THE UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF PHYSICS

PHYS3550 GENERAL RELATIVITY - ASSIGNMENT 1, 2013
Lecturer: John Webb
Date set: 18 April 2013
Due: 9 May 2013, by 5pm
Submit paper solutions only please – not by email.
Please submit solutions either direct to me in lecture period, or place in my mailbox in room OMB G61 (Administrative Office, School of Physics, Old Main Building). Late submissions will have marks deducted. Solutions will be sent out on 16 May – no marks will be awarded after that date.

1. Use the Lorentz transformation formulae to derive:

(a) The time dilation formula.
(b) The Lorentz contraction formulae.

Do this by identifying the pairs of events whose separations (in time or space) are to be compared, and then using the Lorentz transformation to accomplish the algebra that was used for the invariant hyperbolae.

2. Make a careful sketch or graph as asked below, indicating how you derived or constructed your results for each part.

(a) In the space-time diagram of O, draw the basis vectors $\vec{e}_0$ and $\vec{e}_1$.
(b) Draw the corresponding basis vectors of observer $\bar{O}$, who moves with speed 0.6 in the positive x direction relative to O.

3. The following matrix gives a Lorentz transformation from O to $\bar{O}$:

$$\begin{pmatrix} 1.25 & 0 & 0 & 0.75 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0.75 & 0 & 0 & 1.25 \end{pmatrix}$$

(a) What is the velocity (speed and direction) of $\bar{O}$ relative to O?
(b) What is the inverse matrix to the given one?
(c) Find the components in O of a vector $\vec{A} \xrightarrow[\bar{O}]{} (1,2,0,0)$

4.

(a) Find the energy, rest mass, & three-velocity $\vec{v}$ of a particle whose four-momentum has the components (4,1,1,0) kg.

(b) The collision of two particles of four-momenta

$$\vec{p}_1 \rightarrow (3,-1,0,0)\ kg, \qquad \vec{p}_2 \rightarrow (2,1,1,0)\ kg$$

results in the destruction of the two particles and the production of three new ones, two of which have four-momenta

$$\vec{p}_3 \rightarrow (1,1,0,0)\ kg, \qquad \vec{p}_4 \rightarrow (1,-1/2,0,0)\ kg$$

Find the four-momentum, energy, rest mass, and three-velocity of the third particle produced. Find the CM frame's three-velocity.

5.

(a) Let frame $\bar{O}$ move with speed v in the x-direction relative to *O*. Let a photon have frequency ν in *O* and move at an angle θ with respect to *O*'s x-axis. Show that the frequencies in *O* and $\bar{O}$ are related by

$$\frac{\bar{\nu}}{\nu} = \frac{(1 - \mathrm{v}\cos\theta)}{(1 - \mathrm{v}^2)^{1/2}}$$

(b) Even when the motion of the photon is perpendicular to the x axis ($\theta = \pi/2$) there is a frequency shift. This is called the **transverse Doppler shift**, and arises because of the time dilation. At what angle θ does the photon have to move so that there is **no** Doppler shift between O and $\bar{O}$**?**

(c) Use the two equations

$$-\vec{p}\cdot\vec{U}_{obs} = \bar{E}$$
$$E = h\nu$$

to calculate the frequency ratio above.