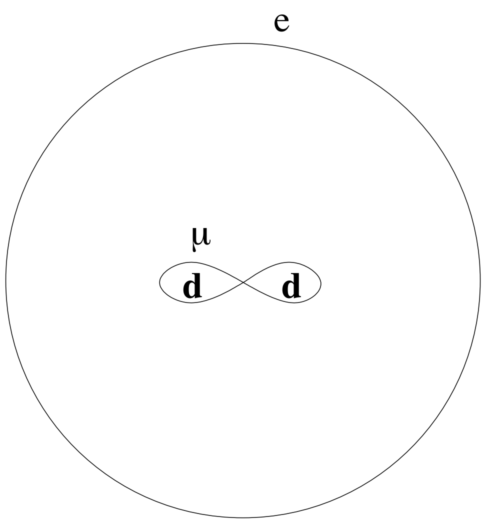
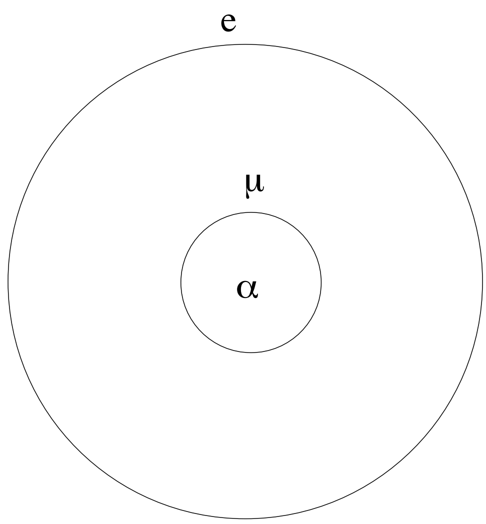
Question 1

Consider and compare two neutral systems shown below schematically. The first one consists of two deuterons (deuteron is a nucleus that consists of a proton and a neutron), a negative muon, and an electron. The deuterons are bound by the muon into a $dd\mu$ molecule. The electron is orbiting around the molecule.
The second system consists of α-particle (α-particle is a nucleus that consists of two protons and two neutrons), a negative muon, and an electron. The mass of the muon is $m_\mu \approx 200 m_e$





and masses of the proton and the neutron are $m_p \approx m_n \approx 2000 m_e$, where m_e is the electron mass. The atomic unit of energy is $\frac{m_e e^4}{\hbar^2} \approx 27.2 eV$ and the atomic unit of length is $a_B = \frac{\hbar^2}{m_e e^2} \approx 0.53\ 10^{-8} cm$. Disregard spins and magnetic interactions of the particles. Assume also that nuclear binding is infinitely strong, so disregard intranuclear excitations.

Estimate parametrically and numerically the following quantities

System I
1) Size of $dd\mu$ molecule.
2) Size of the entire system.
3) Dissociation energy of $dd\mu$ molecule.
4) Vibrational energy of $dd\mu$ molecule.
5) Rotational energy of $dd\mu$ molecule.
6) Compare these energies with the excitation energy of the electron and hence draw schematically the spectrum of the system.

System II
1) Size of the $\alpha\mu$ atom.
2) Size of the entire system.
3) Ionization energy of the $\alpha\mu$ atom.
4) Compare these energies with the excitation energy of the electron and hence draw schematically the spectrum of the system.

Question 2

The Hamiltonian of a charged particle in a static magnetic field is

$$H = \frac{(\mathbf{p} - \frac{e}{c}\mathbf{A})^2}{2m} \ .$$

a) Calculate the velocity operator

$$\hat{\mathbf{v}} = \dot{\mathbf{r}} = \frac{i}{\hbar}[H, \mathbf{r}] \ ,$$

and hence show that

$$\hat{\mathbf{v}} = \frac{1}{m}(\mathbf{p} - \frac{e}{c}\mathbf{A}) \ .$$

b) Consider an uniform magnetic field directed along the z-axis, $\mathbf{B} = (0, 0, B)$. Calculate the following commutators

$$[\hat{v}_x, \hat{v}_y]$$
$$[\hat{v}_y, \hat{v}_z]$$
$$[\hat{v}_z, \hat{v}_x]$$

You can perform the calculation in the gauge invariant form or alternatively you can use any particular gauge you like.

c) Comment on physical implications of values of commutators found in question **b**.