THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

FINAL EXAMINATION

PHYS3031 – Advanced Optics and Nuclear Physics, Paper 2

Session 2, 2014

1. Time allowed – 2 hours
2. Total number of questions – 5
3. Total marks available – 100
4. Answer ALL questions. If math presents a difficulty use physical arguments and plain English.
5. **Answer Part A (questions 1, 2) in one booklet and Part B (questions 3, 4, 5) in a separate booklet.**
6. QUESTIONS ARE NOT OF EQUAL VALUE.
   Marks available for each question are shown in the examination paper.
7. University-approved calculators may be used.
8. All answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.
9. This paper may be retained by the candidate.

Useful Formulae

- Radial Schrödinger equation for a central potential, letting $\psi(r,\theta,\phi) = \frac{R_l(r)}{r} Y_{lm}(\theta,\phi)$:

$$\frac{d^2 R_l(r)}{dr^2} + \frac{2m}{\hbar^2}\left(E - V(r) - \frac{\hbar^2 l(l+1)}{2mr^2}\right) R_l(r) = 0\ .$$

- Density of states formula:

$$dn = \frac{4\pi p^2}{(2\pi\hbar)^3}\, dp$$

- $E^2 = m^2c^4 + p^2c^2$

- Wavefunction of K-shell electron ($1s$ electron):

$$\psi(r) = \sqrt{\frac{Z^3}{\pi a_B^3}} \exp(-Zr/a_B), \qquad a_B = \frac{\hbar^2}{m_e e^2}$$

- $m_n = 939.5656\,\mathrm{MeV}$
- $m_p = 938.2723\,\mathrm{MeV}$
- $m_e = 0.5110\,\mathrm{MeV}$

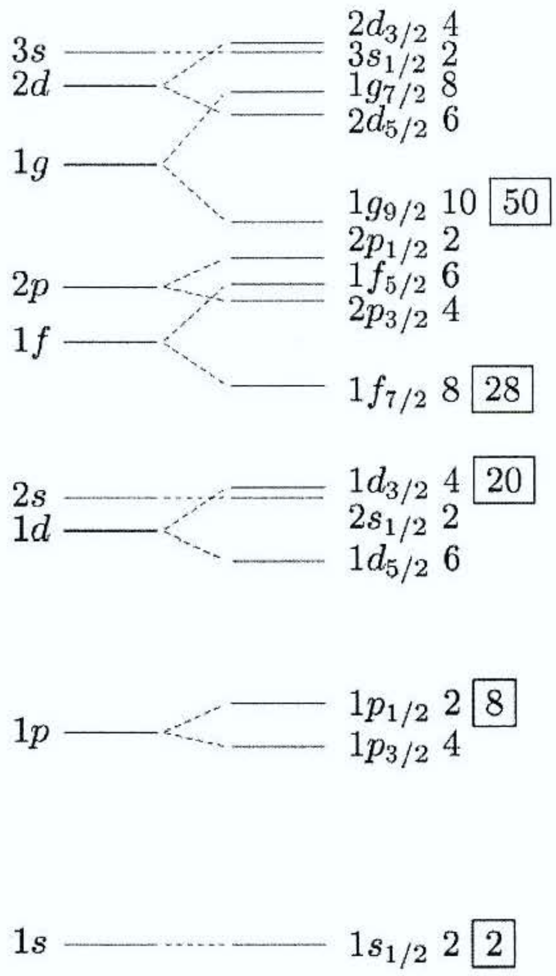

Figure 1: Shell model energy levels.

Part A (answer in a separate booklet)

Nuclear units (fm, MeV, for distances and energies) are adopted: $\hbar c = 197$ MeV $\cdot$ fm. Please be brief, only the most necessary facts and topics should be discussed.

Question 1 (25 marks)

Nuclear structure and nuclear forces

(a) Consider a simple phenomenological estimate for the dependence of the nuclear radius r_n on the number of nucleons A,

$$r_n = 1.2\, A^{1/3}\, \mathrm{fm}\,. \tag{1}$$

Explain briefly the physical reasons prompting this dependence.

(b) Using Eq. (1) find a simple estimate for the typical separation between nucleons in nuclei.

(c) Estimate further the typical kinetic energy and velocity of nucleons in nuclei, as well as the potential energy of a nucleon.

(d) Remember that the radius of the charge distribution in deuterium is

$$r_d \approx 2.1\, A^{1/3}\, \mathrm{fm}\,.$$

Find from this equation an estimate for the pion mass.

(e) Present the formula for the Yukawa potential, which describes the interaction between two heavy particles (having mass M), arising due to exchange of a light scalar particle with mass m, $m \ll M$.
Explain briefly how the Yukawa potential depends on the mass m; explain in particular its behaviour when $m \to 0$, and discuss also the opposite limit of large m (presuming that this "large limit" for m is still restricted by $m < M$ which makes the Yukawa formula applicable).
Outline briefly the role played by the Yukawa-type interaction in nuclear physics.

Question 2 (25 marks)

Semi-phenomenological models

(a) Explain very briefly the physical reasons warranting qualitative applicability of the Fermi-gas model for description of particular nuclear properties. Give examples of those physical quantities that can be estimated within this model.
Briefly outline fundamental shortcomings of this model, illustrating them by some nuclear properties that cannot be evaluated within the model.

(b) Using the Fermi-gas model calculate the kinetic energy of a spherically symmetric nucleus, present explicitly its dependence on the number of nucleons A and protons Z.
Using the Fermi-gas model find the value of Z that provides a minimum for the kinetic energy of a nucleus having a given number A of nucleons.
(Hint: If numerical coefficients (2, 3, π, etc) present a difficulty do not dwell on them. What is important is to explain the dependence on essential parameters A, Z, as well as to explain where the scale of the kinetic energy is coming from.)

(c) Outline very briefly physical reasons favouring validity of the Weizsäcker liquid drop model (semi-empirical mass formula).

(d) Present the Weizsäcker formula for the binding energy explicitly, explaining what is called (1) the mass term, (2) surface term, (3) Coulomb term, (4) asymmetry term, (5) pairing term. Give physical reasons for their signs. Provide a typical scale for the coefficients in each term of the Weizsäcker formula. (Note that only a typical scale for these coefficients is required, not the exact values.)

(e) Using the Weizsäcker formula find an expression for Z which minimises the energy of nuclei with a given value for A.
(Hint: disregard for simplicity the last, pairing term in the Weizsäcker formula.)

Part B (answer in a separate booklet)

Question 3 (10 marks)

Neutron star

(a) Starting from the well known nuclear radius formula $r = 1.2A^{1/3}$ fm, find the approximate density of nuclear matter. Hence approximate the radius of a neutron star with mass $2M_{\odot}$ ($M_{\odot} \approx 2 \times 10^{30}$ kg). Express your answer in units an engineer could understand.

(b) In the collapse of an ordinary star to a neutron star, there is a point where it becomes energetically favourable for the protons and electrons to fuse into neutrons:

$$p + e^{-} \rightarrow n + \nu_e$$

By considering the protons, neutrons, and electrons of the neutron star in the Fermi gas approximation, estimate the relative abundance of protons to neutrons.

[Note: neutron stars are actually inhomogenous, and generally much more complex than this simple model.]

Question 4 (15 marks)

Shell model

The ground state of the nucleus ${}^{39}_{19}$K has quantum numbers $J^{\pi} = 3/2^{+}$.

(a) Using the attached picture of single particle energy levels (Fig. 1), find the shell model configuration for the ground state.

(b) The first excitation of this nucleus has quantum numbers $1/2^{+}$. Find the shell model configuration for this state.

(c) The energy of the excitation which is the spin-orbit partner to the ground state is 4.51 MeV. Quantum numbers of this state are $5/2^{+}$. What is the shell model configuration of this state?

(d) Using the experimental value of the spin-orbit splitting $\Delta E = 4.51$ MeV and averaging the spin-orbital contribution to the Hamiltonian $H_{ls} = a\,(\boldsymbol{l} \cdot \boldsymbol{s})$ over appropriate states, find the value of the spin-orbit constant a. Compare this value with the usual estimate $a \approx -\frac{20}{A^{2/3}}$ MeV.

Question 5 (25 marks)

Beta decay

The β^{-} decay of ${}^{116}_{49}$In can be studied by neutron bombardment of ^{115}In.

(a) Use the attached shell model diagram (Fig. 1) to find the angular momentum and parity of the ground state of ${}^{115}_{49}$In.

(b) After neutron capture and subsequent relaxation to the ground state, the angular momentum and parity of the odd-odd nucleus ${}^{116}_{49}$In is 1^{+}. Assuming that the proton structure is the same as it was in ^{115}In (from part a), in which shell do you expect the valence neutron to be? Justify your answer.

(c) ^{116}In is unstable, and β-decays to $^{116}_{50}$Sn with half-life $\tau_{1/2} = 14.1$ sec,

$$^{116}\text{In} \rightarrow {}^{116}\text{Sn} + e^{-} + \bar{\nu}\,.$$

The binding energy of $^{116}_{49}$In is 986.188 MeV, and the binding energy of $^{116}_{50}$Sn is 988.680 MeV. What is the energy released, Q, in the β-decay? In a few words, explain why this β-decay has such a high energy.

(d) Starting from the Fermi golden rule

$$w = (2\pi)^4 \left|V_{if}\right|^2 \rho_f \, \delta(E_i - E_f)\, \delta(\vec{k}_i - \vec{k}_f)$$

where ρ_f is the final density of states

$$\rho_f = \frac{d^3k_A}{(2\pi)^3}\frac{d^3k_e}{(2\pi)^3}\frac{d^3k_\nu}{(2\pi)^3} \times \text{polarization, etc,}$$

and neglecting all constants, derive the form of the spectrum of emitted electrons

$$\frac{dw}{d\epsilon} \sim \epsilon^2 (Q-\epsilon)^2$$

where ϵ is the energy of the emitted electron. Explain the physical reasoning behind each step and justify any assumptions used.

(e) Integrating over all emitted electron energies from $\epsilon = 0$ to Q, one obtains the total probability of β-decay $w \sim Q^5$. Using this and the level structure below, explain why the β-decay proceeds to the ground state of Sn. What type of β-decay (Fermi or Gamow-Teller or both) is this?

Table 1: Level structure of $^{116}_{50}$Sn.

E_{level} (keV)	$J\pi$	$\tau_{1/2}$
0	0+	STABLE
1294	2+	0.374 ps
1757	0+	44 ps
2027	0+	160 ps
2112	2+	1.89 ps
2225	2+	2.4 ps
2266	3−	0.34 ps
2366	5−	348 ns
2391	4+	0.28 ps
2529	4+	< 100 ps
	...	

(f) The first excited state of $^{116}_{49}$In is a 5^+ level at 127 keV. It also β-decays to $^{116}_{50}$Sn. Predict which level of Sn is the final state. What type of β-decay is this? Assuming that all constants and matrix elements are approximately the same as in the ground state decay, which has lifetime 14.1 sec, estimate the lifetime of the excited state.
[Note: The measured lifetime of the excited state is 54.3 min.]

— End of Exam —