Time allowed – 55 minutes (start 9:05 end 10:00)
Total number of questions – 3
Total number of marks – 20
Answer ALL questions
The questions are of equal value – each question is worth 10 marks
This examination paper has 3 pages.

This paper may be retained by the candidate
Portable battery-powered electronic calculators (without alphabetic keyboards) may be used.
All answers must be in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.
The following information is supplied as an aid to memory.

Boltzmann’s constant $k_B = 1.38 \times 10^{-23} \text{ J/K}$
Avogadro’s number $N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$
Real gas constant $R = 8.314 \text{ J/K.mol}$

Specific heat of liquid H$_2$O = 4.18 J/gK
Latent heat of the liquid-solid transition for H$_2$O = 333 J/g
Adiabatic constant for N$_2$ $\gamma = 1.4$
Molar mass of air = 29 g/mol

Ideal gas equation $PV = nRT = Nk_BT$

Maxwell’s velocity distribution

$$D(v) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} 4\pi v^2 \exp\left(-\frac{mv^2}{2k_B T}\right)$$
Question 1 (4 Marks)

Two identical bubbles of gas form at the bottom of a lake, then rise to the surface. Because the pressure is much lower at the surface than at the bottom, both bubbles expand as they rise. Bubble A rises very quickly, whereas bubble B rises slowly because it is impeded by a tangle of seaweed.

(a) Explain which bubble undergoes an adiabatic expansion, which undergoes a more isothermal expansion, and why.

(b) Which of the two bubbles is larger by the time it reaches the surface? Briefly explain your answer.

Assume that all of the water in the lake is at a constant temperature from bottom to top. Note also, that the pressure of a bubble of gas increases linearly with depth when immersed in a liquid. (Hint: In tackling this problem, it might help to think about the first law).

Question 2 (8 Marks)

Maxwell’s velocity distribution gives the probability density $D(v)$ as a function of particle velocity $v$ for gases.

(a) A vessel contains 1 L of Helium gas (molar mass = 4g) at 25°C. Using the equipartition principle (i.e., $\langle \frac{1}{2}mv^2 \rangle = 3/2k_BT$), calculate the root mean square velocity $v_{\text{rms}} = \langle v^2 \rangle^{1/2}$ for the He molecules in the gas.

(b) What is the probability of a randomly chosen molecule having a particular velocity between $v$ and $v + dv$? Briefly discuss with one or two paragraphs and a sketched graph of the probability density $D(v)$ versus velocity $v$. Indicate on your graph how the probability density varies with $v$ in the limit of $v \to 0$ and $v \to \infty$, and the locations of the most-probable velocity $v_{m.p.}$ and the root-mean-square velocity $v_{\text{rms}}$.

(c) Obtain an expression for the most probable velocity $v_{m.p.}$, and calculate $v_{m.p.}$ for the gas in (a). Show that $v_{\text{rms}}$ is 22.5% greater than $v_{m.p.}$?

Question 3 (8 Marks)

The first law of thermodynamics is usually written as $dU = Q + W$.

(a) Define each of the terms $dU$, $W$ and $Q$ and briefly explain the meaning and significance of the first law.

(b) In a few sentences, clearly explain the differences between temperature, heat and internal energy.

(c) Briefly discuss how compressive work in a gas is given by the expression $W = -\int PdV$.

(d) 1 L of air at 1 atm and 298 K is expanded isothermally to a pressure of 0.5 atm. Calculate the work done in this process and comment on the meaning of the sign of the result.