SOLUTION to T1R S2 2000

Question 1 (16 Marks)

(a) The velocity is
\[ v = \frac{dx}{dt} = 1.60(1.30) \cos(1.30t - 0.75) \text{ cm/s} \]
and the acceleration is
\[ a = \frac{dv}{dt} = -1.60(1.30)^2 \sin(1.30t - 0.75) \text{ cm}^2/\text{s} \]
\[ = -(1.30)^2 y \]

At t=0 s

Displacement: \[ y = 1.60 \sin(-0.75) = -1.60 \sin 43^\circ = -1.09 \text{ cm} \]
Velocity: \[ v = 2.08 \cos(43^\circ) = 1.52 \text{ cm/s} \]
Acceleration \[ a = -(1.30)^2 (1.09) = 1.84 \text{ cm}^2/\text{s} \]

At t=0.60 s

Displacement: \[ y = 1.60 \sin(0.03) \approx 1.60(0.03) = 0.048 \text{ cm} \]
Velocity: \[ v = 2.08 \cos(0.03) \approx 2.08 \text{ cm/s} \]
Acceleration \[ a = -(1.30)^2 (0.048) = -0.081 \text{ cm}^2/\text{s} \]

(b) A torsional pendulum has a period given by

\[ T = 2\pi \sqrt{\frac{I}{K}} \]

In the question, a torque of 5Nm produces a deflection of 12° which gives a spring constant, \( K \), given by

\[ K = \frac{\text{external torque}}{\text{angular displacement}} = \frac{5\text{Nm}}{12 \times \frac{2\pi}{360}} = 23.9 \text{ Nm/rad} \]

\[ I = \left( \frac{T}{2\pi} \right)^2 K = \left( \frac{0.5s}{2\pi} \right)^2 (23.9) = 0.151 \text{ kgm}^2 \]
**Question 2 (18 Marks)**

Let source 1 emit waves in the positive \( x \)-direction such that

\[
y_1 = y_{1m} \sin 2\pi v_1 (t - \frac{x_1}{v})
\]

and source 2 emit in the negative \( x \)-direction:

\[
y_2 = y_{2m} \sin 2\pi v_2 (t + \frac{x_2}{v})
\]

We are told \( v = 3 \text{ m/s} \). \( x_1, x_2 \) are measured from source 1, source 2 respectively.

Equating \( y_1 \) at \( x_1 = 0 \) to \( y_{1s} \) and \( y_2 \) at \( x_2 = 0 \) to \( y_{s2} \) we find

\[
y_{1m} = 0.06 \text{ m} \quad v_1 = v_2 = 0.5 \text{ Hz} \quad \text{ and } y_{2m} = 0.02 \text{ m}
\]

Superposition of the 2 waves at \( x_1 = 12 \text{ m} \) and \( x_2 = -8 \text{ m} \) gives

\[
y = y_1 + y_2 = 0.06 \sin \pi (t - \frac{12}{3}) + 0.02 \sin \pi (t - \frac{8}{3})
\]

\[
= 0.06 \sin \pi t + 0.02 \sin (\pi t - \frac{2\pi}{3})
\]

\[
= 0.06 \sin \pi t + 0.02 (\sin \pi t \cos \frac{2\pi}{3} - \cos \pi t \sin \frac{2\pi}{3})
\]

\[
= 0.06 \sin \pi t + 0.02 \sin \pi t (-\frac{1}{2}) - \cos \pi t \sin \frac{\sqrt{3}}{2})
\]

\[
= 0.05 \sin \pi t - 0.0173 \cos \pi t
\]

(b) The sound (pressure) wave is given as

\[
p = 1.5 \sin \left(\frac{2\pi}{\lambda}(x - 330 t)\right)
\]
A travelling wave equation in standard form is

\[ y = y_m \sin \left[ 2\pi \nu (t - \frac{x}{v}) \right] \]

with amplitude \( y_m \), frequency \( \nu \) and velocity \( v \). Rewriting the given sound wave in standard form,

\[ p = -1.5 \sin \left[ 2\pi \frac{330}{\lambda} (t - \frac{x}{330}) \right] \]

whence,

(i) \( v = 330 m/s \),

(ii) \( v = \frac{330}{2} = 165 Hz \)

(iii) amplitude \( p_0 = 1.5 Pa \)

(iv) at \( x=1/6 m \) and \( t=0 \), by substituting in,

\[ p = -1.5 \sin \left[ 2\pi \frac{330}{2} (0 - \frac{1/6}{330}) \right] = 1.5 \sin \frac{\pi}{6} = 0.75 Pa \]

**Question 3 (13 Marks)**

The \( m^{th} \) bright fringe due to \( \lambda_1 \) and the \( k^{th} \) bright fringe due to \( \lambda_2 \) are at positions

\[ y_m = \frac{mD\lambda_1}{d} \quad \text{and} \quad y_m' = \frac{kD\lambda_2}{d} \]

For a bright fringe from each wavelength at the same location,

\[ \frac{m}{k} = \frac{\lambda_2}{\lambda_1} = \frac{900}{750} = \frac{6}{5} \]

and \( y_6 = y_5' = \frac{(6)(2)(750 \times 10^{-9})}{2 \times 10^{-3}} = 4.5 \times 10^{-3} m \)
Question 4 (19 Marks)

The diagram required is

We require wavelengths, $\lambda_1$ in air (with refractive index) which fall in the visible range (400nm to 700nm) and interfere firstly, constructively

$$\lambda_1 = \frac{2(n_2 / n_1)t}{m + 1} = \frac{(2)(1.80 / 1)(250 \times 10^{-9})}{m + 1}$$

and $m = 0, 1, 2, \ldots$

or

$$\lambda_1 = \frac{900 \times 10^{-9}}{m + 1} = \frac{900 \text{ nm}}{m + 1}$$

(constructive interference)

for which $m = 1$ gives $\lambda_1 = 450 \text{ nm}$ (blue/indigo), all other wavelengths are outside the visible range, and destructively,

$$\lambda_d = \frac{2(n_2 / n_1)t}{m + \frac{1}{2}} = \frac{(900 \text{ nm})}{m + \frac{1}{2}}$$

(destructive interference)
where \( m = 1 \) giving \( \lambda_1 = 600 \text{ nm} \) (orange), is the only visible wavelength.

The medals will appear blue/violet, since light reflected at the red/orange end of the spectrum will be attenuated and light from the blue/violet end of the spectrum will be most strongly reflected.

**Question 5 (14 Marks)**

(a) (i) The polariser reduces the incident intensity \( I_0 \) to \( I = \frac{I_0}{2} \). The analyser will transmit an intensity \( I' \) given by

\[
I' = I \cos^2 \theta = \frac{I_0}{2} \cos^2 30^\circ = 0.375I_0
\]

(ii) The polariser will transmit an intensity

\[
I = I_0 \cos^2 \theta = I_0 \cos^2 30^\circ = 0.75I_0
\]

which is polarised at 30 degrees to the analyser’s axis, so that the final transmitted intensity is

\[
I_f = I \cos^2 \theta = 0.75I_0 \cos^2 30^\circ = 0.563I_0
\]

(b) Quarter wave plate: For plate thickness \( l \), the optical paths lengths of the ordinary and extraordinary rays are \( l_{\perp} \) and \( l_{||} \) respectively.

The two rays must emerge with a 90\(^\circ\) phase difference so the optical paths must differ by

\[
(k + \frac{1}{4})\lambda_0 \quad \text{with} \quad k=0,1,2,3\ldots
\]

The minimum thickness is given by

\[
\frac{\lambda_0}{4} = l(n_{\perp} - n_{||})
\]

so that

\[
l = \frac{\lambda_0}{4(n_{\perp} - n_{||})} = \frac{589 \times 10^{-9}}{4(1.732 - 1.456)} = 534 \times 10^{-9} \text{ m} = 534 \text{ nm}
\]