QUESTION 1 [Marks 10]

A circus performer is fired from a canon at an angle of $\theta = 30°$ to the horizontal with an initial speed of $v_0 = 20\ m.s^{-1}$. Neglect air resistance and assume constant gravity. Show all working.

(a) The performer's displacement vector (with respect to the origin) as a function of time, t, is given by

$$\underset{\sim}{r} = \left(v_0 \cos(\theta)t\right)\hat{i} + \left(v_0 \sin(\theta)t - \frac{gt^2}{2}\right)\hat{j}$$

Show that the trajectory can be written

$$y = (\tan\theta)x - \left(\frac{g}{\left(2v_0{}^2\cos^2\theta\right)}\right)x^2$$

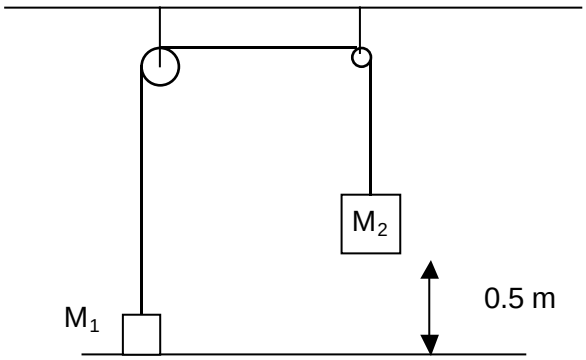
(b) How far does the performer travel before landing on a safety mat on the ground?
{Hint: use the identity $2\sin\theta\cos\theta = \sin 2\theta$ }

(c) By adjusting the firing angle to give maximum range how far can the performer be shot?

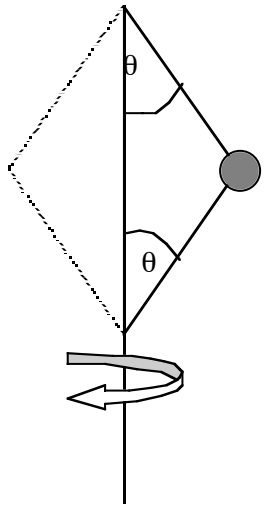
(d) A 15 m high wall is placed at the mid point between the canon and the maximum range (as determined in (c). Does the performer hit the wall if he is fired at the maximum range angle?

QUESTION 2 [Marks 10]

(a) A 3 kg mass, M_1 and a 5 kg mass, M_2 are attached to each other by a massless cord passed over two frictionless pulleys. The pulleys are 1.5m above the floor as shown in the figure. The 3 kg mass is pulled down to the floor so that the 5 kg mass is suspended 0.5 m off the floor. The 3 kg mass is then released.
With what speed is the 5 kg mass travelling when it strikes the floor?



(b)



(b) A small 2 kg mass rotates in a horizontal circular path around a central rod, which is attached by two massless strings. Both strings are 1.5 m long and both make an angle of $\theta = 30°$ to the rod.

(i) Draw a free body diagram showing all forces acting on the mass

(ii) Given that the tension in the upper string is 70 N, what is the tension in the lower string?

(iii) What is the linear velocity of the mass?

QUESTION 3 **[Marks 10]**

A 60 gram bullet is fired from a rifle and collides with a 2 kg block of wood on a horizontal surface in a totally inelastic collision, ie the bullet is embedded in the wood as a result of the collision. The block which is initially at rest slides 16.4 m before coming to rest as a result of the collision. The coefficient of kinetic friction between the block and the surface is 0.2.

(a) Calculate the initial speed of the bullet.
(b) What percentage of total mechanical energy is lost in the collision?
(c) What percentage of total mechanical energy is lost by friction?
(d) What is work done by the friction force?

QUESTION 4 **[Marks 12]**

A mass resting on a horizontal surface is attached to a horizontal massless spring. It is initially extended to a position A which is 3 cm from equilibrium – this requires a force of 12 N. The mass in then further extended to position B which is 10 cm from equilibrium and released. The mass now exhibits simple harmonic motion (ie $x = A\cos(\omega t)$), when is passes point A it has a velocity of 1.0 $m.s^{-1}$.

(a) Show that the velocity as a function of x is given by

$$v = \pm\omega\sqrt{A^2 - x^2}$$

[hint: use the relation: $\sin^2\theta + \cos^2\theta = 1$]

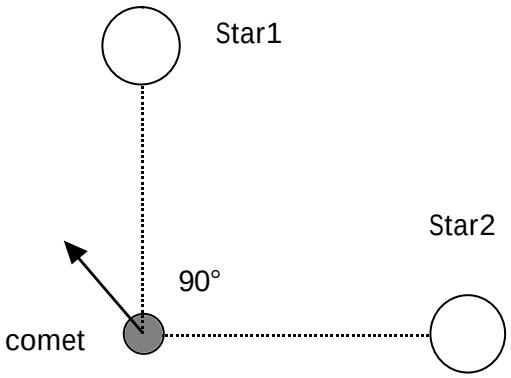
(b) What is the period of the motion ?
(c) What is the force constant of the spring?
(d) What is the mass? {given $T = 2\pi\,(m/k)^{1/2}$ }
(e) What is the elastic potential energy of the mass at point B?
(f) What is the total mechanical energy of the mass spring system?

QUESTION 5 **[Marks 10]**

(a) Calculate the magnitude of the 'free-fall' acceleration at an altitude of 1000 km above the Earths surface. (Earths mass = 5.98×10^{24} and its mean radius = 6370 km).

(b) A half-brick is fired directly upwards from the surface of the Earth at 8 $km.s^{-1}$. How high above the Earths surface will it travel before returning to Earth? Neglect air resistance.

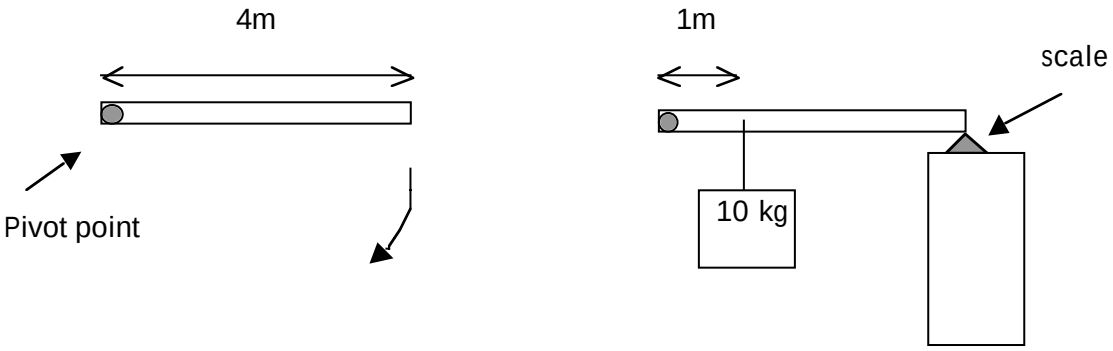
(c)



A comet with a constant velocity directed as shown is equidistant (50×10^6 km) from two stars which form a double star system (diagram not to scale). At the instant shown in the diagram what is the magnitude and direction of the net gravitational force on the comet?
[Star1 mass = 5×10^{30} kg,
star2 mass = 8×10^{30} kg,
comet mass = 5×10^8 kg]

What effect does this gravitational force have on the comet's direction of motion and why?

QUESTION 6 **[Marks 10]**



(a) A uniform solid rod of length 4 m and mass 5 kg is pivoted at one end. It is held initially in the horizontal position and then released. Assume the pivot is frictionless. Using the parallel axis theorem ($I = I_{cm} + Mh^2$), find the angular acceleration of the rod immediately after it is released. Note: The moment of inertia of a rod about a perpendicular line through its centre of mass is given by $I = (1/12)\ ML^2$.

(b) What two conditions must be satisfied for a body to be in equilibrium? What addition condition is required if the body can be said to be in static equilibrium?

(c) The rod is returned to the horizontal position and a 10 kg mass is now attached at the point shown. A fixed scale (reading Newtons) is placed under the free end of the bar. What is the reading on the scale? (The scale will measure the downwards forces enacted on it by the bar).