Physics 1A   PHYS1121 2006-S1 Answers

1.   10 Marks Total

(a) A neutral atom is one with no net charge. The number of electrons is the same as the number of protons.

(b) A negatively charged atom has one or more excess electrons over the number of protons.

(c) Charge 1 +q at (0,*a*), Charge 2 +3q at (*2a, a*), Charge 3 -q at (0, -*a*).

Force between charges q_1 and q_2 a distance r apart is given by $F_{12} = k\frac{q_1 q_2}{r^2}$ directed towards each other if the charges have opposite signs, and away from each other if they have the same sign.

Charges 1 and 2 are *2a* apart.

Thus $F_{12} = k\frac{q\ 3q}{(2a)^2} = 0.75k\left(\frac{q}{a}\right)^2$ directed along the positive x-axis.

Charges 1 and 3 are $\sqrt{(2a)^2 + (2a)^2} = \sqrt{8}a$ apart and the line connecting them is at 45° to the axes.

Thus $F_{23} = k\frac{-q\ 3q}{(\sqrt{8}a)^2} = -0.375k\left(\frac{q}{a}\right)^2$; i.e. directed towards each other.

Thus, taking components along the *x*- and *y*-axes:

x-axis: $F_x = k\left(\frac{q}{a}\right)^2 \left[0.75 - 0.375\cos(45°)\right] = k\left(\frac{q}{a}\right)^2 0.485$

y-axis: $F_y = k\left(\frac{q}{a}\right)^2 \left[-0.375\sin(45°)\right] = -k\left(\frac{q}{a}\right)^2 0.265$

Therefore, the net force on charge *3q* is given by $\underline{F} = k\left(\frac{q}{a}\right)^2 \left[0.485\underline{i} - 0.265\underline{j}\right]$.

2. 6 Marks Total

(a) Applying Gauss's Law, the surface must enclose a positive net total charge, since $\frac{q}{\varepsilon_0} = \Phi > 0$.

(b)
(i) Only the charge inside the radius *R* contributes to the flux.
i.e. applying Gauss's law, $\Phi = \frac{q}{\varepsilon_0}$.

(ii) For a sphere of radius *2a* we must include the total charge from both the ring and the point charge.

Charge on the ring is $2\pi a\lambda$.

Thus, Gauss's law gives $\Phi = \frac{q + 2\pi a\lambda}{\varepsilon_0}$.

3. 9 Marks Total

(a) The potential energy increases. When a charge is made to move in the direction of the field it moves to a region of lower electric potential. Then the product of the negative charge times the lower potential gives a higher potential energy.

(b) The units of linear charge density must be C/m (charge per unit length).

Thus units of α are [charge per unit length / length] = [charge/length^2] = [Cm^{-2}].

The electric potential is given by $V = \int \frac{k.dq}{r}$.

Consider an element of length dx at distance x from the origin.

Its charge, dq, must be $\lambda\, dx = \alpha\, x\, dx$

and the contribution to the potential at the point $x = -D$ therefore $k\, \alpha\, x\, dx\, /\, (x+D)$.

Thus the total potential at the point $x = -D$ is given by (noting that $r = x+D$):

$$V = \int_0^L \frac{k.dq}{r} = \int_0^L \frac{k\lambda}{r} dx = \int_0^L \frac{k\alpha x}{x+D} dx = k\alpha \int_0^L \frac{x}{x+D} dx.$$

Making use of $\int \frac{xdx}{a+1.x} = \frac{x}{1} - \frac{a}{1^2}\ln(a+1.x) = x - a\ln(a+x)$, we have

$$\int_0^L \frac{x}{x+D} dx = [x - D\ln(D+x)]]_0^L = [L - D\ln(D+L) + D\ln(D)] = L - D\ln\left(1 + L/D\right).$$

Thus $V = k\alpha\left(L - D\ln[1 + L/D]\right)$.

4. 10 Marks Total

(a) Nothing happens to the charge if the wires are disconnected – the capacitors remain charged.

If the wires are now connected to each other, the charges can move along the wires until the entire conductor is at a single potential, and the capacitor discharged. There is now no net charge on the capacitor.

(b)

(i) The potential energy of a capacitor is given by $U=0.5CV^2$.

Thus for the two capacitors, each charged to a potential difference ΔV, we have
$U = 0.5C(\Delta V)^2 + 0.5C(\Delta V)^2 = C(\Delta V)^2$.

(ii) The altered capacitor has capacitance $C' = C/2$.

The potential across each capacitor must be the same, since they are in parallel.

The total charge, Q, is the same as before. Let $\Delta V'$ be the potential difference across the capacitors after the separation.

Hence, since $Q=CV$, we have $Q = C\Delta V + C\Delta V$ (before) $= C\Delta V' + C/2\Delta V'$ (after)

so that $2\Delta V = 3/2\Delta V'$; i.e. $\Delta V' = 4/3\Delta V$

(iii) The potential energy is given by $U=0.5CV^2$

so that the new potential energy is:
$$U' = \frac{1}{2}C(\frac{4\Delta V}{3})^2 + \frac{1}{2}\frac{C}{2}(\frac{4\Delta V}{3})^2 = \frac{16}{9}\left(\frac{1}{2}+\frac{1}{4}\right)C(\Delta V)^2 = \frac{4}{3}C(\Delta V)^2.$$

(iv) The energy has increased by $\left(\frac{4}{3}-1\right)C(\Delta V)^2 = \frac{C}{3}(\Delta V)^2$.

The extra energy comes from the work put into the system when the plates of the capacitor are pulled apart. This requires a force to be applied because the oppositely charged plates attract.

5 11 Marks Total

(a) Applying the right hand rule (motion up, field out of page), the proton experiences a force directed from left to right across the page, and so it veers to the right.

It will proceed to follow a circle in the clockwise direction, as it always experiences a force perpendicular to its direction of motion.

If instead the particle were a negatively-charged electron the path would veer to the left and then continue to move in a circle in the anti-clockwise direction.

At the same speed, the electron's circle would have a much smaller radius.
(this comes from $\frac{mv^2}{r} = qvB$, hence $r = \frac{mv}{qB}$, but they don't need to prove this)

(b) For each segment we have I = 5.00 A and **B** = 0.020 T **j**.

The force on a current carrying wire is given by **F** = I **l** **x** **B**, where **l** is the vector denoting the length and direction of the wire.

Resolve **l** into components along each section of the wire.

(i) For segment *ab* **l** = -0.40 m **j**. Hence **F** = 5.00 x -0.40 x 0.020 **j x j** N = 0 N.

(ii) For segment *bc* **l** = +0.40 m **k.** Hence **F** = 5.00 x 0.40 x 0.020 **k x j** N = -0.040 **i** N.

(iii) For segment *cd* **l** = -0.40 m **i** + 0.40 m **j**. Hence **F** = 5.00 x 0.40 x 0.020 (**-i x j** + **j x j)** N = **-**0.040 **k** N.

(iv) For segment *da* **l** = +0.40 m **i** - 0.40 m **k**. Hence **F** = 5.00 x 0.40 x 0.020 **(i x j - k x j)** N = 0.040 **(k + i)** N.

6. 6 Marks Total

From Ampere's law, the magnetic field at point *a* is given by $B_a = \dfrac{\mu_0 I_a}{2\pi r_a}$ where I_a is the net current through the area of the circle of radius r_a, which in this case is 1.00A out of the page.

Hence $B_a = \dfrac{(4\pi 10^{-7}\ \ Tm/A)(1.00\ \ A)}{2\pi(1.00\times 10^{-3}\ \ m)} = 2.00\ \ 10^{-4} = 200\ \ \mu T$ towards the top of the page (direction from right hand rule).

Similarly, at point *b*: $B_b = \dfrac{\mu_0 I_b}{2\pi r_b}$ where I_b is the net current through the area of the circle of radius r_b, which in this case is 1.00-3.00 A = -2.00 A into the page.

Therefore $B_b = \dfrac{(4\pi 10^{-7}\ \ Tm/A)(2.00\ \ A)}{2\pi(3.00\times 10^{-3}\ \ m)} = 133\ \ \mu T$ towards the bottom of the page.

7. 8 Marks Total

(a) Faraday's Law of Induction states that $\varepsilon = -\frac{d\Phi}{dt}$; i.e. the emf, ε, generated in a circuit is proportional to minus the rate of change of magnetic flux, Φ, through that circuit.

(b) $\varepsilon = -\frac{d\Phi}{dt}$ with $\Phi = BA$, and A the area of the flux linked.

Thus $\varepsilon = -B\frac{dA}{dt}$

where $\frac{dA}{dt}$ is the rate of change of area = av (as only side a is cutting new flux).

So $\varepsilon = -Bav$.

Now $\varepsilon = IR$ for a current I, so that $IR = Bav$, taking the absolute value.

In equilibrium (i.e. constant speed, v), the weight balances the opposing force.

The force on a current carrying conductor is given $F = BIa$,

and from Lenz's law it must be upwards (so as to oppose the change).

Thus $mg = BIa$ with $I = Bav/R$ (from above).

Hence $\frac{B^2 av}{R} = mg$

or $B = \sqrt{\frac{mgR}{a^2 v}} = \sqrt{\frac{0.5\ \ 9.8\ \ 2}{1^2\ \ 8}} = 1.11T$.