Phys 1121 T1 2004

Question 1. (16 marks)

A scientist is standing at ground level, next to a very deep well (a well is a vertical hole in the ground, with water at the bottom). She drops a stone and measures the time between releasing the stone and hearing the sound it makes when it reaches the bottom.

i) Draw a clear displacement-time graph for the position of the falling stone (you may neglect air resistance). On the diagram, indicate the depth h of the well and the time T_1 taken for the stone to fall to the bottom.

ii) Showing your working, relate the depth h to T_1 and to other relevant constants.

iii) The well is in fact 78 m deep. Take $g = 9.8$ ms^{-2} and calculate T_1.

iv) On the same displacement-time graph, show the displacement of the sound wave pulse that travels from the bottom to the top of the well. Your graph need not be to scale.

v) Taking the speed of sound to be 344 ms^{-1}, calculate T_2, the time taken for the sound to travel from the bottom of the well to reach the scientist at the top. Show T_2 on your graph.

vi) State the time T between release of the stone and arrival of the sound. Think carefully about the number of significant figures.

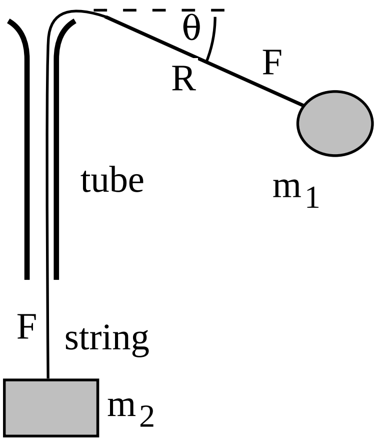
The scientist, as it happens, doesn't have a stop watch and can only estimate the time to the nearest second. Further, because of this imprecision and because she is solving the problem in her head, she neglects the time taken for the sound signal to reach her. For the same reason, she uses $g \cong 10$ ms^{-2}.

vii) What value does the scientist get for the depth of the well?

viii) Comment on the relative importance of the errors involved in (a) neglecting the time of travel of sound, (b) approximating the value of g and (c) measurement error.

Question 2. (14 marks)

a)



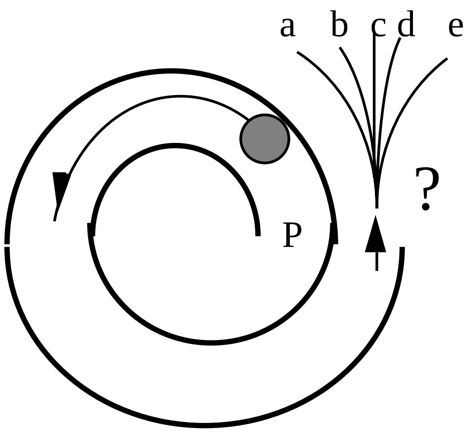
Two masses, m_1 and m_2, are attached to opposite ends of a string that passes through a tube, whose upper end has been smoothed to reduce the sliding friction with the string. The tube is held vertically and stationary. m_1 is caused to travel in a horizontal circle, in such a way that m_2 does not move. Neglecting the friction between string and tube,

i) Derive an equation for θ in terms of m_1 and m_2.

ii) Derive an expression for the period T of the circular motion of m_1.

iii) State the direction for the normal force N exerted by the tube on the string, and derive an expression for N in terms of F. (You may find a it helpful to draw a diagram)

b)

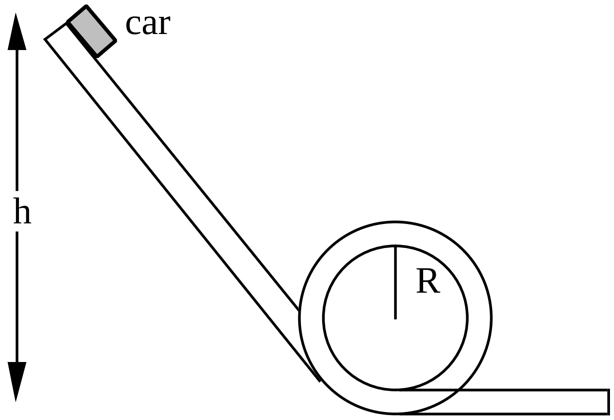


The picture shows the top view of a vertical wall, shaped in a spiral, mounted on a hard, smooth, horizontal surface. A ball (the shaded circle in the diagram), initially placed at point P, is given a push that makes it roll around the spiral, following the path indicated.

i) When the ball leaves the spiral at the point marked "?", which of the paths (a, b, c, d, e) best approximates the path of the ball? You may neglect air resistance and friction between the ball and the horizontal surface.

ii) Explain your answer. Your explanation should include a physical law. If your explanation is expressed in equations, define the symbols in teh equations.

Question 3 (10 marks)



A toy racing car is placed on a track, which has the shape shown in the diagram. It includes a loop, which is approximately circular with radius R. The wheels of the car have negligible mass, and turn without friction on their axle. You may also neglect air resistance. The dimensions of the car are much smaller than R.

Showing all working, determine the minimum height h from which the car may be released so that it maintains contact with the track throughout the trip.

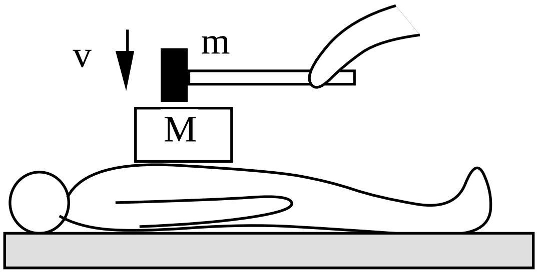
Question 4. (17 marks)

i) An apple, attached to a tree a distance of 6370 km from the centre of the Earth, falls to the ground, and appears to accelerate at 9.80 ms^{-2}. The average Earth-moon distance is 3.84×10^8 m. Making the approximation that the Earth is an inertial frame, using these two data and the inverse square law of gravitation, but *without using a value for the gravitational constant G or the mass of the Earth,* determine the period of the moon's orbit around the Earth. Express your answer in days. Give at least one reason why your answer might differ from a lunar month (29.5 days).

ii) The International Space Station has an orbital period of 91.8 minutes. The mass of the Earth is 5.98×10^{24} kg and its radius is 6.37×10^6 m. $G = 6.673 \times 10^{-11}$ N m^2 kg^{-2}. From these data and the law of universal gravitation, determine the elevation of the station above the Earth and its speed.

Question 5. (12 marks)

a)



In a circus performance, a clown lies on his back with a brick, mass M, on his chest. An assistant uses a hammer with a mass m = 1.0 kg, to crack the brick. The head of the hammer is travelling vertically down at v = 20 ms^{-1}. The mass of the handle is negligible. The collision between hammer and brick is of extremely short duration. However, because the brick cracks at the surface, the collision is completely inelastic.

i) Derive an expression for the velocity V of the brick plus hammer immediately after the collision with the brick.

ii) In an earlier part of the performance, a selection of audience members with different weights has stood on the clown's chest. The deformation of the chest is proportional to the weight of the person standing, and a 100 kg man produces a depression of 30 mm in his chest. Derive an expression for the spring constant of the clown's chest.

iii) The Occupational Health and Safety Officer for the circus decides that the breaking brick trick should not depress the clown's chest more than 30 mm beyond the resting position of the brick before the collision. Derive a value for the required mass M of the brick. You may neglect the gravitational potential energy associated with deformation of the clown's chest.

iv) Express your answer to part (iii) as an inequality. Describe the reason for the direction of the inequality.

Caution. Do not try this exercise at home.

Question 6. (13 marks)

i) A solid sphere, a disc and a hoop are released from rest and roll down an inclined plane, beginning at height h and ending at height 0. Air resistance is negligible. All have the same radius R. Showing all working, *and stating any assumptions you make*, determine the speed v of one of the objects at the bottom of the plane, in terms of its radius of gyration.

ii) The radii of gyration are

$k_{sphere} = \sqrt{\frac{2}{5}}\,R$ $\qquad k_{disc} = \sqrt{\frac{1}{2}}\,R$ $\qquad k_{hoop} = R$

If they are all released at the same time, state the order of their arrival at the bottom, and briefly explain your reasoning.

iii) In two or three clear sentences, explain why one of these objects is faster than another one *in terms of conservation of mechanical energy.*

iv) Using your answer to part (i), state whether a large sphere or a small sphere would roll faster when released from rest on the plane. In one sentence, explain your answer.