With P = nRT/V but this takes a lot longer.....

iv) Around a cycle there is no change in the internal energy:

$\Delta E_{intA\to B} + \Delta E_{intB\to C} + \Delta E_{intC\to D} + \Delta E_{intD\to A} = 0$

$\Delta E_{intD\to A} = 1.00 \times 1.01 \times 10^5 \times (6.000 - 1.227) \times 10^{-3} - 1203 = -721J$

$\Delta E_{intA\to B} + 620 + 0 - 721 = 0$

$\Delta E_{intA\to B} = 101J$

a)

i)

$3\lambda = 2.40$ m (as there are 6 loops, i.e. 6 half wavelength)

$\lambda = 0.800m$

$v = f\lambda = 50 \times 0.800 = 40ms^{-1}$

ii)

$v = \sqrt{\frac{T}{\mu}}$

$T = mg = \mu v^2$

$\Rightarrow m = \frac{0.234\times10^{-3}\times40^2}{9.8} = 0.0382kg = 38.2g$

iii) for next harmonic $\lambda = \frac{2.40}{7/2} = 0.6857m$

$v = 50 \times 0.6857 = 34.2857$

$\Rightarrow m = \frac{\mu v^2}{g} = \frac{0.234\times10^{-3}\times34.2857^2}{9.8} = 28.06g$

so need to remove 38.2 - 28.1 = 10.1 g.

b) i) As observed frequency is less than emitted frequency and the observer is stationary the source (plane) is moving away from Stan, ie. Receding.

ii)

$f' = f(\frac{c}{c+v_s})$

$\frac{2}{3}f = f(\frac{c}{c+v_s})$

$\frac{2}{3} = \frac{c}{c(1+v_s/c)}$

$\Rightarrow v_s = \frac{1}{2}c = 170 \text{ ms}^{-1}$

iii)

$40.0\text{km/h} = 11.11\text{ms}^{-1}$

$f' = f_p(\frac{c+v_o}{c+v_s})$

$= f_p(\frac{340+11.11}{340+170})$

$= 0.688f_p = 0.69f_p$ (2 sig. fig.) either 2 or 3 sig figs is acceptable.

c)

i)

Make use of conservation of momentum.

$mv = (m + M)v'$

$v' = \frac{10.5\times10^{-3}\times630}{5.7}$

$= 1.16 \text{ ms}^{-1} = 1.2 \text{ ms}^{-1}$(2 sig. fig)

ii)

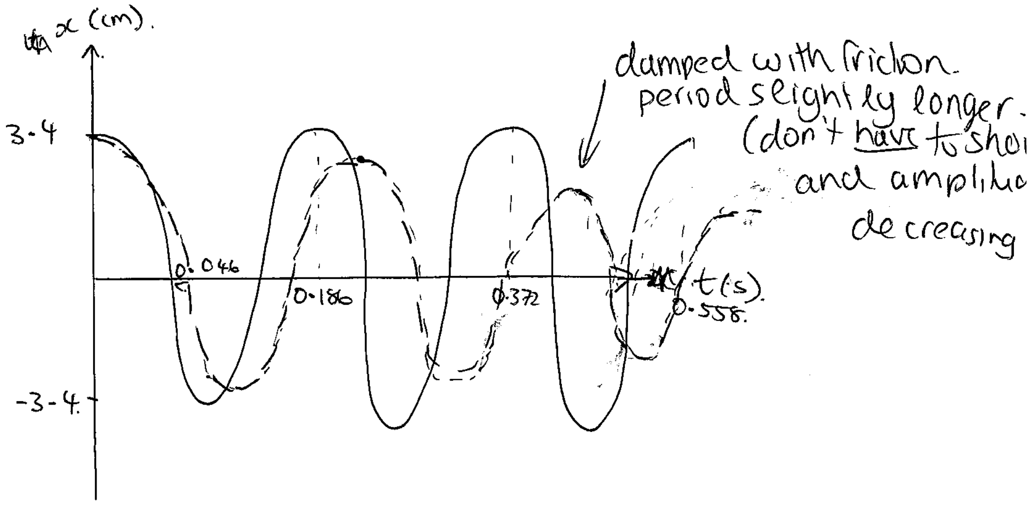
The maximum potential energy is equal to the maximum kinetic energy

$\frac{1}{2}kA^2 = \frac{1}{2}mv^2$

$6500 \times A^2 = 5.7 \times 1.16^2$

$A = 0.03436$ m = 3.4 cm (2 sig. fig)

iii) and iv)



$$\omega^2 = \frac{k}{m} = \frac{6500}{5.7} = 1140.$$

$\Rightarrow f = 5.37\ Hz \Rightarrow T = 0.186\ s.$