THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

FINAL EXAMINATION: JUNE 2014

PHYS3011 & PHYS3230
Electrodynamics

Time allowed – 2 hours
Total number of questions – 5
Answer ALL 5 questions

Answer Part 1 (Questions 1 and 2) in one answer book

and Part 2 (Questions 3, 4, 5) in a separate answer book

All questions are of equal value

Candidates must supply their own, university-approved calculator.

All answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching, or graphical work.

Candidates may keep this question paper.
Part 1

Question 1 (20 marks)

Consider an electromagnetic wave propagating in vacuum (i.e., no sources).

(a) Explain the meaning of these equations

\[ \frac{\partial u}{\partial t} + \nabla \cdot \tilde{S} = 0, \]  
\[ u = \frac{\varepsilon_0}{2} |\tilde{E}|^2 + \frac{1}{2\mu_0} |\tilde{B}|^2, \]  
\[ \tilde{S} = \frac{1}{\mu_0} \tilde{E} \times \tilde{B} \]

for the system. Be sure to explain the meaning of each quantity appearing in the equations.

(b) Show that equation (1) can be derived from Maxwell’s equations.

(c) Suppose the electromagnetic wave under consideration is a plane wave with wave vector \( \tilde{k} \). Write down the corresponding \( \tilde{E} \) and \( \tilde{B} \) fields, and evaluate \( \tilde{S} \) explicitly.

Question 2 (20 marks)

(a) Consider the linear transformation

\[ x'^\mu = \Lambda^\mu_\nu x^\nu. \]

What are the conditions on the matrix \( \Lambda \) for this to be a Lorentz transformation? Derive an expression for the inverse transform \( \Lambda^{-1} \) in terms of \( \Lambda^T \), the transpose of \( \Lambda \).

(b) Show that the continuity equation,

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot \tilde{J} = 0, \]

can be derived from the field equation \( \partial_\mu F^{\mu\nu} = \mu_0 J^\nu \).

(c) Suppose an inertial reference frame \( S' \) is moving away from a frame \( S \) with velocity \( v \) in the positive \( x_1 \) direction. If the observer in \( S \) measures a static charge distribution

\[ \rho(\vec{x}) = Q \exp \left( -\frac{x_1x_i}{a} \right), \]

where \( i = 1, 2, 3 \), find the charge and current distributions as measured by the observer in \( S' \). Discuss the nonrelativistic limit of your result.

(Continued overleaf)
Part 2

Remember to use a new answer book for questions in Part 2.

Question 3 (20 marks)

The wave equation for a plane wave moving in the positive x-direction in a conducting medium is given in the formula sheet. The solution is also given, and the equation for $k$ in a good conductor.

(a) By substituting the given solution in the wave equation, find the general equation for $k$, and show it reduces to the given form in a good conductor.

(b) If $k = \alpha + i \beta$, find the values of $\alpha$ and $\beta$ in terms of $\mu_0$, $\epsilon_0$, $\sigma$ and $\omega$. \[ \text{NB } \sqrt{i} = \frac{(1 + i)}{\sqrt{2}} \]

(c) Find the limiting values of $\alpha$ and $\beta$ for a very good conductor and for a very poor conductor.

(d) Find the skin depth in the conducting medium.

(e) Find the refractive index of the conducting medium.

Question 4 (20 marks)

(a) Explain the terms “retarded potential” and “Hertzian dipole”.

The $E$ field from a short oscillating electric dipole, $p = p_0 \cos \omega t$, at the origin and aligned along the $z$ axis, is given by

$$E(r, \theta, t) = -\frac{\mu_0 p_0 \omega^3 \sin \theta}{4\pi r} \cos[\omega(t - r/c)] \hat{\theta},$$

where $r$ is large compared to the size of the dipole, and to the wavelength of the radiation.

(b) Using the plane-wave approximation, valid for large $r$, write down the corresponding expression for $B$ (both magnitude and direction). (Note that the direction of propagation, $\hat{k}$, is along $\hat{r}$.)

(c) Hence, (i) find the Poynting vector for this radiation, and (ii) the total power radiated in all directions.

(Continued overleaf)
Question 5 (20 marks)

(a) Starting from the boundary conditions for \( \mathbf{E} \), \( \mathbf{D} \), \( \mathbf{B} \) and \( \mathbf{H} \) at the boundary of two dielectrics, and remembering that \( \mathbf{E} = \nu \mathbf{B} \) in a dielectric, show that the ratio of the reflected power to the incident power for a wave at normal incidence from a vacuum onto a medium of refractive index \( n \) and permeability \( \mu = \mu_0 \) is

\[
R = \left( \frac{n - 1}{n + 1} \right)^2,
\]

as given on the formula sheet.

(b) An observer, wearing polarising glasses to completely absorb horizontally polarised light, is looking at light reflected from the surface of the sea at an angle of 45°. Calculate the ratio of light intensity he sees with and without these glasses. (NB this is not the ratio to the intensity of the incident light.) The refractive index of water is 1.33.
Capacitance of isolated sphere is \( C = \frac{4\pi \varepsilon_0}{9} \).

Capacitance of parallel-plate capacitor is \( C = \frac{\varepsilon_0}{d} \).

Stored energy \( U = \frac{1}{2} CV^2 \).

Total charge \( Q = C V \).

\( \nabla \cdot \mathbf{D} = \rho \)  
Capacitive Poisson equation:  
\[
\frac{\nabla \cdot \mathbf{D}}{\rho} = \frac{Q_0}{\varepsilon_0}
\]

For a spherical cavity:  
\[ \frac{\partial \phi}{\partial r} + \frac{\partial \phi}{\partial r} = 0 \]

For a disc-shaped cavity:  
\[ \frac{\partial \phi}{\partial r} + \frac{\partial \phi}{\partial r} = \frac{Q_0}{\varepsilon_0} \]

Charge density: \( \rho \)  
\( \nabla \cdot \mathbf{D} = \rho \)

Duality: \( \phi = \nabla \cdot \mathbf{D} \)

Coulomb's law:  
\[ \mathbf{F}_{\mathbf{E}} = \frac{q_1 q_2}{4\pi \varepsilon_0 r^2} \]

Venn diagram:  
\[ A \cup B \]

Current density: \( J \)  
\( \nabla \times \mathbf{E} = \mu_0 J \)

Divergence theorem:  
\[ \int_{V} \nabla \cdot \mathbf{D} \, dV = \int_{S} \mathbf{D} \cdot d\mathbf{A} \]

Volume of sphere:  
\[ V = \frac{4}{3} \pi r^3 \]

Volume of cube:  
\[ V = a^3 \]

Usual Formule:  
[PHYS3011/PHYS3230]
\[
\begin{align*}
\theta &= \frac{\cos \left( \frac{1}{2} \phi + \frac{1}{2} \phi \right)}{\cos \frac{1}{2} \phi} = \tan^2 \frac{\theta}{2} \\
\text{Refraction:} & \quad \text{incident angle} \quad \text{in vacuum} \quad \text{折射率} \quad \text{medium} \\
\text{Reflection:} & \quad \theta = \theta \\
\text{Reflection and refraction at interface between two dielectrics} \\
\text{Maxwell's equation of a medium:} \\
\n\begin{align*}
\mathbf{B} \times \mathbf{\Delta} &= 0 & \mathbf{\Delta} &= \mathbf{\Delta} \\
\mathbf{B} &= \mathbf{\Delta} \times \mathbf{\Delta} & d &= \mathbf{\Delta} \\
\text{Maxwell's equations in vacuum} \\
\mathbf{c} \times (\mathbf{B} + \mathbf{\Phi}) &= \mathbf{E} \\
\mathbf{E} \times (\mathbf{c} + \mathbf{\Phi}) &= \mathbf{\Delta} \\
\text{Lorentz force law:} \\
\n\begin{align*}
\frac{\partial \mathbf{E}}{\partial t} + \mathbf{c} \times \mathbf{\Phi} &= \mathbf{\Delta} \times \mathbf{\Delta} & 0 &= \mathbf{\Delta} \\
\frac{\partial \mathbf{\Phi}}{\partial t} &= \mathbf{\Delta} \times \mathbf{\Delta} & \mathbf{\Phi} &= \mathbf{\Delta} \\
\text{In a vacuum:} \\
\text{Maxwell's equations} \\
\n\begin{align*}
\mathbf{B} \cdot \mathbf{c} &= \mathbf{E} \cdot \mathbf{c} \\
\text{Boundary conditions:} \\
\mathbf{c} \cdot \mathbf{c} &= \mathbf{c} \cdot \mathbf{c} \\
\mathbf{\Phi} &= \mathbf{\Phi} \\
\text{Self Inductance of a solenoid:} \\
\mathbf{\Phi} &= \mathbf{\Phi} \\
\text{Mutual Inductance:} \\
\mathbf{\Phi} &= \mathbf{\Phi} \\
\text{Inductance} \\
\mathbf{H} &= \mathbf{H} \quad \text{and} \\
\mathbf{B} &= \mathbf{B} \quad \text{boundary} \\
\frac{\partial}{\partial t} (\mathbf{\Phi}) &= \mathbf{\Phi} \\
\text{Amper's law becomes:} \\
0 &= \mathbf{B} \cdot \mathbf{\Delta} + \mathbf{H} \cdot \mathbf{\Delta} \\
\mathbf{B} &= \mathbf{B} \\
\text{Maxwell's equation} \\
\mathbf{B} \cdot \mathbf{\Delta} &= \mathbf{E} \cdot \mathbf{\Delta} \\
\text{Ohm's law:} \\
\mathbf{J} &= \mathbf{\Phi} \\
\text{Joint heating:} \text{ power dissipated} \\
\mathbf{J} &= \mathbf{J} \\
\text{Kirchhoff's laws:} \\
0 &= \mathbf{\Phi} \\
\text{Ohm's law:} \\
\mathbf{J} &= \mathbf{J} \\
\text{DC Circuits} \\
\end{align*}
\end{align*}
\end{align*}
\]
In both cases, A is to B, and both are \( \mathbf{E} \) to \( \mathbf{E} \). and \( \mathbf{B} = \mathbf{B} \).

\[ \phi, \rho = \rho \frac{\sin \theta}{\cos \theta} = \rho \frac{\sin \theta}{\cos \theta} \]

**Wave equation**:

\[ \frac{\partial^2 \psi}{\partial t^2} - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial x^2} = 0 \]

**Transmission lines**:

\[ \frac{d^2 \psi}{d x^2} - \frac{\psi}{c^2} = \frac{\psi}{c^2} \]

**Reflection from metallic normal incidence**:

\[ \frac{z}{z} = \frac{1}{z} = \frac{z}{z} \]

**Skin depth**:

\[ \delta = \frac{1}{\sqrt{2\pi c \rho}} \]

**Wave equation**:

\[ \frac{\partial^2 \psi}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial x^2} = 0 \]

**Transmission lines**:

\[ \frac{d^2 \psi}{d x^2} - \frac{\psi}{c^2} = \frac{\psi}{c^2} \]

For any incident, \( \frac{\theta}{\theta} = a \cdot \frac{\psi}{\psi} + \frac{\psi}{\psi} = \psi \cdot \frac{\psi}{\psi} \)

**Characteristics line**:

\[ \frac{d^2 \psi}{d \zeta^2} - \frac{\psi}{\zeta} = 0 \]

**Transmission lines**:

\[ \frac{d^2 \psi}{d \zeta^2} - \frac{\psi}{\zeta} = 0 \]

**Wave equation**:

\[ \frac{\partial^2 \psi}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial x^2} = 0 \]

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**Transmission lines**:

\[ \frac{d^2 \psi}{d x^2} - \frac{\psi}{c^2} = \frac{\psi}{c^2} \]
\[
\begin{pmatrix}
\frac{\partial \theta}{\partial \phi} & \frac{\partial \phi}{\partial \theta} & 1 - \Delta = n \theta' \phi \equiv z \\
0 & 0 & -c^2 g - c \rho \Delta \\
0 & -c \rho & 0 \\
0 & 0 & -c^2 g - c \rho \\
0 & 0 & 0
\end{pmatrix}
= n V_\theta - \lambda V_\phi = \eta_{\theta \phi}
\]

Heuristic electrodynamics

\[
(\frac{d^2 \phi}{d t^2}) = n \eta_{\mu\nu} \equiv \quad d
\]

(\phi \cdot \psi) \psi = \frac{e p}{\sigma \bar{\epsilon}} \equiv \eta^\mu

4-velocity and 4-momentum

\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix} = \eta_{\mu
u}
\]

\[
\operatorname{exp}_{\mu} \eta_{\mu} = \epsilon_{\mu}
\]

Invariant integral and the Minkowski metric

\[
\begin{pmatrix}
\epsilon^x & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix} = \epsilon^x
\]

The \( \xi \) direction in the inertial frame \( S \).

Lorentz transformation for an inertial frame \( S \) moving with speed \( \alpha \) in