Phys 1121 T1 2004

Question 1. (16 marks)

A scientist is standing at ground level, next to a very deep well (a well is a vertical hole in the ground, with water at the bottom). She drops a stone and measures the time between releasing the stone and hearing the sound it makes when it reaches the bottom.

i) Draw a clear displacement-time graph for the position of the falling stone (you may neglect air resistance). On the diagram, indicate the depth h of the well and the time T_1 taken for the stone to fall to the bottom.

ii) Showing your working, relate the depth h to T_1 and to other relevant constants.

iii) The well is in fact 78 m deep. Take $g = 9.8\ ms^{-2}$ and calculate T_1.

iv) On the same displacement-time graph, show the displacement of the sound wave pulse that travels from the bottom to the top of the well. Your graph need not be to scale.

v) Taking the speed of sound to be 344 ms^{-1}, calculate T_2, the time taken for the sound to travel from the bottom of the well to reach the scientist at the top. Show T_2 on your graph.

vi) State the time T between release of the stone and arrival of the sound. Think carefully about the number of significant figures.

The scientist, as it happens, doesn't have a stop watch and can only estimate the time to the nearest second. Further, because of this imprecision and because she is solving the problem in her head, she neglects the time taken for the sound signal to reach her. For the same reason, she uses $g \cong 10\ ms^{-2}$.

vii) What value does the scientist get for the depth of the well?

viii) Comment on the relative importance of the errors involved in (a) neglecting the time of travel of sound, (b) approximating the value of g and (c) measurement error.

Question 1.

i) Depending on the choice of origin, the graph might look like these. The algebra below is for the upper case.

ii) $y = y_o + v_{yo}t + \frac{1}{2}a_y t^2$

$= h + 0 - \frac{1}{2}gt^2$

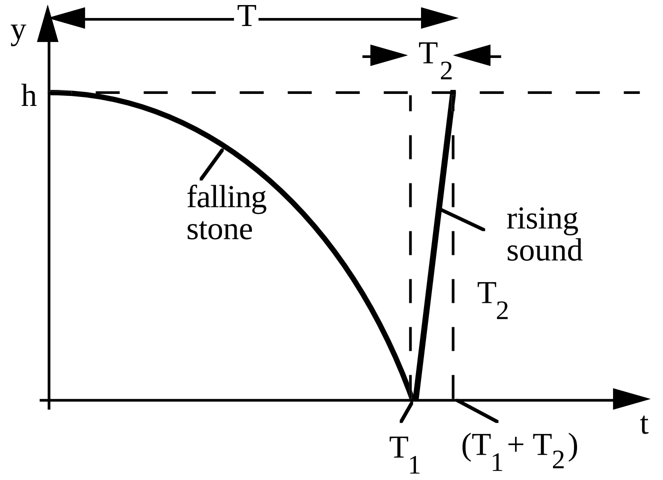
hits the bottom when y = 0, so

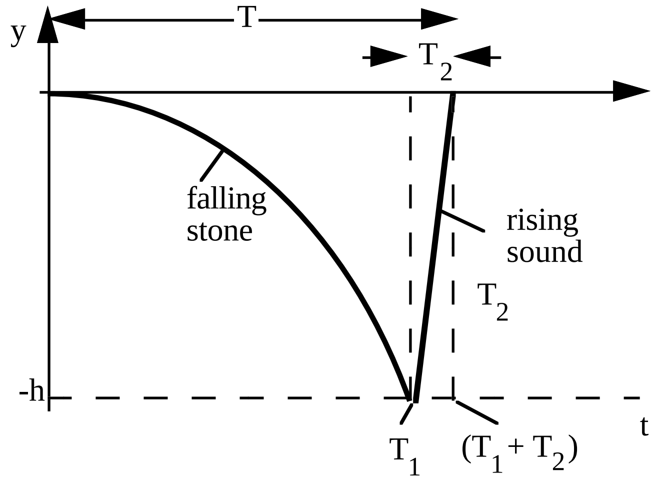
$0 = h - \frac{1}{2}gT_1^2$

$\therefore \quad T_1^2 = \frac{2h}{g}$

$T_1 = \sqrt{\frac{2h}{g}}$ (4 marks)

iii) h = 78 m $\rightarrow$ $T_1 = 4.0$ s. (1mark)





(5 marks for diagram)

v) speed = distance travelled/time taken, so $T_2 = h/v_s = 0.23$ s. (1 mark)

vi) $T = T_1 + T_2 = 3.99$ s + 0.227 s $\rightarrow$ T = 4.2 s. (2 significant figures) (2 marks, incl sig figs)

vii) She says $0 = h - \frac{1}{2}gT_1^2$

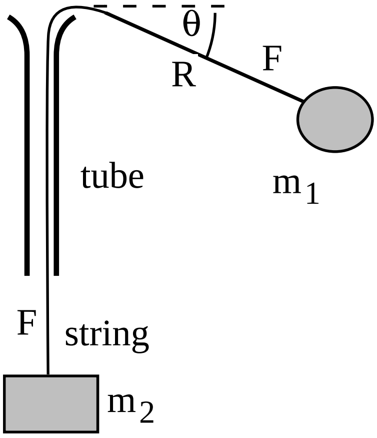
$\therefore \quad h = \frac{1}{2}gT_1^2 \cong \frac{1}{2}gT^2.$

So she calculates $h \cong \frac{1}{2}(10\text{ms}^{-2})(4\text{ s})^2 = 80$ m. (3 marks)

viii) Her time estimate is 4.0 ± 0.5 s, an error of 13%. Neglecting the time for the sound to travel (6%) and taking 9.8 ≅ 10 (2%) are small errors by comparison. <She is lucky to have worked out an answer so close to the precise one.> (*Any reasonable comment about the accuracy earns two marks.*) (2 marks)

Question 2. (14 marks)

a)



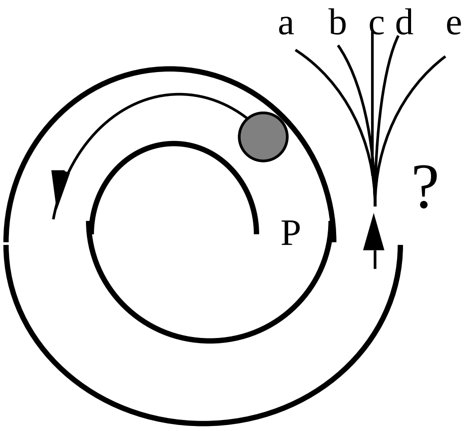
Two masses, m_1 and m_2, are attached to opposite ends of a string that passes through a tube, whose upper end has been smoothed to reduce the sliding friction with the string. The tube is held vertically and stationary. m_1 is caused to travel in a horizontal circle, in such a way that m_2 does not move. Neglecting the friction between string and tube,

i) Derive an equation for θ in terms of m_1 and m_2.

ii) Derive an expression for the period T of the circular motion of m_1.

iii) State the direction for the normal force N exerted by the tube on the string, and derive an expression for N in terms of F. (You may find a it helpful to draw a diagram)

b)



The picture shows the top view of a vertical wall, shaped in a spiral, mounted on a hard, smooth, horizontal surface. A ball (the shaded circle in the diagram), initially placed at point P, is given a push that makes it roll around the spiral, following the path indicated.

i) When the ball leaves the spiral at the point marked "?", which of the paths (a, b, c, d, e) best approximates the path of the ball? You may neglect air resistance and friction between the ball and the horizontal surface.

ii) Explain your answer. Your explanation should include a physical law. If your explanation is expressed in equations, define the symbols in teh equations.

Q2

i) Newton 2 for m_2: $F = m_2g$ (1 mark)

Newton 2 for m_1 (vertical): $0 = m_2a_{vert} = F.\sin\theta - m_1g = m_2g.\sin\theta - m_1g$

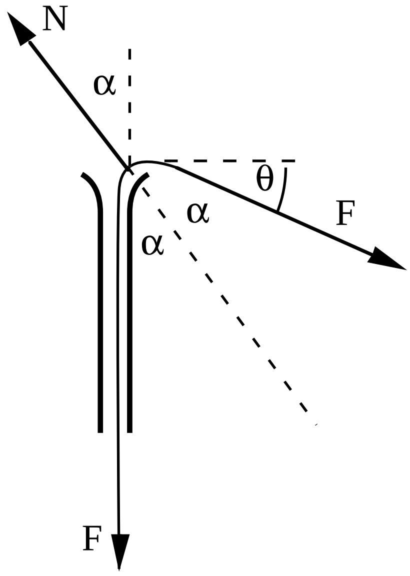
$$\sin\theta = \frac{m_1}{m_2}$$ (3 marks)

ii) Newton 2 for m_1 (horizontal): $m_1a_{centrip} = m_1r\omega^2 = F.\cos\theta = m_2g.\cos\theta$

substitute for r and ω:

$$m_1(R\cos\theta)\left(\frac{2\pi}{T}\right)^2 = m_2g.\cos\theta$$

$$T = 2\pi\sqrt{\frac{m_1R}{m_2g}}$$ (4 marks)



iii) The tension in both ends of the string is F, so, from symmetry, the line of N bisects the angle between the segments of the string. So N is at an angle to the vertical

$\alpha = (90° - \theta)/2$

$N = 2F\cos\alpha$. (3 marks)

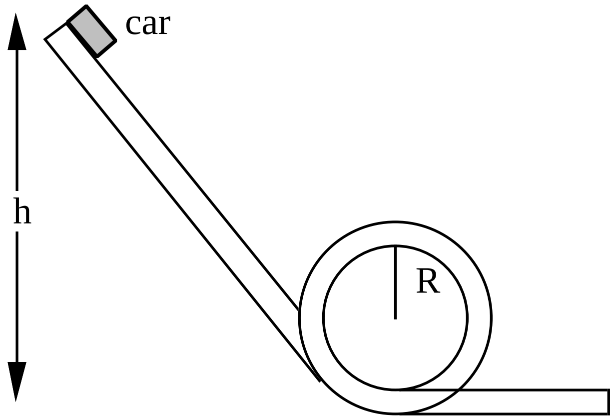
b) Newton's law: (in an inertial frame) a body continues to travel in a straight line unless acted upon by a force. Once it leaves the spiral, there are no horizontal forces on it, so it travels in a straight line in the horizontal plane.

OR. Newton's law: **F** = m**a** where **F** is the total force, m the mass and **a** the resultant acceleration. Here **F** = 0 so **a** = 0 so the velocity is constant, so the ball travels in a straight line.

(3 marks)

<Bonus mark if a student explains this: *In practice, if air resistance or friction on a deformable surface are non-negligible, the spin of the ball will cause it to follow a path like d, but here these effects are neglected.*>

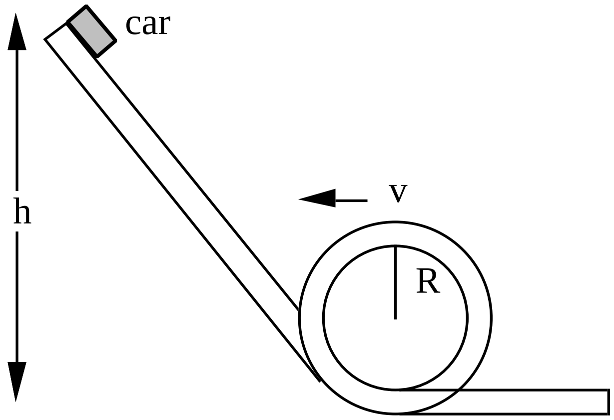
Question 3 (10 marks)



A toy racing car is placed on a track, which has the shape shown in the diagram. It includes a loop, which is approximately circular with radius R. The wheels of the car have negligible mass, and turn without friction on their axle. You may also neglect air resistance. The dimensions of the car are much smaller than R.

i) Showing all working, determine the minimum height h from which the car may be released so that it maintains contact with the track throughout the trip.

Question 3



i) v must be sufficiently great that the centripital force at the top of the loop at least equals the weight of the car. (Faster than this, a downwards normal force is required.)

No non-conservative forces do work, so conservation of mechanical energy applies:

$$U_i + K_i = U_f + K_f$$

$$mgh + 0 = mg.2R + \frac{1}{2}mv^2$$

$$v^2 = 2g(h - 2R)$$ (4 marks)

It loses contact when normal force = 0.

$N + mg = F_{centrip} = mv^2/R$ i.e. falls when $mg = mv^2/R$

$v^2 = gR$ (3 marks)

Therefore it just falls off if h satisfies

$gR = v^2 = 2g(h - 2R)$

$5gR = 2\ gh$ $\therefore$ $h_{min} = 5R/2$ (3 marks)

Question 4. (17 marks)

i) An apple, attached to a tree a distance of 6370 km from the centre of the Earth, falls to the ground, and appears to accelerate at 9.80 ms^{-2}. The average Earth-moon distance is 3.84×10^8 m. Making the approximation that the Earth is an inertial frame, using these two data and the inverse square law of gravitation, but*without using a value for the gravitational constant G or the mass of the Earth,* determine the period of the moon's orbit around the Earth. Express your answer in days. Give at least one reason why your answer might differ from a lunar month (29.5 days).

i) $a_g = \frac{F}{m} = \frac{\text{const}}{r^2}$

When r = 6.37 10^6 m, $a_g \cong g = 9.80$ ms^{-2}, so const = $a_g r^2 \cong 3.98\ 10^{14}$ $m^3 s^{-2}$.

$$a_{moon} = r_{moon}\omega^2 = \frac{\text{const}}{r_{moon}{}^2}$$

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{r_{moon}{}^3}{\text{const}}} = \ldots \cong 27.4 \text{ days.}$$ (7 marks)

This is approximately equal to the lunar month.

However, a_g is the acceleration in a frame of reference that accelerates around the Earth's axis of rotation. The real acceleration of the apple is greater than than this by $r_{earth}\omega_{earth}{}^2$.

Furthermore, during a month, the Earth moves ~360°/13 around the sun, so the lunar month is longer than T by about (14/13).

(2 marks for either. Plus a bonus mark for anyone who gets both.)

ii) The International Space Station has an orbital period of 91.8 minutes. The mass of the Earth is 5.98×10^{24} kg and its radius is 6.37×10^6 m. G = 6.673×10^{-11}N m^2 kg^{-2}. From these data and the law of universal gravitation, determine the elevation of the station above the Earth and its speed.

$$|F| = \frac{GMm}{r^2} = ma_{centrip} = mr\omega^2$$

$$r^3 = \frac{GM}{\omega^2} = \frac{GMT^2}{2^2\pi^2}$$

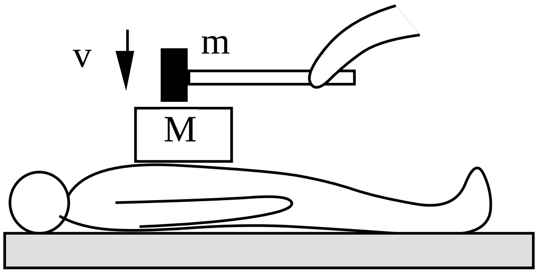
$$r = \sqrt[3]{\frac{GMT^2}{2^2\pi^2}} = 6.74\ 10^6 \text{ m}$$ (6 marks)

$$h = r - r_{Earth} = 370 \text{ km.}$$ (1 mark)

$$v = \frac{2\pi r}{T} = 7.7 \text{ km.s}^{-1}.$$ (1 mark)

Question 5. (12 marks)

a)



In a circus performance, a clown lies on his back with a brick, mass M, on his chest. An assistant uses a hammer with a mass m = 1.0 kg, to crack the brick. The head of the hammer is travelling vertically down at v = 20 ms^{-1}. The mass of the handle is negligible. The collision between hammer and brick is of extremely short duration. However, because the brick cracks at the surface, the collision is completely inelastic.

i) Derive an expression for the velocity V of the brick plus hammer immediately after the collision with the brick.

ii) In an earlier part of the performance, a selection of audience members with different weights has stood on the clown's chest. The deformation of the chest is proportional to the weight of the person standing, and a 100 kg man produces a depression of 30 mm in his chest. Derive an expression for the spring constant of the clown's chest.

iii) The Occupational Health and Safety Officer for the circus decides that the breaking brick trick should not depress the clown's chest more than 30 mm beyond the resting position of the brick before the collision. Derive a value for the required mass M of the brick. You may neglect the gravitational potential energy associated with deformation of the clown's chest.

iv) Express your answer to part (iii) as an inequality. Describe the reason for the direction of the inequality.

Caution. Do not try this exercise at home.

Question 5.

i) During the brief collision, large contact forces act, so external forces are neglected. So momentum of hammer plus brick is conserved. In the vertical direction: $p_{initial} = p_{final}$.

$$mv = (m+M)V,$$

so
$$V = \frac{m}{m+M} v.$$
(3 marks)

ii) From the proportionality of load to deformation, the clown's chest obeys Hooke's law: F = – kx.

Here $k = \frac{\text{weight of 100 kg man}}{\text{deformatio}} = \frac{980\ \text{N}}{30\ \text{mm}} = 33\ \text{kN.m}^{-1}$. (3 marks)

iii) The clown's chest has been shown to obey Hooke's law, so we assume it acts like a spring, and provides a conservative restoring force. Therefore mechanical energy will be conserved in the deformation that follows the collision. Neglecting gravitational potential energy, we have:

$$U_i + K_i = U_f + K_f$$

$$0 + \frac{1}{2}(M+m)V^2 = \frac{1}{2} kx_{max}^2 + 0$$

$$M+m\left(\frac{m}{m+M} v\right)^2 = kx_{max}^2$$

$$\frac{m^2}{m+M} v^2 = kx_{max}^2 \qquad (*)$$

$$m + M = \frac{m^2v^2}{kx_{max}^2}$$

$$M = \frac{m^2v^2}{kx_{max}^2} - m = 13\ \text{kg}.$$
(6 marks)

(Anyone whose sympathy for the clown prompts him/her to point out that this already compresses the chest by 4 mm, and that they OHS Officer should be warned that the total deformation is now 34 mm, should get a bonus mark.)

iv) M > 13 kg. From (*), if M < 13 kg, the energy of the brick after the collision will cause excessive deformation of the clown's chest.

Question 6. (13 marks)

i) A solid sphere, a disc and a hoop are released from rest and roll down an inclined plane, beginning at height h and ending at height 0. Air resistance is negligible. All have the same radius R. Showing all working, *and stating any assumptions you make*, determine the speed v of one of the objects at the bottom of the plane, in terms of its radius of gyration.

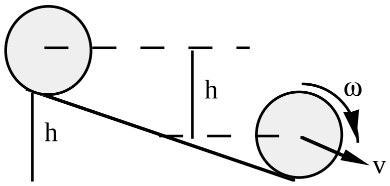
ii) The radii of gyration are

$k_{sphere} = \sqrt{\frac{2}{5}}\,R$ $\quad\quad k_{disc} = \sqrt{\frac{1}{2}}\,R$ $\quad\quad k_{hoop} = R$

If they are all released at the same time, state the order of their arrival at the bottom, and briefly explain your reasoning.

iii) In two or three clear sentences, explain why one of these objects is faster than another one *in terms of conservation of mechanical energy.*

iv) Using your answer to part (i), state whether a large sphere or a small sphere would roll faster when released from rest on the plane. In one sentence, explain your answer.

i) 

Rolling: point of application of friction stationary ∴ non-conservative forces do no work ∴

$$U_f + K_f = U_i + K_i$$

$$0 + \left(\frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2\right) = Mgh + 0$$

rolling ∴ $\omega = \frac{v}{R}$ and write $I = Mk^2$

$$\frac{1}{2} Mv^2 + \frac{1}{2} Mk^2 \frac{v^2}{R^2} = Mgh$$

$$\frac{1}{2} v^2\left(1 + \frac{k^2}{R^2}\right) = gh$$

$$v = \sqrt{\frac{2gh}{1 + k^2/R^2}}$$ (7 marks)

ii) from (i), v decreases increasing ratio k/R

$$\frac{k_{hoop}}{R} = 1 > \frac{k_{disc}}{R} = \sqrt{\frac{1}{2}} > \frac{k_{sphere}}{R} = \sqrt{\frac{2}{5}}$$

∴ $v_{hoop} < v_{disc} < v_{sphere}$

∴ sphere arrives before disc, which arrives before hoop. (3 marks)

iii) During the descent, gravitational potential energy is converted into kinetic energy of two types: translational and rotational. The equation above shows that the ratio of rotational to translational kinetic energy is $(k/R)^2$, so objects with large k/R ratios have proportionally less translational kinetic energy and so lower (translational) speed, all else equal. (2 marks)

iv) the answer to part (i) does not depend on mass m or radius R, so the large and small spheres should travel equally quickly. (1 mark)