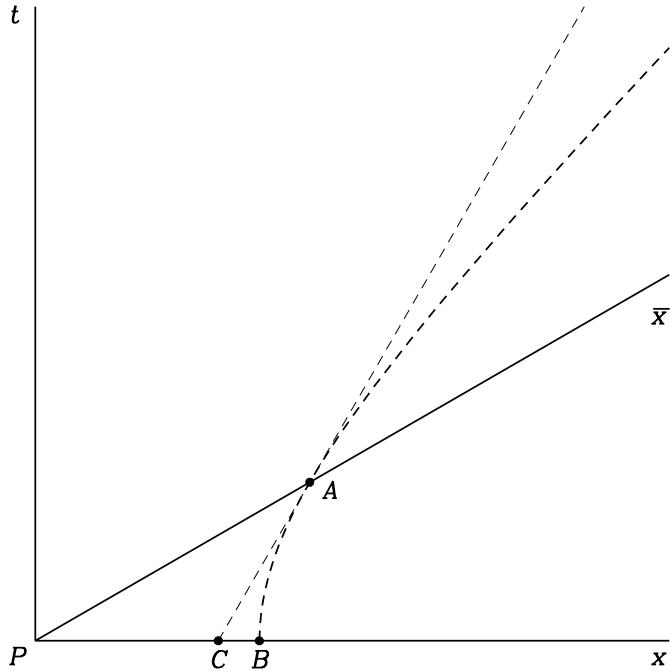


## Question 1

In the space-time diagram shown,  $t$  and  $x$  are co-ordinates as measured by observer  $O$ , with event  $P$  as origin. The co-ordinates of point  $A$  are  $t = 1/3$ ,  $x = 2/3$ . The curve  $AB$  is an invariant hyperbola, and  $AC$  is a straight line tangent to the hyperbola at  $A$ .



- (a) Determine the co-ordinates of the event  $B$ .
- (b) Write down the equation in  $x$  and  $t$  describing the curve  $AB$ . Differentiate this equation to determine the gradient of this curve, and hence determine the slope of the line  $AC$ .
- (c) Determine the co-ordinates of the event  $C$ .
- (d) If  $PA$  is the  $\bar{x}$  axis for an observer  $\bar{O}$ , determine his velocity relative to  $O$ .
- (e) If the events  $P$  and  $A$  correspond to the two ends of a rod in  $\bar{O}$  at time  $\bar{t} = 0$ , determine the length of the rod as seen by observers  $\bar{O}$  and  $O$ . Hence show that the rod exhibits the expected Lorentz contraction in frame  $O$ .

## Question 2

(a) A general expression for the stress-energy tensor is

$$T(\tilde{dx}^\alpha, \tilde{dx}^\beta) = T^{\alpha\beta}$$

Briefly explain what each symbol in this expression is.

(b) Briefly explain the difference between *dust* and *perfect fluids*.

(c) A group of particles are all moving in the same direction with the same velocity  $v$  relative to an inertial frame  $O$ . What type of particles are these? What are the components of the stress-energy tensor in the MCRF? What are the components in frame  $O$ ? Justify your answers.

(d) Consider the same group of particles as in part (c) above, but with a modified velocity distribution: they now move *isotropically* outwards from a common origin, (i.e. uniformly outwards in all directions), relative to frame  $O$ . Each particle has the same speed  $v$ .

(i) Re-write the components of the stress-energy tensor for this situation. To do this, explicitly average over all possible orientations of the velocity.

(ii) Which components are non-zero? Show all working in your answer.