

THE UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF PHYSICS
FINAL EXAMINATION

PHYS 4103
SOLID STATE PHYSICS

PS: These questions represent the magnetism section of the course; should take 1 hours of the exams and should count on 1/3 of the final mark of the exam

Question 1:

For Gadolinium (Gd), a rare earth solid with atomic density n , the atoms have partially filled 4f shells with seven electrons:

1. Can you calculate S_{Gd} , L_{Gd} and J_{Gd} of the atoms of this rare earth crystal just by using Hund's rules?
2. Which one of the following formulas **(a)**, **(b)** and **(c)** is the Brillouin function introduced to describe the magnetisation of a paramagnetic/ferromagnetic systems under the influence of a magnetic field H :
 - (a) $B_J(y) = \frac{2J}{2J+1} \coth(\frac{1}{2J}y) - \frac{1}{2J+1} \coth(\frac{1}{2J}y)$
 - (b) $B_J(y) = \frac{2J+1}{2J} \coth(\frac{2J+1}{2J}y) - \frac{1}{2J} \coth(\frac{1}{2J}y)$
 - (c) $B_J(y) = \frac{2J+1}{2J} \operatorname{tgh}(\frac{2J+1}{2J}y) - \frac{1}{2J} \operatorname{tgh}(\frac{1}{2J}y)$
3. Once you know J_{Gd} , can you insert its value in the right $B_J(y)$ formula and can you briefly discuss why Gd can behave as a paramagnetic material?
4. The y in the $B_{J_{Gd}}(y)$ function will have a certain dependence on the temperature, T , and on the magnetic field, H . For certain T **or** H values (i.e.: $T_{saturation}$, $H_{saturation}$) the material will be in the magnetic saturation regime. Can you estimate for which $T_{saturation}$ or $H_{saturation}$ saturation will arise? Can you calculate the value of magnetisation for saturation, $M_{saturation} = ng\mu_B J_{Gd} B_{J_{Gd}}(y = y_{saturation})$? (hint: you don't need to get the numerical result but the final value of $M_{saturation}$ in function of n , g and μ_B)
5. As all paramagnetic materials, Gd will obey to Curie Law (i.e.: $\chi_m(T) = M/H \propto 1/T$). Is this true for all temperatures? In the case this is not always true, can you indicate for which range of temperatures Curie law will fails in the correct description of this system?

Question 2:

1. For a magnetic material, in which the electronic configuration of the last shells of atoms leads to a positive value for the exchange integral (J), we can imagine to be in the hypothetical case (not possible in real systems) of having J with a long range of action (e.g.: $J \propto 1/(distance)$). Oppositely to the short range of action and the exponential dependence over the distance (i.e.: $J \propto \exp[-(distance)]$) observed in real systems. Without the need of calculations, but just by using the intuitive approach discussed in the lectures, can you give a rough estimation of how many Weiss domains this hypothetical material would have?
2. Taking into account that the Curie temperature (T_C) of a certain ferromagnetic material is 2000K, can you give an estimation of the molecular field in this material? (order of magnitude is fine).
3. For a ferromagnetic material in the $T < T_C$ regime, both equations **(a)** and **(b)** can be true ($B_J(y)$ being the Brillouin function):

$$\text{(a) } M(T)/M_0 = B_J(y), \text{ (b) } M(T)/M_0 \cong \frac{y}{3} \frac{T}{T_C} \quad (1)$$

Can you discuss how Eq. **(a)** and Eq. **(b)** can be used to find a value for the spontaneous magnetisation. One or two paragraphs are sufficient to discuss this matter. (hint: sketch the two equations!)

The following may be needed $k_B = 8.617 * 10^{-5} eV/K$, $\mu_B = 5.788 * 10^{-5} eV/T$.