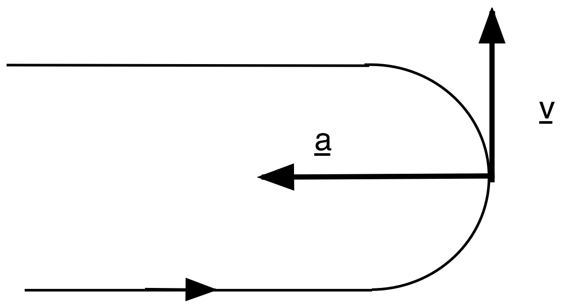
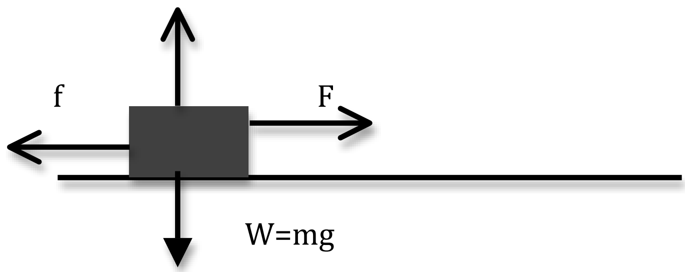
Exam Question 1 PHYS 1121 2013, Session 2



a) i) $\mathbf{v} = v\mathbf{j}$

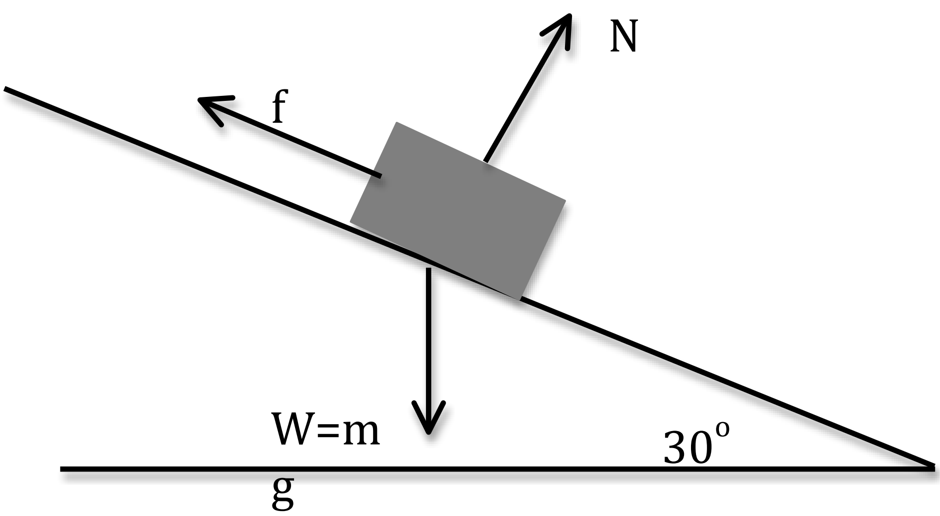
ii) $\mathbf{a} = -\frac{v^2}{r}\mathbf{i}$

b)



c) 0 ms^{-2}

d) i)



The direction of the acceleration is parallel to the ramp, in the downward direction.

The forces in the y-dir (perpendicular to ramp) are

N, and mg cos 30°. There is no motion in the y-dir, so

$$\sum F_y = N - mg\cos\theta$$
$$0 = N - mg\cos\theta$$
$$N = mg\cos\theta$$

In the x-dir

$$\sum F_x = N - mg\sin\theta$$
$$ma_x = mg\sin\theta - f$$
$$ma_x = mg\sin\theta - \mu N$$
$$ma_x = mg\sin\theta - \mu mg\cos\theta$$
$$a_x = g\sin\theta - \mu g\cos\theta$$
$$= 9.8\sin(30°) - 0.40(9.8)\cos(30°)$$
$$= 1.5 ms^{-2}$$

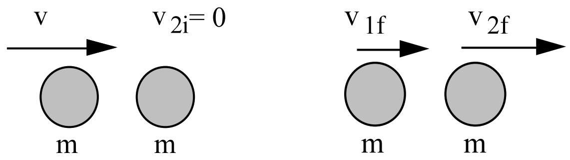
As the only acceleration is in the x-direction, this is the magnitude of the total acceleration.

Q2

a) i) If non-conservative forces do no work, mechanical energy is conserved.

ii) In a completely elastic collision, mechanical energy is conserved.

iii) All initial motion is in x direction, so no y momentum. After the collision the second particle travels in the x direction. So the second particle must also travel in the x direction, if at all.



neglect external forces ⇒ $p_i = p_f$

$$mv + 0 = mv_1 + mv_2 \qquad \text{(i)}$$

Elastic collision so mechanical energy conserved

$$\frac{1}{2} mv^2 + 0 = \frac{1}{2} mv_1{}^2 + \frac{1}{2} mv_2{}^2 \qquad \text{(ii)}$$

(i) –> $v_{2f} = v - v_{1f}$ (iii)

substitute in (ii) –>

$$\frac{1}{2} mv^2 = \frac{1}{2} mv_1{}^2 + \frac{1}{2} m(v^2 + v_1{}^2 - 2vv_1)$$

$$\therefore \quad 0 = v_1{}^2 - vv_1$$

$$0 = v_1(v_1 - v) \qquad \textit{2 solutions}$$

Either: $v_1 = 0$ and (iii)–> $v_2 = v$

i.e. 1st stops dead, all p and K transferred to m_2

or: $v_{1f} = v$ and (iii)–> $v_2 = 0$ i.e. missed it.

We are told that v_2 is non-zero so we keep the first solution: $v_1 = 0$ and $v_2 = v$

b) It rolls without slipping, so friction does no work, so mechanical energy is conserved.

$K_i + U_i = K_f + U_f$ so

$\frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 = 0 + mgh$

Equation sheet has I for a sphere = $2/5\ mr^2$. Rolling so $\omega = v/r$

$\frac{1}{2} mv^2 + \frac{1}{2} (2/5\ mr^2)(v/r)^2 = mgh$

$\frac{1}{2} mv^2 + 1/5\ mv^2 = mgh$

$(7/10)v^2 = gh$

$h = 7v^2/10g$

c) $I = \int_{body} r^2 \, dm$

Write $M = \lambda L$ where the mass per unit length $\lambda = M/L$

So for an element of mass $dM = \lambda dx$

$$I = \int_{x=0}^{L} x^2 \lambda dx = \frac{1}{3}\lambda(L^3 - 0) \qquad \text{but } \lambda L = M \text{ so}$$

$$I = \frac{1}{3}ML^2.$$

i) $a_c = r\omega^2 = r\left(\frac{2\pi}{T}\right)^2$

ii) magnitude of gravitational force between star and planet

= centripetal force on the star. Equating magnitudes:

$$Ma_c = |F_g|$$

$$Mr\left(\frac{2\pi}{T}\right)^2 = \frac{GMm}{r^2}$$

$$m = \left(\frac{2\pi}{T}\right)^2 \frac{r^3}{G}$$

iii)

EITHER From Newton's third law, the gravitational forces on the star and the planet have equal magnitude. They orbit their common centre of mass with the same period *T*. Hence

$$Mr\left(\frac{2\pi}{T}\right)^2 = mR'\left(\frac{2\pi}{T}\right)^2$$

OR They orbit around their centre of mass. If it is at the origin, then $\Sigma m_i r_i = 0$

Both of these give mR' = Mr.

From the diagram, R = r + R' = r(1 + R'/r) = r(1 + M/m)

Solutions: Question 4 1121 T2 2013

a) i) 3
ii) all translational, in the x, y and z directions.
iii)
$$Q = nc_V \Delta T$$
$$= 15.0 \times \frac{1}{2} \times 3 \times 8.314 \times 10$$
$$= 1870 \text{ J (3 sig fig)}$$

b) i)
$$\Delta L = \alpha L \Delta T$$
$$= 24 \times 10^{-6} \times 1.00 \times 160$$
$$= 3.84 \times 10^{-3} \text{ cm}$$

ii)
$$\Delta A = 2\alpha A \Delta T$$
$$= 2 \times 24 \times 10^{-6} \times 1.00 \times 160$$
$$= 0.154 \text{ cm}^2$$

iii)
$$Q = mc\Delta T$$
$$= 1 \times 20 \times 2.70 \times 10^{-3} \times 910 \times 160$$
$$= 7862 \text{ J}$$
$$= 7860 \text{ J (3 sig fig)}$$

c) i)
$$PV = nRT$$
$$T = \frac{PV}{nR} = \frac{5.51 \times 1.01 \times 10^5 \times 20 \times 10^{-3}}{2.5 \times 8.314}$$
$$= 535 \text{ K}$$

ii)
$$W = -\int P dV$$
$$= P\Delta V$$
$$= 1.53 \times 1.01 \times 10^5 \times 30 \times 10^{-3}$$
$$= 4636\text{J}$$
$$= 4640\text{J is done on the gas}$$

iii)
$$\Delta E_{int} = \frac{f}{2} nR\Delta T = 2.50 \times \frac{5}{2} \times 8.314 \times (534.49 - 148.69)$$
$$= 20099 \text{ J}$$
$$= 20100 \text{ J (3 sig fig)}$$

iv)
$$\text{adiabatic } \Rightarrow PV^\gamma = \text{constant}$$
$$\gamma = \frac{7/2}{5/2} = 1.4$$
$$PV^{1.4} = 5.51 \times 1.01 \times 10^5 \times (20 \times 10^{-3})^{1.4}$$
$$= 2328 \text{ Pa m}^{4.2}$$
$$= 2330 \text{ Pa m}^{4.2} \text{ (3 sig fig)}$$

v) One way is to add the changed in internal energy around the cycle:

$$W = \Delta E_{int} = -(\Delta E_{intC\to A} + \Delta E_{intB\to C})$$
$$\Delta E_{intB\to C} = Q_{B\to C} + W_{B\to C} = -16200 + 4640 = -11560$$
$$\Rightarrow W = -(20099 - 11560)$$
$$= -8540 \text{ J}$$

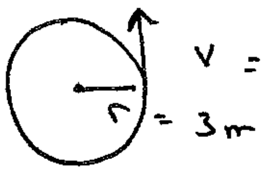
Alternatively you could do the integral (harder though…):

$$W = -\int PdV$$
$$= -\int_{20L}^{50L} \frac{2330}{V^{1.4}} dV$$
$$= \frac{2330}{0.4}[V^{-0.4}]_{20\times 10^{-3}}^{50\times 10^{-3}}$$
$$= 5825 \times [3.3 - 4.78]$$
$$= -8562 \text{ J}$$
$$= -8560 \text{ J (3 sig fig)}$$

The slight difference in the answers is down to how many significant figures were kept in the working. Either answer is acceptable.

PHYS 1121 2013-2 Q5 | Total 25 Marks

(a)



② (i) $T = \frac{2\pi r}{v} = \frac{2\pi \cdot 3}{4} = \frac{3}{2}\pi = 4.7$ m/s to 2SF

$f = \frac{1}{T} = 0.21$ Hz to 2SF

③ (ii) $(x, y) = (0, 3m)$ at $t = 0s$.

let $x = A \sin(\omega t + \phi_1)$, $y = A \cos(\omega t + \phi_2)$ } ⇒ $A = 3$ since $x_{max}, y_{max} = 3$

Then x: $0 = A \sin(\phi_1)$ ⇒ $\phi_1 = 0$

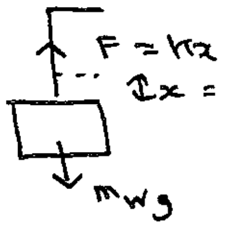
y: $3 = A \cos(\phi_2) = 3 \cos \phi_2$ } ⇒ $\cos \phi_2 = 1$ ⇒ $\phi_2 = 0$

ie $x = 3 \sin \omega t$, $y = 3 \cos \omega t$ } with $\omega = 2\pi f = \frac{2\pi \cdot 2}{3\pi} = \frac{4}{3} = 1.3 s$

(b) $m_p = 4.50\text{g}$

$m_w = 1.63\text{kg}$

④ (i)



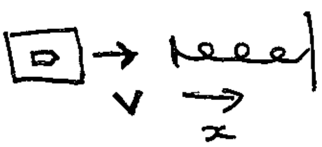
We have $F = kx = m_w g$ in equilibrium

$$\Rightarrow k = \frac{m_w g}{\Delta x} = \frac{1.63 \times 9.81}{0.140} \text{ N/m}$$

$$= ~~113.52~~ \; 114.22 \text{ N/m}$$

$\underline{k = 114 \text{ N/m}}$ to 3 SF

⑤ (ii)



$x = 13.0\text{cm}$

Let bullet + block move at V when reaches spring

Assuming energy conservation: [Motion across frictionless not relevant]

KE lost by bullet/block = PE gained by spring on compression

ie $\frac{1}{2}(m_b + m_w)V^2 = \frac{1}{2}kx^2$

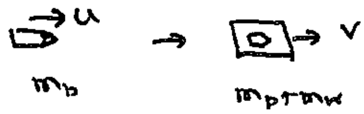
$$\Rightarrow V^2 = \frac{kx^2}{m_b + m_w} = \frac{114 \times (0.13)^2}{[1.63 + 0.0045]}$$

$$= 1.181 \quad (\text{m/s})^2$$

$$\Rightarrow V = 1.087 \text{ m/s}$$

$$\underline{= 1.09} \text{ m/s to 2SF}$$

④ (iii) Now conserving momentum between bullet and bullet-block

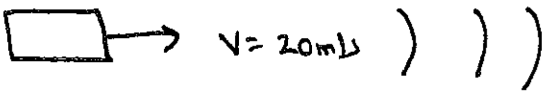


$m_b u = (m_b + m_w)V$

$$\Rightarrow u = \left(\frac{m_b + m_w}{m_b}\right)V = \left(\frac{0.0045 + 1.63}{0.0045}\right)1.087 = 394.7 \text{ m/s}$$

$= 395$ m/s to 3SF

(c)



$\leftarrow$ $V_W = 10.0$ m/s

$f = 900$ Hz

$\rightarrow$ $c = 343$ m/s

④ (i) For the man the frequencies are unchanged as he is at rest wrt to the source

ie $f_{man} = 900.0$ Hz

For the Woman apply doppler shift formula for a moving source

$$f' = f\left(\frac{V + V_o}{V - V_s}\right)$$ where V_o speed observer $= 0$
V_s – source
V – sound

$$= 900\left(\frac{343}{343 - 20}\right) \text{ Hz}$$

$= 955.7$ Hz

$= 956$ Hz to 3SF.

③ (ii) Speed of sound is now effectively $c = V + W_w$
(V_w wind speed)

So doppler formula becomes

$$f' = f\left(\frac{V + V_w}{V + V_w - V_s}\right) = 900\left(\frac{343 + 10}{343 + 10 - 20}\right)$$

$= 954.05$ Hz

For woman $= 954$ Hz to 3SF

For man, frequency is unchanged.