PHYS3230/3011 Electrodynamics 2015: Mid-session test
Thursday April 16, 2015, 5–6pm

Question 1
Consider the scalar and vector potentials

\[
\phi_X(r,t) = 0, \quad A_X(r,t) = -\frac{1}{4\pi \varepsilon_0} \frac{qt^2}{r^2} \hat{r},
\]

in some gauge \( X \).

(a) Find the E and B fields.

(b) Find the corresponding charge and current distributions.

(c) Find a gauge function \( \lambda(r,t) \) which transforms \( \phi_X \) and \( A_X \) to the Coulomb gauge.

(d) Compute the scalar potential \( \phi_C \) in the Coulomb gauge.

Hints: \( \nabla \cdot \left( \frac{\mathbf{r}}{r^2} \right) = 4\pi \delta^{(3)}(r) \), and \( \nabla(1/r) = -\hat{r}/r^2 \).

Question 2
Consider an elliptically polarised plane wave of angular frequency \( \omega \) and wavenumber \( k \) propagating in free space in the \( z \)-direction.

(a) Write down the expression for this plane wave.

(b) Find the corresponding magnetic field and Poynting vector.

(c) Calculate the intensity of this wave crossing a surface parallel to the \( xy \)-plane.

(d) Suppose the wave strikes, at right angle, the surface of a medium with permittivity \( \varepsilon \) and permeability \( \mu \). How much energy is transmitted across the surface per unit area per unit time?

Hint: \( \int_0^T dt \cos^2(a-t) = \int_0^T dt \sin^2(a-t) = T/2 \).