Total for this exam is 40 marks. Time allowed 50 mins. This in-session test is worth 20% of the total course mark.

**Question 1** Gallium Arsenide: crystal structure, lattice vibrations and heat capacity

Consider the semiconductor Gallium Arsenide (GaAs) which crystallizes in the important zincblende structure.

(i) What is the Bravais lattice type for this structure? (2 marks)

(ii) How many lattice points are there per primitive unit cell? (2 marks)

(iii) Write down the basis (4 marks)

(iv) If the zincblende structure of semiconductors were close-packed (it isn’t, but let’s claim that it is for the purposes of this question) then calculate the maximum theoretical packing factor using the close-packed hard sphere model; show all working (6 marks)

(v) If the primitive lattice vectors for the Zincblende structure are

\[ \mathbf{a} = \frac{a}{2}(\hat{x} + \hat{y}) \quad \mathbf{b} = \frac{a}{2}(\hat{y} + \hat{z}) \quad \mathbf{c} = \frac{a}{2}(\hat{z} + \hat{x}) \]

calculate the reciprocal lattice vectors \( \mathbf{a}^* \), \( \mathbf{b}^* \), \( \mathbf{c}^* \) (8 marks)

(b) Now consider a one-dimensional chain of alternating Ga and As atoms.

(i) Sketch and fully label the dispersion relations for this chain of atoms with diatomic basis. (6 marks)

(ii) Mark clearly and label on your sketch the first Brillouin zone and the region from which we can find the speed of sound (4 marks)

(c) The interatomic spacing of a 1D lattice of atoms of mass \( m = 3.7 \times 10^{-27} \) kg is 3.0 Å. The effective force constant representing the chemical bonds between the atoms is \( K = 1.5 \times 10^{-2} \) Nm\(^{-1}\).

(i) Calculate the maximum frequency which can be supported by this lattice (4 marks) and,

(ii) calculate the frequency of waves having wavevector 90% of the maximum wavevector (4 marks).

(End)
N.B. This is a generic PHYS3080 data/formula sheet and may contain some additional information you may not necessarily need for this exam

\[ a^* = \frac{2\pi(bxc)}{a(bxc)} \]

\[ \omega^2 = \frac{2K(M + m)}{Mm} \quad \text{and} \quad \omega^2 = \frac{2K}{m} \quad \text{(optic)} \]

\[ \omega^2 = \frac{2K}{M} \quad \text{(acoustic)} \]

\[ e^x = 1 + x + \frac{x^2}{2} \ldots \]

\[ \int_0^{\Theta_B/T} \left( \frac{x^4 e^x}{(e^x - 1)^2} \right) dx = \int_0^{\omega^*} \left( \frac{x^4 e^x}{(e^x - 1)^2} \right) \, dx = \frac{4\pi^4}{15} \]

\[ \dot{Q} = \frac{dQ}{dt} = \kappa A \frac{dT}{dx} \quad C_v = \frac{1}{2} k_B T \text{ mol}^{-1} \text{ per degree of freedom} \]

\[ \kappa = \frac{1}{3} \frac{V}{C} \quad R = \frac{k_B}{N_A} \]

\[ \varepsilon = E_g + \frac{\hbar^2 k^2}{2m_e} \quad \varepsilon = -\frac{\hbar^2 k^2}{2m_b} \quad E_n = -\frac{m^*_e e^4}{8\hbar^2 n^2 \varepsilon_0^2} \]

\[ n_p n_n = n_i^2 = n_p n_p \]

\[ F = q(vxB) \quad I = nAve \quad \nu = -\frac{e\tau}{m_e} \quad E = \sigma E \quad \sigma = ne\mu \quad \mu = \frac{\nu_d}{E} \]

\[ \varepsilon_0 = 8.854 \times 10^{-12} \text{ Fm}^{-1} \quad N_A = 6.023 \times 10^{26} \text{ (kg.mol)}^{-1} \quad e = 1.6 \times 10^{-19} \text{ C} \]

\[ h = 6.63 \times 10^{-34} \text{ Js} \quad \hbar = 1.05 \times 10^{-34} \text{ Js} \quad \hbar^2 = 1.11 \times 10^{-68} \text{ J}^2 \text{ s}^2 \]

\[ j = j_0 \sin \left[ \frac{2e}{h} \left( V_0 t + \frac{v}{\omega} \sin(\omega t) \right) + \delta_0 \right] \quad V_0 = \frac{nh\omega}{2e} = \frac{nhv}{2e} \]

\[ n_{\text{phonon}} \sim \exp(-\Theta_D/T) \quad \lambda_{\text{phonon}} \sim \exp(+\Theta_D/T) \]

\[ k_F = \left( \frac{3\pi^2 N}{V} \right)^{1/3} \]

\[ \xi_0 = \frac{\hbar v_F}{\pi \Delta(0)} \quad V_0 = \frac{nh\omega}{2e} = n\nu \Phi \]