

### Question 1

a) Your car is stopped on the side of a straight highway. The road is busy: many cars are going past, all of them at 100 km per hour. It's a sunny day and the solar cells are clean, so assume that your car accelerates at a constant rate  $a = 1.9 \text{ m.s}^{-2}$  from zero until it reaches a final speed  $v_f = 100 \text{ km per hour}$ . You need a long gap between cars to be able to accelerate to 100 km per hour to join the stream of the traffic. In this question you will work out how long.

- i) Sketch a displacement - time or  $x(t)$  graph showing the position of a car accelerating from rest to  $v_f$  at constant acceleration  $a$ , and then continuing at constant speed  $v_f$ .
- ii) On the same graph, show the displacement of a following car. This is a car, which travels at a constant speed  $v_f$  at all times and which, when your car has finished accelerating, is a safe distance  $L$  behind yours. Show  $L$  clearly on the graph.
- iii) Below or above your displacement graph, and using the same scale for the time axis, sketch a velocity - time ( $v(t)$ ) graph for the two cars. Show  $v_f$  on the graph.
- iv) Some authorities judge that, in good conditions, the safe distance  $L$  between cars on a highway is the distance travelled by a car in 2 seconds\*. What is  $L$  for this case?
- v) Assume that you start accelerating when the car ahead is a distance  $L$  in front of you and finish accelerating when the car behind (which travels at constant speed  $v_f$ ) is a distance  $L$  behind you. (It is not required, but it may help to draw the  $x(t)$  for the car ahead of yours, as well.)

Showing your working, calculate the minimum necessary distance between the car in front of you and the car behind you.

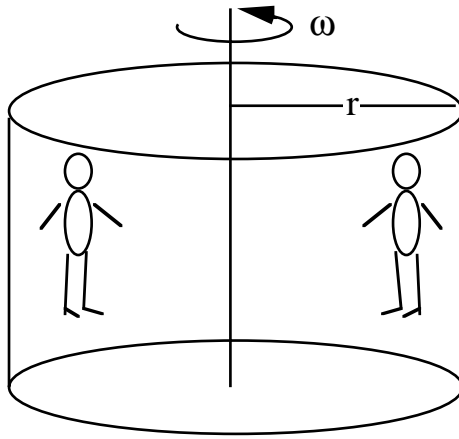
Hint 1: you may wish to find out how far (say  $d_1$ ) you travel while accelerating, and how far the following car travels while you are accelerating (say  $d_2$ ).

Hint 2: look carefully at your diagram.

- b) With respect to the ground, the wind is blowing from the North East at speed  $v_w = 15 \text{ km per hour}$ . You are bicycling South at speed  $v$  (with respect to the ground).
  - i) Relative to you, the wind is coming directly from the East. Determine your speed  $v$ .
  - ii) Relative to a second cyclist, also travelling South, the wind is coming directly from the SouthEast. Determine her speed (with respect to the ground).

\* A comment for road safety but not for marks. The safe distance is a minimum for good conditions. In poor visibility, leave larger gaps. The timing requires judgment and the acceleration  $1.9 \text{ m.s}^{-2}$  over this distance is not always achievable. Consider this calculation as an *underestimate*.

### Question 2

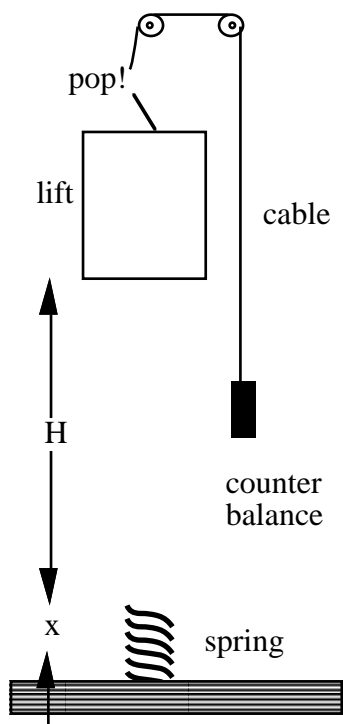


The sketch represents a fairground ride called the Gravitron. Jack and Jill stand against the inside, vertical wall of a cylindrical chamber, radius  $r$ , that is initially stationary.

When the chamber rotates about its axis, Jack and Jill discover that they need not touch the floor: the vertical wall alone stops them from falling.

- In several clear sentences and with the aid of at least one vector diagram, explain the origin of the force that stops Jack and Jill from falling.
- On your vector diagram, show the contact force that the wall exerts on Jack.
- Derive an expression for the minimum angular frequency  $\omega$  that will suffice to stop them from falling. The coefficients of kinetic and static friction between the wall and their clothes are  $\mu_k$  and  $\mu_s$ .
- If  $r = 3.0$  m,  $\mu_s = 0.28$  and  $\mu_k = 0.21$ , calculate the minimum number of revolutions per minute that the chamber must make to prevent them from falling.

### Question 3



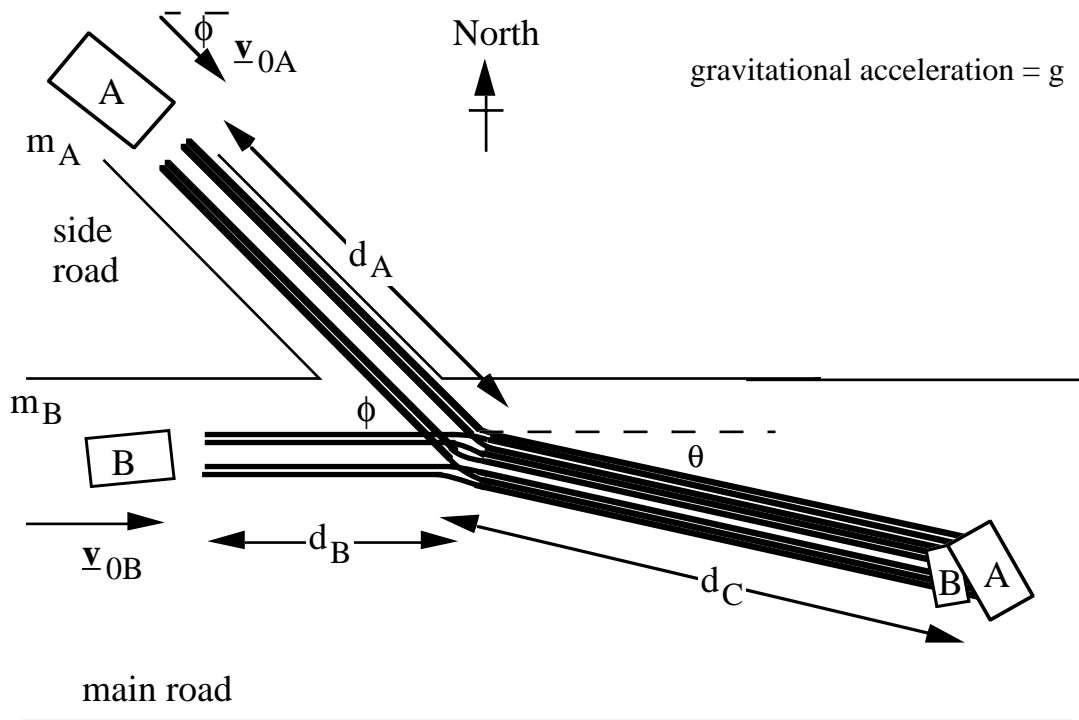
As a safety precaution, you decide to install a large spring in the bottom of a lift shaft. (A lift is the same thing as an elevator. The shaft is the volume in which it travels.) The mass of the spring is negligible compared to the mass  $M$  of the lift (and, happily, there are no passengers in the lift for this problem). Assume that the spring constant is  $k$ , and the spring obeys Hooke's law for the range considered in this problem.

Suppose that bottom of the lift is a distance  $H$  above the spring when the cable breaks, while the lift is stationary. It then falls and hits the spring. Air resistance and friction are negligible.

- Explaining your reasoning, derive an expression for the maximum compression  $x_m$  of the spring. To simplify the algebra, you may assume that  $H \gg x_m$ .
- Briefly explain why the dimensions (or units) in your equation are correct.
- When will the acceleration of the lift be greatest? Explain your answer briefly. You may assume that  $kx_m > 2Mg$ .
- Derive an expression for the greatest acceleration  $a_{\max}$  in terms of the parameters of the problem.

#### Question 4

A traffic accident investigator finds a scene represented by the following schematic. She makes the following observations and deductions:



Car A has skidded a distance  $d_A$  along a side street before colliding with car B. Vehicle B has skidded a distance  $d_B$  along a main street before colliding with car A. Squashed together during the collision, the two wrecked cars have skidded together, sideways but without rotating. They have skidded a distance  $d_C$ , at an angle  $\theta$  to the main street and have come to rest together as shown. From the black marks left on the street, we know that all four wheels on both vehicles skidded both before and after the collision. Neither street has any slope.

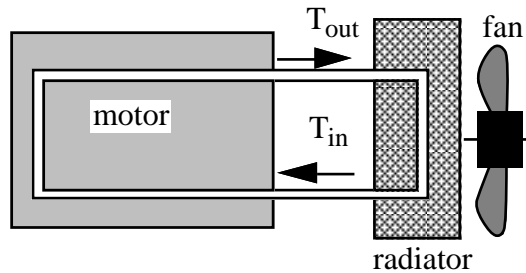
The investigator assumes that the initial velocities  $\underline{v}_{0A}$  and  $\underline{v}_{0B}$ , before either vehicle started skidding, are in the directions shown. For legal reasons, she wishes to calculate their magnitudes,  $v_{0A}$  and  $v_{0B}$ . The masses of the cars A and B are  $m_A$  and  $m_B$ , respectively. The coefficients of static and kinetic friction between the rubber and the street are  $\mu_s$  and  $\mu_k$ , respectively.

- Explaining any assumptions and reasoning, derive an expression for the momentum  $\underline{p}_c$  of the two cars together, after the collision, in terms of  $m_A$ ,  $m_B$ ,  $d_C$ ,  $g$  and the appropriate  $\mu$ .  
(Hint: What forces are acting during the combined skid?)
- Between the time when either of the cars begins to skid and the time when they come to rest, is there any stage where conservation of mechanical energy is an appropriate approximation? If so, explain when and why. If not, explain why not.
- Between the time of the cars beginning to skid and the time when they come to rest, is there any stage where conservation of momentum is an appropriate approximation? If so, explain when and why. If not, explain why not.
- Explain in several sentences how you would deduce the speed  $v_{0A}$  of car A, before it started skidding. You do not have to derive an expression: just state clearly the arguments you would use. You may set out your argument in point form if you wish.

*(In practice, the wreckage usually rotates about a vertical axis, giving rise to loops and sometimes discontinuities in the skid marks, but the principles are similar.)*

## Question 5

a)



An internal combustion engine (the motor in a car) generates heat at a rate of 20 kW. It is cooled by water that flows in a continuous circuit through a radiator and the motor, as shown in the very simplified sketch. (The radiator dissipates the heat because cool air is forced through the radiator by a fan. This detail is not relevant to our problem.)

The density of water is  $1000 \text{ kg.m}^{-3}$ , its specific heat capacity is  $4.2 \text{ kJ.kg}^{-1}.\text{K}^{-1}$ , its latent heat of vaporisation is  $2.3 \text{ MJ.kg}^{-1}$  and it boils at  $100^\circ\text{C}$ .

- i) If the temperature of the water going into the motor is  $50^\circ\text{C}$  and the temperature of the water coming out of the motor is  $85^\circ\text{C}$ , what is the rate of flow of water through the motor? (Express your answer in litres per minute.)
- ii) Due to a malfunction, flow of the cooling system ceases. The motor comprises 110 kg of a metal whose specific heat capacity is  $0.43 \text{ kJ.kg}^{-1}.\text{K}^{-1}$ , and it contains 3 litres of cooling water. Assume that the motor and the water are all at the same temperature (this is a severe oversimplification). The motor continues to produce heat at 20 kW, and loses it at a negligible rate. Determine long does it take the temperature to rise from  $85^\circ\text{C}$  to  $100^\circ\text{C}$ .
- iii) From the time when the water starts to boil, how long is it before the 3 litres of water is all boiled away?
- iv) Once the water is boiled away, what is the rate of temperature rise in the motor? Express your answer in  $^\circ\text{C}$  per minute.

- b) For a gas, which is greater: the specific heat at constant volume  $c_V$  or the specific heat at constant pressure  $c_P$ ? Explain your answer in about four or five clear sentences.