Physics 1A PHYS1121 2007-S1 Answers

1. 10 Marks Total

(a) Coulomb's law is given by $\mathbf{F}_e = k_e \frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$ where $\mathbf{F}_e$ is the electric force between two charges, q_1 and q_2, a distance r apart. k_e is Coulomb's constant. $\hat{\mathbf{r}}$ is the unit vector directed between q_1 and q_2.

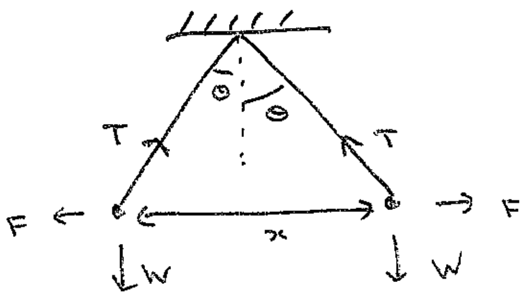
Coulomb's law states that the electric force between two charges depends on the product of the charges and inversely on the square of their separation.

The constant of proportionality is known as Coulomb's constant.

The direction of the force is along the line joining the charges, such that for two charges of the same sign it is repulsive (directed directly away from each other), whereas if the charges are of opposite sign, then it is directed towards each other.

Note OK to have used $k_e = \frac{1}{4\pi\varepsilon_0}$ instead [permittivity of free space].

(b)



(i) There are three forces acting on each charge, the electrostatic repulsion between the two charges, ***F***, its weight due to gravity, ***W***, and the tension in the silk thread, ***T***.

These must be balanced for the charges to the in equilibrium.

We have $W=mg$, acting vertically, for the action of gravity,
and $F = k_e q^2/x^2$, acting horizontally, for the electric force.
Thus, resolving the forces:

Horizontally: $T\sin(\theta) = F = \frac{k_e q^2}{x^2}$ (1)

Vertically: $T\cos(\theta) = W = mg$ (2)

Eliminating T by dividing (1) by (2): $\tan(\theta) = \frac{k_e q^2}{mgx^2}$

For small θ, $\tan(\theta) = \sin(\theta) = (x/2)/l$, where x is the separation.

Thus, on re-arranging, $x^3 = \left(\frac{2lk_e q^2}{mg}\right)$ or $x = \left(\frac{2lk_e q^2}{mg}\right)^{1/3}$.

(ii) We have $q^2 = \left(\frac{mgx^3}{2lk_e}\right)$ so that $q = \left(\frac{mgx^3}{2lk_e}\right)^{0.5} = \sqrt{\frac{0.01\times 9.80\times 0.030^3}{2\times 2.00\times 8.9875\times 10^9}} = 8.579\times x10^{-9}\text{C}$.

i.e. $q = 8.58\text{nC}$ to 3 significant figures.

2. 10 Marks Total

(a) Gauss's law for electric fields states that the total electric flux passing through any closed surface is given by the net charge within that surface divided by ε_0, the permittivity of free space. [or by ε, the permittivity of the medium]

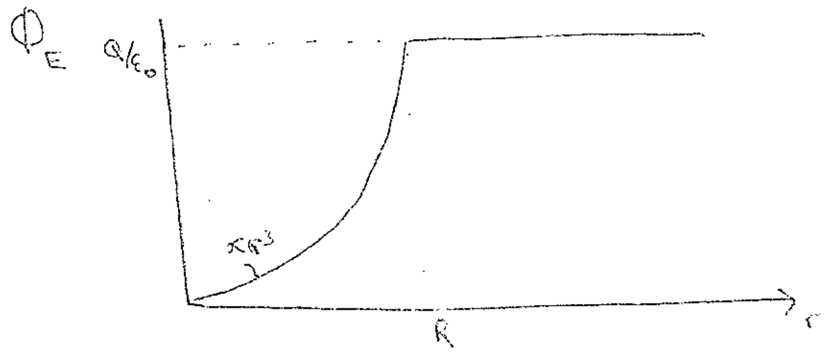
(b) We apply Gauss's Law, $\Phi = Q/\varepsilon_0$, where Q is the total charge enclosed within a surface.

The charge is uniformly spread throughout the non-conducting sphere, so the charge within radius r is given by $q(r) = Q(r/R)^3$.

Hence (i) for $r < R$ $\Phi(r) = Q(r/R)^3/\varepsilon_0$.

For (ii) $r > R$ the charge enclosed is Q for all r, so that $\Phi(r) = Q/\varepsilon_0$.

GRAPH

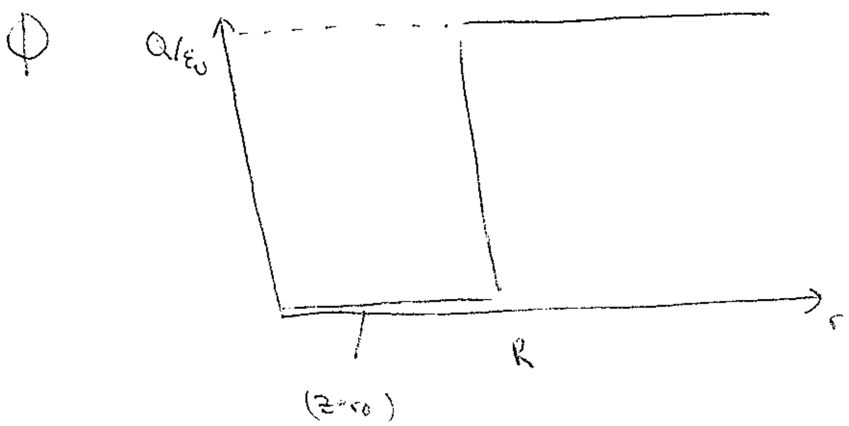


(c) For a conducting sphere all the charge will be found on the surface of the sphere (equally distributed across the surface), with none inside the sphere.

Hence for part (i) there will be no electric flux for any $r < R$ as there is no charge enclosed.

For part (ii), $r > R$, the charge enclosed remains Q and so the result will remain unchanged, $\Phi(r) = Q/\varepsilon_0$.

(d) GRAPH.



3. 7 Marks Total

(a) The potential energy increases.

The change in potential energy is given by $U = q.V$ where V is the potential difference.

When a charge is made to move opposite to the direction of an electric field, it moves to a region of higher electric potential, so that $V > 0$.

Since q is positive, and $V > 0$, then $U > 0$. Thus the potential energy increases.

(b) The electric potential is given by $V = \int \frac{k_e.dq}{r}$.

Consider a circular element of width dr and radius r in the annulus.

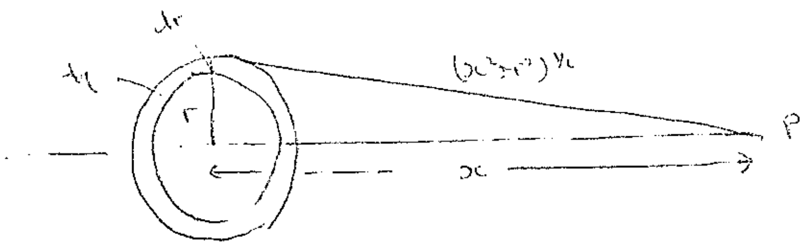
The area element is given by $dA = 2\pi r dr$.

The charge, dq, on it must therefore be $\sigma\, dA = \sigma 2\pi r\, dr$.

Hence at point P, a distance $(x^2+r^2)^{0.5}$ from the element, the contribution to the potential is

$$dV = \frac{k_e dq}{\sqrt{x^2+r^2}}. \text{ Hence } \quad V = 2\pi\sigma k_e \int_a^b \frac{rdr}{\sqrt{r^2+x^2}} = 2\pi\sigma k_e \left[\sqrt{r^2+x^2}\right]_a^b$$

$$\therefore V = 2\pi\sigma k_e \left[\sqrt{b^2+x^2} - \sqrt{a^2+x^2}\right]$$



4. 10 Marks Total

(a) A capacitor stores energy in the electric field. Even after the applied voltage is turned off the capacitor retains this energy – unless there is an external resistance connected through which the capacitor may discharge.

Thus, if one was to touch the terminals you would be providing such a resistance path. The capacitor would discharge through you, carrying a large current for a short period of time.

To make the capacitor safe it could be discharged through a conductor, such as a screw driver. [Note that you should only be touching the insulating handles while this is being done.]

(b)

(i) The potential energy of a capacitor is given by $U=0.5CV^2$.

Thus for the two capacitors, each charged to a potential difference V, we have initially $U = 0.5CV^2 + 0.5CV^2 = CV^2$.

(ii) Since $C = \varepsilon_0 A/d$ then halving the plate spacing doubles the capacitance of a capacitor. The altered capacitor thus has capacitance $C' = 2C$.

The potential difference across each capacitor must be the same, since they are in parallel.

The total charge, Q, is the same as before, from conservation of charge.

Let V' be the potential difference across the capacitors after the separation.

Hence, since $Q=CV$, we have $Q = CV + CV$ (before) $= CV' + C'V'$ (after) $= CV' + 2CV'$

so that $2V = 3V'$; i.e. $V' = 2/3.V$

(iii) The potential energy is given by $U=0.5CV^2$

so that the new potential energy is:
$$U' = \frac{1}{2}C(\frac{2V}{3})^2 + \frac{1}{2}2C(\frac{2V}{3})^2 = \frac{2}{3}CV^2.$$

(iv) The energy has decreased by $\left(1-\frac{2}{3}\right)CV^2 = \frac{C}{3}V^2$.

The energy has been lost from the system because energy is released when the plates of the capacitor are moved together. This happens because the oppositely charged plates attract and so bringing them together decreases their potential energy.

5 6 Marks Total

(a) Pick any three of the following:

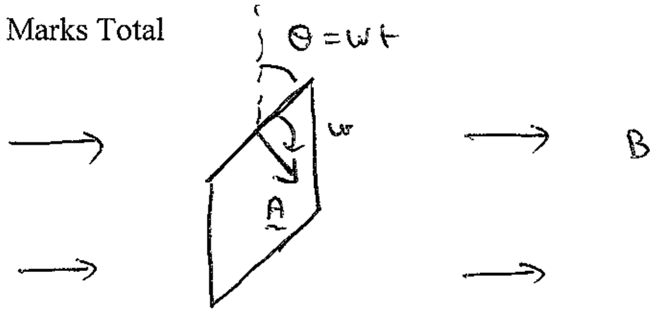
1. Both can alter the velocity of the particle.
2. Both exert forces which are proportional to the charge of the particle.
3. The forces are in opposite directions for particles of opposite charge.
4. Both are proportional to the magnitude of the field.
5. Both depend on the direction of the field.

(b)

1. The direction of the electric force is parallel (or anti-parallel) to the direction of the electric field. The direction of the magnetic force is perpendicular to the direction of the magnetic field.
2. Magnetic forces only act on moving charges, whereas electric forces can also act on stationary charges.

6. 11 Marks Total

(a)



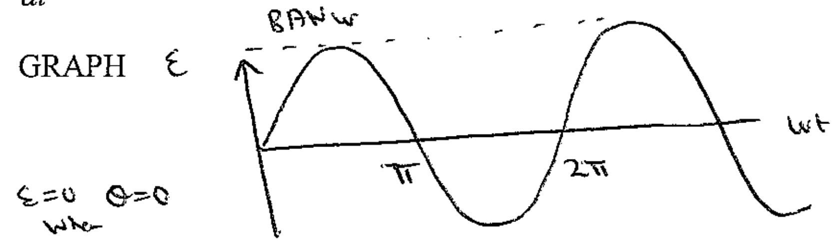
Suppose the axis of the coil makes an angle θ with the direction of the magnetic field at time *t*. Then $\theta = \omega t$.

The magnetic flux Φ passing through one turn of the coil is then given by $\Phi = BA\cos(\theta)$.
For N coils this is $\Phi = BAN\cos(\theta)$.

Applying Faraday's Law, the emf generated in the circuit is given by $\varepsilon = -\frac{d\Phi}{dt} = -\frac{d}{dt}[BAN\cos(\theta)]$.

$\therefore \varepsilon = -\frac{d}{dt}[BAN\cos(\omega t)] = BAN\omega\sin(\omega t)$.

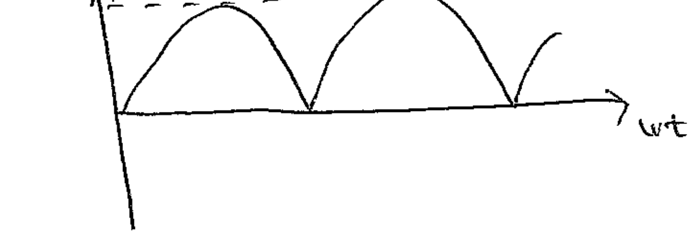
(b) GRAPH



(i) The magnitude of the emf is maximum when sin(θ) = 1, which occurs when θ = 90° or 270°. This occurs when the *plane* of the coil is orientated along the direction of the magnetic field. At this angle, while there is no magnetic flux through the coil, the rate of change of magnetic flux is at its maximum.

(ii) The magnitude of the emf is minimum when sin(θ) = 0, which occurs when θ = 0° or 180°. This occurs when the *plane* of the coil is orientated perpendicular to the direction of the magnetic field. At this angle, while there is the maximum magnetic flux passing through the coil, the rate of change of magnetic flux is at its minimum.

(c) A direct current can be produced by placing a commutator [or split ring] in the circuit so that the direction of the current through the circuit reverses every half revolution, at the point where the emf itself is zero. Hence the emf will always have a positive value, as in the sketch:



7. 11 Marks Total

(a) The oscillations decay rapidly.
As the metal disk swings the amount of magnetic flux passing through it varies.

Hence by Faraday's Law an emf is induced in it, which causes a current to circulate inside it.

This current will produce a magnetic field. By Lenz's law it will act in a direction to oppose the change producing it.

Hence the induced field in the plates will be in the opposite direction to the applied field. Since its polarity is different to the applied field, it will be repelled.

This gives rise to a repulsive force which opposes the motion, quickly bringing it to rest.

Cutting a series of slots into the plate will prevent the formation of large current loops. Hence the induced field will be small and so the repulsive force small. The disk will swing more freely.

(b) The self-induced emf, ε_L, in a circuit is proportional to the rate of change of current, I, in a circuit, and is given by $\varepsilon_L = -L\frac{dI}{dt}$, where L is a constant known as the inductance of the circuit.

By Faraday's Law we also have that $\varepsilon_L = -N\frac{d\Phi_B}{dt}$, where Φ_B is magnetic flux enclosed by the circuit and N is the number of turns in the coil.

Equating the two equations, we have $\varepsilon_L = -L\frac{dI}{dt} = -N\frac{d\Phi_B}{dt}$.

Hence $LdI = Nd\Phi_B$, on integrating with respect to dt.

Integrating again, $LI = N\Phi_B$ [I=0 if Φ_B=0].

$\therefore L = \frac{N\Phi_B}{I}$.

(c) The magnetic field of a long, ideal solenoid is given by $B = \frac{\mu_0 NI}{l}$ where N is the number of turns, l is the length, and I is the current passing through it.

The inductance is $L = \frac{N\Phi_B}{I}$ with $\Phi_B = BA$.

Hence $L = \frac{NBA}{I} = \frac{\mu_0 N^2 IA}{Il} = \frac{\mu_0 N^2 A}{l}$.

Now $N = l/d$ where d is the thickness of the wire, so that $L = \frac{\mu_0 l^2 A}{ld^2} = \frac{\mu_0 lA}{d^2}$.

Hence $L = \frac{4\pi 10^{-7} \times 2 \times 50 \times 10^{-4}}{0.0025^2}\text{H} = 2.01 \times 10^{-3}$ H.