Midsession test

Duration: 50 minutes
University-approved calculators may be used.

Question 1  (15 marks)

(a) Using whatever units you like, find the approximate density of nuclear matter. Hence approximate the radius of a neutron star with mass \(2M_\odot\) \((M_\odot \approx 2 \times 10^{30} \text{kg})\). Express your answer in humanish units.

(b) Calculate the Coulomb energy density of a uniformly charged sphere with charge \(Q\) and radius \(R\) (Hint: in Gaussian units, the energy between two point charges is \(E = q_1q_2/r\)). Hence estimate the electrostatic contribution to the proton mass (in MeV).

(c) The nuclear magneton is

\[
\mu_N = \frac{e\hbar}{2m_Nc} \quad \text{(Gaussian units)}
\]

\[
= \frac{e\hbar}{2m_N} \quad \text{(SI units)}
\]

Estimate the "Zeeman splitting" of the nuclear spectrum in a 5 Tesla field. What is the order of magnitude of the required frequency resolution, \(\Delta \omega/\omega\), needed to observe this splitting in a spectral line with frequency \(\hbar\omega = 1 \text{ MeV}\)? How does this compare to usual atomic Zeeman splitting, \(\Delta \omega/\omega\), in the same field?

Question 2  (15 marks)

(a) Using the attached picture of single particle energy levels, find the shell model configuration for the ground state of \(^{41}\text{Sc}\) and \(^{40}\text{Ca}\). What are the parities and spins of these nuclei?

(b) Calculate the magnetic moment of these nuclei using the formula

\[
\mu = \begin{cases} 
\mu_N \left[ g_l(j - 1/2) + \frac{1}{2}g_s \right], & j = l + \frac{1}{2} \\
\mu_N \left[ g_l\frac{j(j+3/2)}{j+1} - \frac{1}{2}\frac{j}{j+1}g_s \right], & j = l - \frac{1}{2}
\end{cases}
\]

where \(g_l = 1\), \(g_s = 5.6\) for a proton and \(g_l = 0\), \(g_s = -3.8\) for a neutron.
(c) The first two excited states of $^{41}_{20}$Ca have quantum numbers $J^P = 3/2^-$ and $3/2^+$. Find the shell model configurations for these states (they have energies of 1.943 MeV and 2.010 MeV, respectively).

(d) The spin-orbit partner of the $^{41}_{20}$Ca ground state has energy 2.577 MeV. By calculating the expectation value of the spin-orbit contribution to the Hamiltonian $H_{ls} = a (l \cdot s)$ in the ground state and this excited state, find the value of the spin-orbit constant $a$ for this spin-orbit pair.