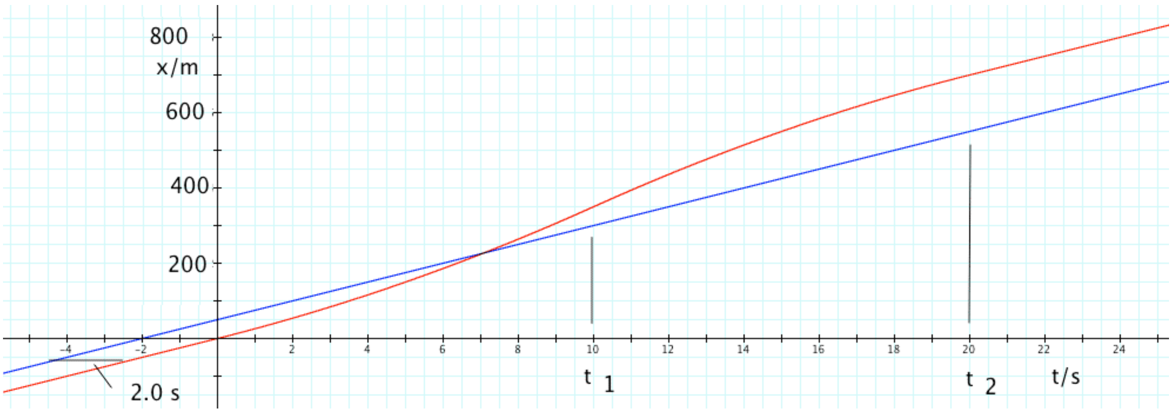
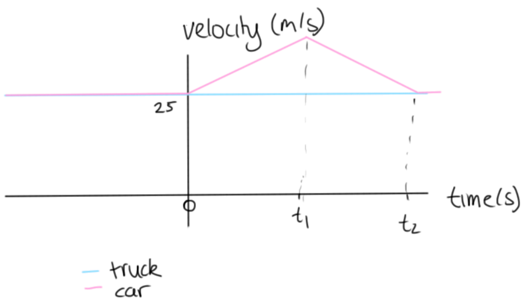
Question 1

i) (a sketch similar to this)



ii)



iii) v_0 = 90 k.p.h. = (90,000 m /3600 s) = 25 $m.s^{-1}$

Car starts at x = 0, truck starts at x = d

where d = (2 s * v_0) = 2 s * 25 $m.s^{-1}$ = 50 m.

iv) $x_{truck} = v_0t + d$

$x_{car} = v_0t + \frac{1}{2}at^2$ while acceleration is positive

car is 50 m ahead when

$(v_0t + \frac{1}{2}at^2) - (v_0t + d) = 50$ m

$\frac{1}{2}at_1^2 = 100$ m

$t_1 = \sqrt{2 * 100\text{m} / 2.0\text{m.s}} = 10$ s

v) $x_1 = v_0t_1 + \frac{1}{2}at_1^2 = 25*10 + \frac{1}{2}*2*10^2 = 350$ m

vi) See graphs in parts (i) and (ii)

Question 2

a)

i) $\boldsymbol{v}_w = -v_w \cos\theta\,\mathbf{i} - v_w \sin\theta\,\mathbf{j}$

ii) $\boldsymbol{v}_w = \boldsymbol{v} + \boldsymbol{v}'$ where $\boldsymbol{v}'$ is the apparent wind velocity

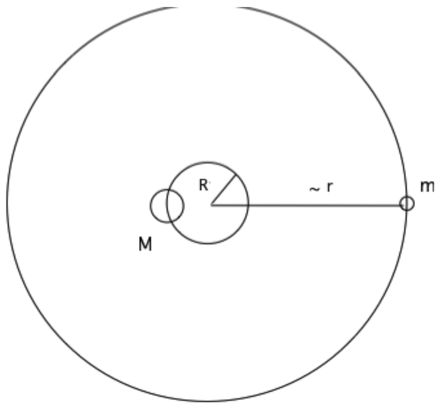
$\boldsymbol{v}' = \boldsymbol{v}_w - \boldsymbol{v} = -v_w \cos\theta\,\mathbf{i} - v_w \sin\theta\,\mathbf{j} - v\,\mathbf{j}$

$\boldsymbol{v}' = -v_w \cos\theta\,\mathbf{i} - (v + v_w \sin\theta)\,\mathbf{j}$

iii) $v' = (v_w^2 \cos^2\theta + (v^2 + 2vv_w \sin\theta + v_w^2 \sin^2\theta))^{1/2}$

$= (v_w^2 + v^2 + 2vv_w \sin\theta)^{1/2}$

b)



i)

Note that the radius of the planet's orbit is approximately a circle with radius r, because r >> R

ii) Either:

Newton's 3rd law:

|Force on star| = |force on planet|

$Ma_{star} = ma_{planet}$

$MR\omega^2 = mr\omega^2$

$m = MR/r$

or

Newton's 2nd law and law of gravitation

$Ma_{star} = |F_{grav}|$

$MR\omega^2 = GMm/r^2$

$m = r^2R\omega^2/G = 4\pi^2r^2R/GT^2$

or anything else correct.

Question 3

i) Before the collision, there are no non-conservative forces acting, so mechanical energy is conserved.

$U_i + K_i = U_f + K_f$ or just $E_i = F_f$

Immediately before the collision, mass *m* is travelling horizontally with speed *v*. So

$mgR + 0 = 0 + ½mv^2$

so $v = \sqrt{2gR}$

ii) During the collision, no external horizontal forces act, therefore momentum is conserved. Take the speed of the combined object as *V*. So

$mv = (m + M)V.$

$V = mv/(m + M)$

$= m\sqrt{2gR}/(m + M)$ or $= \sqrt{2gR}/(1 + M/m)$

4 a)

|Heat energy gained by water and bucket| - |heat energy lost by axe head| = 0

$$
\begin{aligned}
&m_w c_w (T_f - T_{wi}) + m_b c_s (T_f - T_{wi}) + m_{ah} c_s (T_f - T_{ahi}) = 0 \\
&12.5 \times 4186 \times (T_f - 24.3) + 0.500 \times 456 \times (T_f - 24.3) + \\
&+ 0.755 \times 456 \times (T_f - 337) = 0 \\
&\Rightarrow T_f (344.28 + 52325 + 228) = 1406830 \\
&T_f = \frac{1406830}{52553} = 26.60 \\
&T_f = 26.6^o\text{C (3 sig. fig)}
\end{aligned}
$$

b) i) This is a diatomic gas at approximately room temperature. It has 5 degrees of freedom. 3 of these degrees of freedom correspond to the translational movement of the particles (one in x, one in y and one in z direction) and 2 correspond to the rotational movement of the molecules (abut the two axes perpendicular to the line joining the two atoms).

ii) This gas is kept at a constant volume by the air tight seals so:

$$Q = nC_V \Delta T$$

Need to calculate *n* using the ideal gas law:

$$
\begin{aligned}
&n = \frac{PV}{RT} \\
&= \frac{1.01 \times 10^5 \times 5.00 \times 5.00 \times 2.00}{8.314 \times 273.15} \\
&= 2223.72
\end{aligned}
$$

Alternatively use the fact that at 0.00 ^{o}C 1 mol of ideal gas takes up 2.241 × 10^{-2} m^3 to calculate *n*, this gives *n* = 2 231 mols.

$$
\begin{aligned}
&Q = 2223 \times (\tfrac{1}{2} f R) \times 24.5 = 1132 kJ \\
&= 1130\text{kJ (3 sig. fig)}
\end{aligned}
$$

Or if using *n* calculated the second way you get Q = 1136 kJ = 1140 kJ (3 sig fig) which is also correct.

c) i)

$$
\begin{aligned}
&W = -\int P.dV \\
&\text{which is just the area under the rectangle} \\
&W = -3.00 \times 1.01 \times 10^5 \times (2.000 - 0.6347) \times 10^{-3} \\
&= -413.685 J = -414\text{J (3 sig fig)}
\end{aligned}
$$

ii)

$$
\begin{aligned}
&\Delta E_{int B \to C} = Q_{B \to C} + W_{B \to C} \\
&= 1033 - 414 = 619 J
\end{aligned}
$$

iii)

$$
\begin{aligned}
&\Delta E_{int C \to D} = 0 \text{ isothermal process} \\
&W_{C \to D} = \Delta E_{int C \to D} - Q_{C \to D} \\
&= 0 - 664 = -664 J
\end{aligned}
$$

Alternatively could integrate

$$W_{C \to D} = -\int P.dV$$

With P = nRT/V but this takes a lot longer…..

iv) Around a cycle there is no change in the internal energy:

$$\Delta E_{intA\rightarrow B} + \Delta E_{intB\rightarrow C} + \Delta E_{intC\rightarrow D} + \Delta E_{intD\rightarrow A} = 0$$
$$\Delta E_{intD\rightarrow A} = 1.00 \times 1.01 \times 10^5 \times (6.000 - 1.227) \times 10^{-3} - 1203 = -721J$$
$$\Delta E_{intA\rightarrow B} + 620 + 0 - 721 = 0$$
$$\Delta E_{intA\rightarrow B} = 101J$$

5 a)

i)

maximum potential energy stored in spring $= \frac{1}{2}kA^2$

$\Rightarrow k = \frac{2\times1.00}{0.100^2} = 200Nm^{-1}$

ii)

$m = \frac{k}{\omega^2}$

maximum speed $= A\omega$

$\Rightarrow \omega = \frac{1.20}{0.100} = 12.0\text{rads}^{-1}$

$\Rightarrow m = \frac{200}{12.0^2} = 1.39kg$

iii)

$f = \frac{\omega}{2\pi}$

$= \frac{12.0}{2\pi} = 1.91\text{Hz}$

iv)

$x = A\sin\omega t$

$= 0.100\sin(12.0t)$

must be sin as it starts at the origin.

b)

i)

$3\lambda = 2.40$ m (as there are 6 loops, i.e. 6 half wavelength)

$\lambda = 0.800m$

$v = f\lambda = 50 \times 0.800 = 40ms^{-1}$

ii)

$v = \sqrt{\frac{T}{\mu}}$

$T = mg = \mu v^2$

$\Rightarrow m = \frac{0.234\times10^{-3}\times40^2}{9.8} = 0.0382kg = 38.2g$

iii)

for next harmonic $\lambda = \frac{2.40}{7/2} = 0.6857m$

$v = 50 \times 0.6857 = 34.2857$

$\Rightarrow m = \frac{\mu v^2}{g} = \frac{0.234\times10^{-3}\times34.2857^2}{9.8} = 28.06g$

so need to remove 38.2 - 28.1 = 10.1 g.

c) i) As observed frequency is less than emitted frequency and the observer is stationary the source (plane) is moving away from Stan, ie. Receding.

ii)

$f' = f(\frac{c}{c+v_s})$

$\frac{2}{3}f = f(\frac{c}{c+v_s})$

$\frac{2}{3} = \frac{c}{c(1+v_s/c)}$

$\Rightarrow v_s = \frac{1}{2}c = 170 \text{ ms}^{-1}$

iii)

$40.0\text{km/h} = 11.11\text{ms}^{-1}$

$f' = f_p(\frac{c+v_o}{c+v_s})$

$= f_p(\frac{340+11.11}{340+170})$

$= 0.688f_p = 0.69f_p$ (2 sig. fig.) either 2 or 3 sig figs is acceptable.