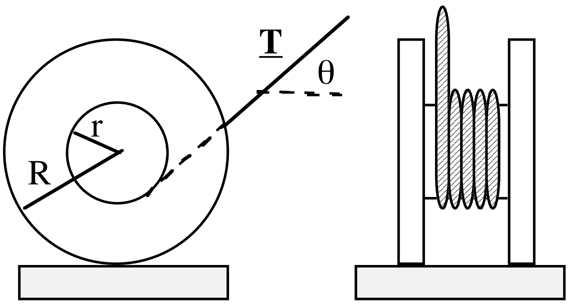
Question 1.



A string is attached to the drum (radius r) of a spool (radius R) as shown in side and end views here. (A spool is device for storing string, thread etc.) A tension T is applied to the string at angle θ above the horizontal. The coefficients of kinetic and static friction between floor and spool are μ_k and μ_s respectively. We are interested in whether and when the spool will move left or right, and how this depends on the nature of the floor.

a) Draw a free body diagram for the spool, showing the forces acting on it, for the case when it is in mechanical equilibrium. (The force vectors need not be to scale, but they should be in approximately the correct direction.)

b) If it slides or skids, in which direction will it move, when pulled by the string?

c) If it rolls, in which direction will it move, when pulled by the string?

d) Showing all working, calculate the critical value of θ (call it θ_c) at which the condition goes from rolling to skidding.

e) If $\theta > \theta_c$, and you pull sufficiently hard on the string, which way does the spool move? (No explanation is required.)

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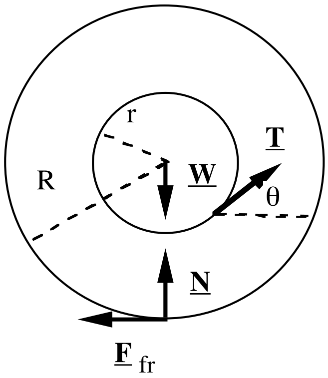
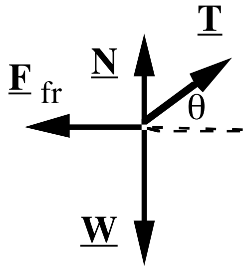




b) If it slides (kinetic friction $\underline{\mathbf{F}}_k$) it moves right.

c) If it rolls (static friction $\underline{\mathbf{F}}_s$) it moves left.

d) In mechanical equilibrium, no acceleration, so

N2 vertical: $T \sin\theta + N = W$ (i)

N2 horizontal: $T \cos\theta = F_{fr}$ (ii)

Also no rotational acceleration, so torques add to zero:

$Tr = F_{fr}R$ (iii)

At point of sliding, $F_{fr} = \mu_s N$ (iv)

Substitute (iv) in (ii) and (iii) gives: $T \cos\theta_c = \mu_s N$

and $Tr = \mu_s NR$

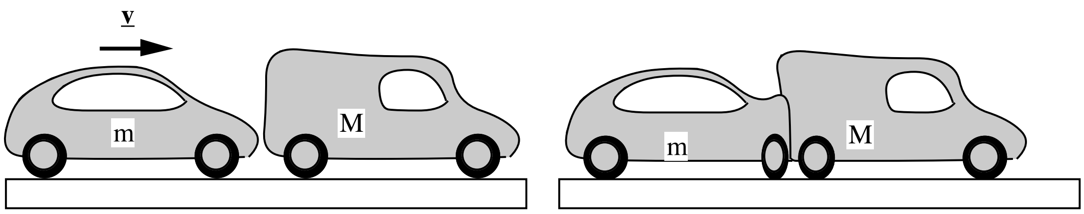
dividing these gives: $\cos\theta_c = \frac{r}{R}$ so $\theta_c = \cos^{-1}\frac{r}{R}$

e) Either: it goes left

or: very hard and it goes up (and a bit to the left then to the right)

Question 2.

a)



A car, mass m = 800 kg, travelling at speed v, collides with the rear of a van, mass M = 1600 kg. The two vehicles remain in contact and travel a distance D = 2.2 m along the road, in the same direction as the velocity of the car. All eight wheels (four on each vehicle) leave skid marks for the full distance D. The coefficients for kinetic and static friction are $\mu_k = 0.80$ and $\mu_s = 0.95$ respectively.

a) Showing your working, and noting any approximations you make or principles you use, derive an algebraic expression relating v to D and to the other data in this problem.

b) Calculate the speed of the car before the collision. Express your answer in kilometers per hour.

c) If the collision takes 100 ms, estimate the magnitude of the average force acting on the car during the collision.

d) Estimate the magnitude of the average force that, during the collision, acts on a 70 kg person firmly held to the seat of the car by seatbelts.

e) Imagine that you had to explain to a non-physicist the size of this force. Describe it quantitatively in several words.

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f) Suppose that the car were travelling at the same speed, that the collision was completely inelastic, but that, in this second case, the van did not have its brakes on during and after the collision. Would your answers to (c) and (d) be substantially different? In one or two clear sentences, explain your answer.

g) From the definition of work, and considering a particle moving in one dimension, prove the Work-Energy Theorem.

Question 2

a) Assume that the collision is brief, so external forces in the horizontal direction are negligible in comparison with the large internal forces during the collision. Consequently, momentum is conserved.

$$mv + 0 = (m+M)v_t$$

where v_t is the velocity of both immediately after the collision. Hence

$$v_t = \frac{m}{m+M} v$$

There is no vertical acceleration, so the total normal force of road on vehicles $N = (M + m)g$. Because all tires are sliding, the total frictional forces between all tires and the road is

$$F_f = \mu_k N = \mu_k(M + m)g$$

After the collision, the kinetic energy of the vehicles is converted to internal energy by the (external) frictional force, so

$$\frac{1}{2}(M + m)\ v_t^2 = F_f D = \mu_k(M + m)gD$$

$$v_t^2 = 2\mu_k gD \qquad \textit{(alternatively use kinematics with const a)}$$

$$\left(\frac{m}{m+M}\right)^2 v^2 = 2\mu_k gD$$

b) $$v^2 = 2\mu_k gD\left(\frac{m+M}{m}\right)^2 = 2(0.80)(9.8\ \text{m/s}^{-2})(2.2\ \text{m})(2400/800)^2$$

$$v = 17.6\ \text{m.s}^{-1} = 63\ \text{k.p.h.}$$

c) $$v_t = \frac{m}{m+M} v = 5.9\ \text{m.s}^{-1}$$

so the car's velocity is reduced by 11.7 m.s^{-1}. So

$$F_{average}\,\Delta t = \Delta p = m\Delta v$$

so $$|F_{average}| = \frac{m|\Delta v|}{\Delta t} = 94\ \text{kN}$$

d) With good seatbelts, acceleration of person is approximately the same as that of the car, so

$$F_{person} = m_{person}a = m_{person}F_{car}/m_{car} = 8\ \text{kN}$$

e) About the weight of 10 or 12 people, a small car, 1000 peaches (or any other reasonable answer)

(acting briefly on the area where the seatbelts are located).

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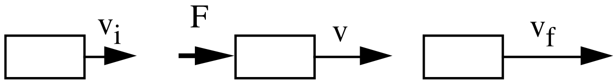
f) Not substantially different. The braking force on the van is probably comparable with its weight, but this external force is much smaller than the large internal forces that act during the collision (the bend and shorten the vehicles). The collision is brief, so during that brief time, the external forces make little difference to the momentum, so the approximate answers in (c) and (d) are unchanged.

g) (Total) force F acts on mass m in x direction.



Work done by F $= \int_i^f Fdx$

and by Newton's second law

$$= \int_i^f m \frac{dv}{dt}\, dx = \int_i^f m \frac{dx}{dt}\, dv$$

$$= \int_i^f m\, v.dv = \left[\frac{1}{2}\, mv^2 \right]_i^f =$$

ΔK

where kinetic energy $K \equiv \frac{1}{2}\, mv^2$

Increase in kinetic energy of body = work done by total force acting on it.